PUBLIC SCHOOL ARITHMETIC

BAKER AND BOURNE

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CAMBRIDGE MATHEMATICAL SERIES

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*** *This Book may be had either with or without Answers*

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PREFACE

In writing this text-book on Arithmetic the authors have aimed at employing and illustrating the methods which have been proved by experience to be the most successful under modern conditions.

Tables have been reduced to as small a compass as practical usefulness allows. The saving of a pupil's time by the universal substitution of a decimal system of tables is a reform not yet realised; but in the various exercises of the book the claims of the metric system have been duly regarded.

Explanations of the steps of his working are repeatedly demanded from the pupil; for teachers know how difficult and how important it is to prevent learners from putting down unexplained strings of figures.

Oral instruction is very freely provided.

The checking of results is encouraged.

The help of Algebra is used whenever it tends to simplification; and an introduction to Logarithms is given.

Perhaps the chief merit claimed lies in the choice of examples. They have been carefully graded; and in the selection of them it has been borne in mind that at certain stages a pupil learns more, in the way of method, from a number of questions which come out easily, than from some of those long, laborious examples which have their use in other directions.

The authors have to thank the Controller of H.M. Stationery Office, the Cambridge Local Examination Syndicate, and the Oxford and Cambridge School Examination Board for permission to use their examination papers.

NOTE

In the latest edition a new table of Standards of Exchange has been included, and a few minor alterations made in the text and in certain examples.

CONTENTS

TABLES

THE abbreviations for the names in the tables are attached to them in brackets.

In the table of money the abbreviations £. *s.* *d.* for the pound, shilling, penny are the initials of *Libra*, *solidus*, *denarius*.

As these are adopted from Latin it is natural to use not *f.* but *q.* (quadrans) for the farthing, if any abbreviation is required; but in most cases the farthings are written as fractions of a penny: $1\ q. = \frac{1}{4}d.$, $2\ q. = 1$ halfpenny $= \frac{1}{2}d.$, $3\ q. = \frac{3}{4}d.$

MONEY.

4 farthings (*q.*)	= 1 penny (*d.*)
12 pence	= 1 shilling (*s.*)
20 shillings	= 1 pound or sovereign (£)

Other coins are the half-sovereign (10*s.*), crown (5*s*), half-crown (2*s.* 6*d.*), florin (2*s.*), sixpence, threepence and halfpenny.

Gold coins, £1 and 10*s.*, have been replaced by notes representing these values.

The guinea (21*s.*), though recognised as a denomination, is not issued as a coin.

TIME.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day
7 days	= 1 week (wk.)
365 days	= 1 year (if it be not a Leap-year)

Leap-year consists of 366 days, the extra day being the 29th of February.

LENGTH OR LINEAR MEASURE.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
*(22 yards	= 1 chain)
220 yards	= 1 furlong (fur.)
8 furlongs	= 1 mile (ml.)
3 miles	= 1 league

Thus a mile = 80 chains = 1760 yards.

6 feet	= 1 fathom (used for measuring depths of water)
4 inches	= 1 hand (used for the height of a horse)

A quarter-chain, known as a pole, is a measure which should not be generally used. One pole, rod, or perch = $5\frac{1}{2}$ yards.

* A chain (used by surveyors) is 22 yards in length, and contains 100 (equal) links.

AREA OR SQUARE MEASURE.

144 square inches = 1 square foot (sq. ft.), for $12^2 = 144$
9 square feet = 1 square yard (sq. yd.), for $3^2 = 9$
10 square chains = 4840 square yards = 1 acre (ac.)
640 acres = 1 square mile

It is also to be noted that 40 square poles (sq. po.) = 1 rood (ro.)
and 4 roods = 1 acre

VOLUME OR CUBIC MEASURE.

1728 cubic inches (c. in.) = 1 cubic foot (c. ft.), for $12^3 = 1728$
27 cubic feet = 1 cubic yard (c. yd.)

CAPACITY.

(For measuring liquids, corn, etc.)

4 gills = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gall.)
2 gallons = 1 peck (pk.)
8 gallons = 4 pecks = 1 bushel (bus.)
8 bushels = 1 quarter

This quarter must not be confused with the quarter in the table of weight.

A gallon of water weighs approximately 10 lbs.; in volume a gallon = roughly $277\frac{1}{4}$ c. in.

APOTHECARIES' MEASURE.

60 minims = 1 fluid drachm (fl. dr.)
8 drachms = 1 fluid ounce (fl. oz.)
20 ounces = 1 pint.

WEIGHT.

AVOIRDUPOIS.

[16 drams (dr.) = 1 ounce (oz.)] (*uncommon*)
16 ounces = 1 pound (lb.)
14 pounds = 1 stone (st.)
28 pounds = 1 quarter (qr.)
112 pounds = 4 quarters = 1 hundredweight (cwt.)
20 hundredweights = 1 ton

NOTE.—1 lb. Avoirdupois = 7000 grains, the grain being the same as that occurring in Troy Weight.

TROY WEIGHT.

(Used for gold, silver, platinum, and precious stones.)

24 grains (gr.) = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz. Troy)
12 ounces = 1 pound (lb. Troy)

Still used by jewellers (1908), though nominally, except the ounce, obsolete by Act of Parliament.

Standard gold (*i.e.* 22 carat gold) contains 22 parts pure gold out of 24; 2 parts being alloy.

APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple (scr.)
3 scruples	= 1 dram (dr.)
8 drams	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

Still used by druggists (1908), though nominally replaced by the Ounce Avoirdupois (containing 437½ grains) and the pound Avoirdupois.

Avoirdupois, Troy, and Apothecaries' Weights are connected by having the grain the same for all.

PAPER MEASURE.

24 sheets	= 1 quire
20 quires	= 1 ream

ANGULAR MEASURE.

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
90 degrees	= 1 right angle

METRIC TABLES.

The convenience of the Metric System is that the different denominations in a table are derived from a fundamental unit by multiplying or dividing successively by 10.

Thus in the Linear Measure we start with a **Metre**, and multiplying by 10, 100, 1000 we get the Decametre, Hectometre, Kilometre; and dividing by 10, 100, 1000 we get the decimetre, centimetre, millimetre.

WEIGHT.

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 Decagram (Dg.)
10 Decagrams	= 1 Hectogram (Hg.)
10 Hectograms	= 1 Kilogram (Kg.)
10 Kilograms	= 1 Myriagram (Mg.)
10 Myriagrams	= 1 Quintal.

CAPACITY.

10 centilitres (cl.)	= 1 decilitre (dl.)
10 decilitres	= 1 litre (l.)
10 litres	= 1 Decalitre (Dl.)
10 Decalitres	= 1 Hectolitre (Hl.)
10 Hectolitres	= 1 Kilolitre (Kl.)

LENGTH.

10 millimetres (mm.)	=1 centimetre (cm.)
10 centimetres	=1 decimetre (dm.)
10 decimetres	=1 metre (m.)
10 metres	=1 Decametre (Dm.)
10 Decametres	=1 Hectometre (Hm.)
10 Hectometres	=1 Kilometre

AREA.

100 square millimetres	=1 square centimetre (sq. cm.)
100 square centimetres	=1 square decimetre (sq. dm.)
100 square decimetres	=1 square metre (sq. m.)
100 square metres	=1 square Decametre =1 are

The **are** is used in measurement of land, according to the following table:

100 centiares	=1 are
100 ares	=1 Hectare

VOLUME.

1000 cubic millimetres (c.mm.)	=1 cubic centimetre (c.cm.)
1000 cubic centimetres	=1 cubic decimetre (c.dm.)
1000 cubic decimetres	=1 cubic metre

One gram = the weight of one cubic centimetre of distilled water at 4° Centigrade).

One litre = the volume of one cubic decimetre.

DECIMAL COINAGE.

Many countries use a decimal coinage; *e.g.* in the United States 100 cents = 1 dollar ($); in both France and Switzerland 100 centimes = 1 franc (f.).

The use of a decimal point simplifies the writing of a sum of money. Thus f.31·25 means 31 francs 25 centimes.

STANDARDS OF EXCHANGE

The rate of exchange between two countries is the value of a coin of the one country expressed in the coinage of the other.

This value may change from day to day; but there is a certain settled scale of values which is called the *Par of Exchange.* This is printed in our newspapers in a convenient form, which shows the value of £1 in terms of the standard coinage or units of the various foreign countries or their cities. For instance the par of exchange between New York and London is quoted as £1 = 4·86⅔ $ which (in spite of the unhappy mixture of decimals and vulgar fractions) evidently means 4 dollars 86⅔ cents.

The following table gives the par value without the daily variations:

Place	Unit	Subdivisions	Method of Quoting	Par of Exchange
Paris . .	Franc	100 centimes	Francs to £	124·21
Zurich .	Franc	100 centimes	Francs to £	25·2215
Milan . .	Lira	100 centesimi	Lire to £	92·45
Brussels .	Belga	5 francs of 100 centimes	Belgas to £	35·00
New York .	Dollar	100 cents	Dollars to £	4·86⅔
Athens .	Drachma	100 lepta	Drachmae to £	375
Madrid .	Peseta	100 centimos	Pes. to £	25·2215
Lisbon .	Escudo	100 centavos	Escu. to £	
Amsterdam .	Florin	100 cents	Fl. to £	12·107
Berlin . .	Reichsmark	100 pfennige	RM. to £	20·45
Vienna .	Schilling	100 groschen	Sch. to £	54·58½
Budapest .	Pengo	100 garas	Pen. to £	27·82
Prague .	Krone	100 heller	Kr. to £	164·25
Warsaw .	Zioty	100 grosz	Zi. to £	43·38
Belgrade .	Dinar	100 paras	Din. to £	25·2215
Oslo . .	Krone	100 öre	Kr. to £	18·150
Stockholm .	Krone	,,	,, ,,	18·150
Copenhagen	Krone	,,	,, ,,	18·150

ARITHMETIC.

I. FIRST FOUR RULES.

1. THOUGH we start with the supposition that the learner has already been taught elementary numeration, notation, simple addition, subtraction, multiplication and division, it is convenient to collect here some **definitions** which have probably been learnt, and to add some notes on **methods of working** in the above-mentioned subjects.

The result of adding quantities or numbers is called their **sum.**

The **symbol** or **sign** used is $+$, called *plus.*

The result of adding five to seven is equal to twelve (their **sum**).

This statement, shortened by the use of signs, would be written

$$7+5=12.$$

The result of subtracting one quantity or number from another is called their **difference**; or it may be called the **remainder.**

If five be taken from seven the result is two.

The sign used is $-$, called *minus.*

Translated into signs, the statement is

$$7-5=2.$$

The quantity to be subtracted is the **subtrahend**; the quantity from which it is to be subtracted, *i.e.* the one to be diminished, is the **minuend.**

NOTE.—*The subtrahend (the quantity to be subtracted) is put* **after** *the minus.*

To some persons it seems to be easier to perform **Complementary Addition** than **Subtraction**; *i.e.* they would rather answer the question, "What must be added to 6 to make it 10?" than the question, "What remains when 6 is taken from 10?"

This will be referred to again under the head of Compound Subtraction, where the method of "giving change in shops" may be found a useful development of this.

Signs =, >, <, ~.

$a=b$ means a is equal to b.
$a>b$,, a is greater than b.
$a<b$,, a is less than b.

A learner sometimes finds a difficulty in remembering which is which in the case of $>$ and $<$. The following may help him. Equality is indicated by two lines equally far apart at their ends. When they are closed at one end to indicate inequality, the wedge-shaped symbol thus formed has its smaller end towards the smaller quantity.

$a \sim b$ means the difference between a and b; *i.e.* the sign $\sim$ demands that the smaller shall be taken from the larger.

"Therefore" is denoted by $\therefore$.

Multiplication is a shortened form of addition.

When 15 has to be multiplied by 7, we might write down seven fifteens in a column, and add them up. The sum would be 105.

Expressed as a multiplication, this would be $15 \times 7 = 105$.

Here the 7 is the **multiplier**, 15 the **multiplicand**, and the result of the multiplication, viz. the 105, is called the **product.**

$$15 \times 7 \times 4 = 420.$$

Here it is directed by the signs that 15 should be multiplied by 7 and the result multiplied by 4. The 420 is called the **continued product.**

The parts of a number which make it up by *multiplication* are called its **factors.**

Thus 5 and 3 are factors of 15; 7 and 11 are factors of 77.

Abstract and Concrete Numbers.

2. A number is **concrete** when it is taken with some unit, **abstract** when it is considered apart from any unit. For instance, 5 pence, 5 yards, 5 sheep would be concrete quantities, but the number 5 taken apart from pence or yards or other objects would be called **abstract**.

A **concrete** number cannot be used as a multiplier.

"5 shillings multiplied by 3" would mean that the 5 shillings had to be written down 3 times, and added up.

"5 shillings multiplied by 3 shillings" would have no meaning.

Division is of two sorts.

(1) When we say 'Divide £15 by 5,' we mean that the £15 has to be separated into 5 equal parts, and the quotient £3 will tell us what is the magnitude of each part.

(2) When we say, 'Divide £15 by £5,' we mean that it has to be discovered how many times the £5 is contained in £15. The quotient tells us the number of times, viz. 3. In this case the quotient is an abstract number.

If 101 be divided by 28, the **quotient** is 3, and there is a **remainder** 17.

$$\begin{array}{r} 3 \\ 28\,\overline{)101} \\ \underline{84} \\ 17 \end{array}$$

101	is called	the	**dividend**,
28	,,	,,	**divisor**,
3	,,	,,	**quotient**,
17	,,	,,	**remainder**.

In every division-question we have

divisor × quotient + remainder = dividend.

An **expression** is a collection of symbols, numbers and signs involving arithmetical operations.

Term. The different parts of an expression connected by the signs plus (+) and minus (−) are called **terms**.

When a number of terms are included within **brackets** ( ) it is understood that the terms within the brackets should be considered *as a whole*.

Thus $8+(9+4)$ means that we first add 4 to 9, and then add the result to 8.

Also, $8 \times (9+4) = 8 \times 13 = 104$.

Instead of $8 \times (9+4)$ we sometimes find $8(9+4)$, the sign of multiplication being understood.

If from 30 we have to subtract 7 and then 5 and then 8, the result will be exactly the same as if we added the 7 and 5 and 8 together and took their sum from 30. In fact, if 30 has to be diminished by 7 and by 5 and by 8, it matters not whether this is done in three steps or in one.

Thus $$30-7-5-8=30-(7+5+8).$$

Exactly the same thing occurs in addition.

$$20+3+4+9=20+(3+4+9).$$

These considerations suggest how an expression may sometimes be simplified by collecting the terms which have + before them, and collecting into another group those with − before them.

Thus

$$\begin{aligned}20+30-7-5-8+3+4+9 &= 20+30+3+4+9-(7+5+8)\\ &= 66-20=46.\end{aligned}$$

Order of Operation indicated by Signs.

3. In every group of numbers connected by signs it must be understood that the multiplications and divisions have to be performed before the additions and subtractions, and moreover that the multiplications and divisions are to be performed in the order in which they occur from left to right, as in (*b*).

Thus (*a*) $$5 \times 3 + 7 \times 2 - 10 \div 2 + 12 \div 4$$

takes the form of $$15+14-5+3$$

by reason of the multiplications and divisions having been performed.

The result is then obtained by performing the additions and subtraction.

Also, (*b*) $72 \div 12 \times 3 = 6 \times 3$ not $72 \div 36$.

(*a*) could only be made to mean something different by the use of brackets.

Grouped in the following way

$$5 \times (3+7) \times 2 - 10 \div (2 + 12 \div 4),$$

it would mean $$5 \times 10 \times 2 - 10 \div (2+3),$$

i.e. $100 - 10 \div 5$, *i.e.* $100-2$, *i.e.* 98.

Multiplication.

4. The Commutative Law. The product of two numbers is not affected by interchanging the multiplier and multiplicand.

$$\begin{array}{ccccccc} * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \end{array}$$

The accompanying diagram shows that $3 \times 7 = 7 \times 3$: for the whole group of objects may be considered (1) as 7 columns containing 3 in each, *i.e.* 3×7; or it may be regarded as 3 rows containing 7 in each, *i.e.* 7×3.

Thus $$3 \times 7 = 7 \times 3.$$

This argument might be applied to any whole numbers.

Similarly, in a continued product such as $3 \times 7 \times 13$ the order of the multiplications does not matter.

The Distributive Law. If the multiplicand is the sum of two or more parts, the product is formed by multiplying the separate parts by the multiplier and adding the results.

Thus $$(2+4+5) \times 3 = 2 \times 3 + 4 \times 3 + 5 \times 3.$$

$$\begin{array}{ccccccccccccc} \bullet & \bullet & \quad & \bullet & \bullet & \bullet & \bullet & \quad & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \quad & \bullet & \bullet & \bullet & \bullet & \quad & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \quad & \bullet & \bullet & \bullet & \bullet & \quad & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

In the diagram the top line consists of $2+4+5$, and so does each of the other two lines.

$$\therefore \text{ the total is } (2+4+5) \times 3.$$

But the total may be regarded as

$$\text{a group } 2 \times 3 + \text{a group } 4 \times 3 + \text{a group } 5 \times 3.$$

$$\therefore (2+4+5) \times 3 = 2 \times 3 + 4 \times 3 + 5 \times 3.$$

Thus the law is proved; for the argument would not be affected if any other whole numbers were chosen.

Multiplication by Factors.

In the proof of the **Commutative Law**, if each of the asterisks contained a number of objects, say n objects, then the total would be 21 times n, i.e. $n \times 21$.

But the top line would be $n \times 7$, and so would the 2nd line and the 3rd.

$$\therefore \text{ the total would be } n \times 7 \text{ occurring 3 times, i.e. } n \times 7 \times 3.$$

Thus $$n \times 21 = n \times 7 \times 3.$$

Thus multiplication by a number may be performed by multiplying successively by the factors of that number.

EXAMPLE. **3174 × 121 = 3174 × 11 × 11.**

$$\begin{array}{r} 3174 \\ 11 \\ \hline 34914 \\ 11 \\ \hline 384054 \\ \hline \end{array}$$

Tests of Accuracy.

5. It is of great importance to make a practice of checking results of arithmetical operations. The following methods will be found useful.

Addition. To test the correctness of the sum of several lines of figures, omit the top line and find the sum of the rest. This result added to the omitted top line should give the total.

A second method is to add from top to bottom in order to check a sum obtained by adding from bottom to top.

Subtraction is tested by **Addition.** The subtrahend added to the result ought to give the minuend.

EXAMPLES I. a.

Find the sums of the following columns (and *check your results*) :

1.	**2.**	**3.**	**4.**	**5.**	**6.**
511	413	297	189	2754	681
108	540	432	304	4444	216
243	135	127	459	5046	999
378	270	163	53	1721	350
					486

7.	**9.**	**10.**	**11.**	**12.**
9472	2016	3096	9665	11371
109	252	500	130528	130165
8971	2448	2664	50157	40073
706	684	2304	2514	5472
49	2880	1804		
	1116	3952		
	3312	1368		
	1548	3528		
	3744	936		
	1980			
	4176			

8.	**13.**	**14.**
99482	14424	23682
893687	118463	4543
2864	36683	9988
152011	5321	131840
346		

15.	16.	17.	18.
176902	31145	27168	12306
20912	191573	20921	81193
23104	155280	153466	34750
166570	44185	14457	12426
22682	71264	16526	10189
15886	13096	146737	52581
20304	11510	4185	113774
24184	44255	5351	20728

19.	20.	21.	22.
613	1239	15142	61244
32787	10632	7150	264496
7456	13461	2578	1675144
7873	30474	2146	1602338
9924	9076	1323	3999762
26047	41749	347	1733124
15474	9655	155	1109059
1234	5190	1884	583850
1827	2649	3416	5048027
12250	4244	8753	1261398
8977	23785	4494	78651
13377	19026	3653	1001897
614	1895	1574	71695
5242	4515	2847	377484
		539	242769
			3533723
			3041228
			260439
			343256
			1458855

EXAMPLES I. b. (*Oral* 1–20.)

1. How many even numbers are there below 80 ?

2. „ odd „ „ „ ?

3. How many numbers are there which contain 4 digits ?

4. „ „ „ „ less than 4 digits ?

5. „ „ „ „ not more than 4 digits ?

6. If a third-class railway carriage holds 60 passengers, 10 in each compartment, how many first-class passengers, 6 in each compartment, would a carriage of the same number of compartments hold ?

7. In the first 100 numbers how many are exactly divisible by 12 ?

8. „ „ „ „ „ „ „ 4 ?

9. „ „ „ „ „ „ „ 7 ?

10. Two persons start simultaneously from points 400 miles apart, and travel towards each other at the rates of 45 and 49 miles an hour. How far are they apart at the end of an hour ?

11. A train 104 ft. long passes through a tunnel 300 ft. long. What distance does it travel from the time when the engine enters till the tail leaves the tunnel?

12. A room 15 ft. wide has a stained border 2 ft. wide all round the room. What is the breadth of the carpeted part?

13. To multiply 35 by 11 we might put down the 3, then the sum of 3 and 5, and then the 5. How is it that this gives the correct result?

14. Multiply 42 by 11 in this way.

15. „ 73 „ „

16. Out of 362 Members of Parliament who voted on a motion, 190 voted for it. What was the majority for it?

17. If 10 of the 190 had voted the other way, what would have been the result?

18. If an ordinary year begins with a Sunday, how many Sundays does it contain?

19. A father is twice as old as his son, and the difference of their ages is 27. What are their ages?

20. A father is 3 times as old as his son, and the difference of their ages is 32. What are their ages?

21. Find $35672 + 4581$ and $35672 - 4581$. Add the results together and subtract 35672.

22. From 2359 subtract 337 seven times. What remains?

23. There are 4 points in a straight line in the order A, B, C, D. From A to C is 768 yards, from B to D 637 yards and from B to C 103 yards. What is the distance from A to D?

24. Find the sum of all the numbers which are greater than 300 and less than 315.

25. The difference of two numbers is 543, and the smaller number is 1160. Find the larger.

26. The difference of two numbers is 1346, and the larger number is 2718. Find the smaller.

27. The sum of two numbers is 83 and their difference 31. Find the numbers.

28. The sum of two numbers is 350, and one of them is 208. What is their difference?

29. In a game of cricket A, B and C score altogether 150, A and C 94, B and C 76. Find the score of each.

30. The population of a place was 17436: a year later it was 17658. Find the excess of births over deaths for the year, supposing that other causes did not affect the population.

EXAMPLES I. c.

Find the following products by using factors :

1. 241×45.	**2.** 846×54.	**3.** 846×324.	**4.** 534×21.
5. 307×27.	**6.** 307×729.	**7.** 1931×25.	**8.** 2700×354.
9. 630×911.	**10.** 640×237.	**11.** 168×271.	**12.** 132×1031.

Note on Multiplication.

Multiplying must be worked from left to right: that is to say, the left-hand digit of the multiplier must be the one used first.

6. This will be found of great importance when approximate work in decimals has to be undertaken, and consequently any learner who has acquired the wrong way ought to give it up and practise the other.

Multiply 8862 by 189.

Right Way.	Inferior Way.
8862	8862
189	189
8862	79758
70896	70896
79758	8862
1674918	1674918

Checks for Multiplication.

1. Work it in a different way. Here we might use the multiplicand as the multiplier and *vice versa*; or we might multiply by 3, then by 9 and then by 7, instead of multiplying by 189.

2. Use the method of "casting out the nines."

3. Use the method of "casting out the elevens."

The remainder, when a number is divided by 9, we will call its "nine-remainder," and similarly for 11.

Mental Tests of Addition and Multiplication by "casting out the nines."

7. The convenience of the test depends on the following principle :

Any number when divided by 9 gives the same remainder as would be obtained by dividing the sum of its digits by 9.

Thus, 1786304, when divided by 9, gives remainder 2; for the sum of the digits is 29.

When 526 is divided by 9, the remainder is 4; for the sum of the digits is 13.

We can express this by saying that the "nine-remainder" of 526 is 4.

Similarly, the nine-remainder of 213 is 6.

∴ in the sum of 526 and 213 the nine-remainder can be got from the sum of 4 and 6, *i.e.* from 10.

∴ the sum of 526 and 213 ought to have a nine-remainder 1.

The sum is 739. Has this a nine-remainder 1? It certainly has; for the sum of the digits of 739 is 19.

Thus the test is satisfied.

We can also use the method for checking the result of multiplication, *e.g.* in the product 526×213.

After the multiplication the nine-remainder of the product ought to be the nine-remainder of 4×6, *i.e.* 6.

Suppose the product was stated to be 114038. We could prove the incorrectness of this.

In	526	the nine-remainder	is	4.
,,	213	,,	,,	,, 6.
,,	114038	,,	,,	,, 8, whereas it should be 6.

The result is evidently wrong. It should actually be 112038.

If the test is satisfied we cannot be *sure* that the result is right; for if the error is a multiple of 9, or if it consists in a misplacing of digits, it will not be detected by this method.

EXAMPLE. **Find the nine-remainder of 206742.**

Here the sum of the digits is 21. Either add the 2 to the 1, *or* divide by 9. The nine-remainder is 3.

Casting out Elevens.

8. To find the remainder which a number leaves when divided by 11, find the sum of all the digits in the odd places, viz. the units, hundreds, etc.; find also the sum of those in the even places, viz. the tens, thousands, etc.; subtract the latter sum from the former or from the former increased by 11 (or a

multiple of 11) if it is the smaller of the two. The result, divided by 11, will give the required remainder.

EXAMPLE 1. **Find the eleven-remainder in 3784156.**

Here $\left.\begin{array}{r} 6+1+8+3=18, \\ 5+4+7=16. \end{array}\right\}$ $18-16=2.$
and

i.e. 3784156, when divided by 11, gives remainder 2.

EXAMPLE 2. **Find the eleven-remainder in 902084156.**

Here $\left.\begin{array}{r} 6+1+8+2+9=26, \\ 5+4+0+0=9. \end{array}\right\}$ $26-9=17=11+6.$
and

i.e. 902084156, when divided by 11, gives remainder 6.

EXAMPLE 3. **Find the eleven-remainder in 38561.**

Here the sum of the odd places $=1+5+3=9,$
,, ,, even ,, $= 6+8=14.$

9 is the less of the two. ∴ add 11.

$20-14=6=$ the required remainder.

This test by elevens may be applied to multiplication, etc., just in the same way as the nine-test, and will detect errors which would escape the other.

EXAMPLES I. d. (*Oral.*)

Find the nine-remainder in

1. 3005. **2.** 761. **3.** 41162. **4.** 531234. **5.** 70203.
6. 618765. **7.** 69122. **8.** 245738. **9.** 546382.

10. Add together the numbers in the first three examples, and test by casting out the nines.

Find the eleven-remainder in the following:

11. 27572. **12.** 16357. **13.** 4030218. **14.** 86071. **15.** 3086705.
16. 572856. **17.** 246837. **18.** 9172463. **19.** 1920512. **20.** 28571.

Test for errors in the following products by the method of elevens:

21. $3255 \times 654 = 2128670.$ **22.** $6436 \times 523 = 3366028.$
23. $5643 \times 7101 = 40070943.$ **24.** $7654 \times 397 = 3038738.$
25. $78847 \times 8803 = 694090141.$ **26.** $65299477 \times 5406 = 353008972662.$

Rough Checks.

9. In very many questions it is worth while to apply a *rough* check to detect glaring errors.

In multiplying 2184 by 1345, suppose we found the answer to be 2937480, there would be nothing absurd on the face of it, for

detect = [illegible]

we can see that the product must be greater than 2000×1300, *i.e.* greater than 2600000 : it must also be less than 2200×1400, *i.e.* less than 3080000.

These *rough* checks must be simple enough to be very rapidly applied, generally as a mental operation.

If, for instance, the product 8862×189 comes out to be 1974918, we know at a glance that something is wrong, for the product must be less than 9000×200; *i.e.* the first two digits of the product must form a number less than 18.

$\therefore$ **1974918** *must* be wrong.

EXAMPLES I. e.

Do the following multiplications, testing each result :

1. 226×18. **2.** 2371×27. **3.** 4409×144. **4.** 206×47.
5. 3136×112. **6.** 976×531. **7.** 1871×7484. **8.** 5423×804.
9. 3939×3939. **10.** 1479×952. **11.** 7654×397. **12.** 6676×406.
13. 18612×8771. **14.** 1342×2769. **15.** 3207×3147.

Multiplying by 5, 25, 125 by means of Division.

10. To multiply by 5 put a cipher on the right and divide by 2 ; for $5 = 10 \div 2$.

To multiply by 25 put 2 ciphers on the right and divide by 4 ; for $25 = 100 \div 4$.

To multiply by 125 put 3 ciphers on the right and divide by 8 ; for $125 = 1000 \div 8$.

EXAMPLES I. f.

Write down the results of the following multiplications, doing the work mentally :

1. 68×5. **2.** 78×5. **3.** 29×5. **4.** 53×5.
5. 248×5. **6.** 737×5. **7.** 1612×5. **8.** 3437×5.
9. 999×5. **10.** 48×25. **11.** 128×25. **12.** 84×25.
13. 528×25. **14.** 161×25. **15.** 693×25. **16.** 806×25.
17. 625×25. **18.** 4876×25. **19.** 64×125. **20.** 72×125.
21. 128×125. **22.** 483×125. **23.** 1228×125.
24. 966×125. **25.** 25×125. **26.** 125×125.
27. 2285×125. **28.** 7249×125. **29.** $4 \times 4 \times 4 \times 5 \times 5$.
30. $4 \times 4 \times 5 \times 5 \times 5$. **31.** $5 \times 5 \times 5 \times 5$. **32.** $5 \times 5 \times 5 \times 5 \times 5$.

Squares, Cubes and Higher Powers.

11. The **square** of a number is the result of multiplying the number by itself.

The **square** of a number n is denoted by n^2, and is called n squared.

Thus $$n \times n = n^2 \text{ and } 7^2 = 7 \times 7 = 49.$$

The **cube** of n is $n \times n \times n$, and is denoted by n^3.

$$6^3 = 6 \times 6 \times 6 = 216.$$

The multiplication table gives the squares of all whole numbers from 1 to 12, as for instance $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, etc.

It is useful to know some squares above 12^2, and all the cubes up to 10^3.

Squares.		Cubes.	
$13^2 = 169.$	$17^2 = 289.$	$1^3 = 1.$	$6^3 = 216.$
$14^2 = 196.$	$18^2 = 324.$	$2^3 = 8.$	$7^3 = 343.$
$15^2 = 225.$	$19^2 = 361.$	$3^3 = 27.$	$8^3 = 512.$
$16^2 = 256.$	$20^2 = 400.$	$4^3 = 64.$	$9^3 = 729.$
		$5^3 = 125.$	$10^3 = 1000.$

In the case of $n \times n$, which is written n^2, and in that of $n \times n \times n$, which is written n^3, the method of abbreviation is obvious; instead of writing down the repetitions of the factor n it is only necessary to write the letter once, and above it put the number of times that it occurs.

So $n \times n \times n \times n$ is written n^4 and is called the **4th power** of n. The number 4 written above is called the **index**.

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7.$$

$$1 \text{ million} = 10^6.$$

Square Root. The **square root** of a number is that which, multiplied by itself, gives the original number.

The symbol $\sqrt{\ }$ denotes square root.

$$\sqrt{49} = 7, \text{ for } 7^2 = 49.$$

Cube Root. The **cube root** of a number is that whose cube is equal to the original number.

Cube root is denoted by the symbol $\sqrt[3]{\ }$.

$$\sqrt[3]{125} = 5, \text{ for } 5^3 = 125.$$

Mental Work in Multiplication.

12. Write down the result of 23×47.

To do this without writing any intermediate lines of figures is quite easy if it is merely remembered that

the units in the product are found by multiplying units by units, the tens in the product come from tens × units and units × tens, and the hundreds come from multiplying tens by tens.

From unit figure × unit we get 21 units.

From tens × units + units × tens we get $2 \times 7 + 3 \times 4$; *i.e.* 26 tens.

From tens × tens we get 8 hundreds.

Thus	23	$3 \times 7 = 21$:—carry 2.
	47	$2 \times 7 + 3 \times 4 +$ the carried $2 = 28$:—carry 2.
	1081	$2 \times 4 +$ the carried $2 = 10$.

With a little practice it is easy to write down products of numbers of two digits, and to apply the same method to numbers of 3 digits.

EXAMPLES I. g.

Write down the following products, doing the work mentally:

1. 18×13. **2.** 19×18. **3.** 27×27. **4.** 34×76.
5. 73×84. **6.** 56×61. **7.** 82×36. **8.** 93×77.

EXAMPLES I. h. (*Oral.*)

Express with indices:

1. 3×3. 2. $5 \times 5 \times 5$. 3. $2 \times 2 \times 2 \times 2$. 4. 7×7.
5. $3 \times 3 \times 3 \times 3$. 6. $7 \times 7 \times 7$. 7. $3 \times 3 \times 7 \times 7$. 8. $5 \times 5 \times 3 \times 3 \times 3$.
9. $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$. 10. $3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$.
11. 49. 12. 25. 13. 8. 14. 16. 15. 81.
16. 32. 17. 343. 18. 243. 19. 10000.

Give the value of:

20. $17-(11-6)$. 21. $15-(8-6)$. 22. $15-(8+6)$. 23. $10-2 \times 3$.
24. $1+7 \times 3$. 25. $18 \div 6+3$. 26. $(10-2) \times 3$. 27. $(1+7) \times 3$.
28. $18 \div (6+3)$. 29. $4+7-5 \times 2$. 30. $8 \times 3+11$. 31. $9 \times 7-5$.
32. $(3+7) \times 5-6 \times (3+5)$. 33. $(3^2+4^2) \div 5^2$. 34. 6^2+8^2.
35. 2^5. 36. 3^4. 37. 5^3. 38. 2^5-3^3. 39. 7^2-6^2.
40. $1^2+2^2+3^2+4^2$. 41. $(7+5) \times 2^3$. 42. 9^2-8^2. 43. $1^3+2^3+3^3+4^3$.
44. $\sqrt{81}$. 45. $\sqrt[3]{27}$. 46. $\sqrt[3]{125}$. 47. $\sqrt{121}$. 48. $\sqrt[3]{8}$.
49. $\sqrt{144}$. 50. $\sqrt[3]{64}$. 51. $\sqrt[3]{216}$. 52. $\sqrt[3]{729}$.

Checks for Division.

13. **Division** may be tested by **Multiplication** or by **Division**.

We know that **Divisor** $\times$ **Quotient** $=$ **Dividend** $-$ **Remainder**.

$\therefore$ we can perform this multiplication as a check; or, after subtracting the remainder from the dividend, we can divide by the quotient and observe whether we get the divisor as result.

Division checked by casting out Nines or Elevens.

For example, 13730 divided by 257 gives quotient 53 and remainder 109.

In 257, 53, 109 the eleven-remainders are 4, 9, 10.

$\therefore$ the eleven-remainder in **divisor** $\times$ **quotient** $+$ **remainder** is found from $4 \times 9 + 10$, and is therefore 2.

The eleven-remainder in the dividend is also 2.

Division.

14. As an instance of Long Division, take the following:

Divide 6424698 by 1782.

$$
\begin{array}{r l l}
 & 3605 & \text{Quotient} \\
1782\,) & 6424698 & \\
 & \underline{5346} & 1^{\text{st}} \text{ partial product} \\
 & 10786 & \\
 & \underline{10692} & 2^{\text{nd}} \text{ partial product} \\
 & 9498 & \\
 & \underline{8910} & \\
 & 588 & \text{Remainder}
\end{array}
$$

Here it is to be noticed that the quotient is arranged *above* the dividend.

We have first to see how many figures of the dividend (counting from the left) we must take to form a number containing the divisor 1782. The number is 6424. This is called the *first partial dividend*; and, as it contains 3 times the divisor, we place a 3 directly above the last digit of the 6424.

$1782 \times 3 = 5346$, the *first partial product.*

To the remainder we annex the 6 brought down from the dividend.

The 10786 thus obtained contains 6 times the divisor, and leaves a remainder 94.

To this we annex the 9 from the dividend and put down the corresponding figure of the quotient, which is 0, since 949 does not contain 1782.

On bringing down the final 8 we get 5 for the last figure of the quotient, and 588 for remainder.

The zeros which are omitted for the sake of shortness would, if inserted, make the soundness of the whole process clear.

Thus $6424698 = 5346000 + 1069200 + 8910 + 588$;

$\therefore$ the quotient

$$\begin{aligned} &= 5346000 \div 1782 + 1069200 \div 1782 + 8910 \div 1782 \\ &= 3000 + 600 + 5 \\ &= 3605; \text{ and the remainder is } 588. \end{aligned}$$

15. The Italian method consists in omitting all the partial products by doing in one step the multiplication *and* subtraction.

For instance, instead of putting down the 1st partial product 5346, one would write the difference between it and the 6424.

The written work would appear as follows:

```
          3605
1782 ) 6424698
        10786
          9498
           588
```

The Italian method has the great disadvantage of making it difficult to check mistakes in the working.

Division by Factors: Formation of the Complete Remainder.

E.g. $678 \div 35$.

16. By dividing by 5, we find that 678 units become 135 groups of 5, and 3 units besides.

By dividing further by 7, we get 19 groups of 35, and there remain 2 groups of 5 + the 3 units mentioned before.

$$\text{Thus}\quad \begin{array}{r|l} 5 & \text{678 units} \\ \hline 7 & \text{135 groups of five} + 3 \text{ units} \\ \hline & \text{19 groups of } 35 + 2 \text{ groups of } 5 + 3 \text{ units} \\ \hline \end{array}$$

∴ the quotient is 19, and the remainder $= 5 \times 2 + 3$.

The work is put down as follows:

$$\begin{array}{r|l} 5 & 678 \\ \hline 7 & 135 \ldots 3 \\ \hline & 19 \ldots 2 \\ \hline \end{array}$$

Complete remainder
$= 2^{\text{nd}}$ remainder $\times$ 1^{st} divisor + 1^{st} remainder
$= 2 \times 5 + 3$
$= 13$.

If the divisor were 315, *i.e.* $5 \times 7 \times 9$, the work would be

$$35\begin{cases} 5 \\ 7 \end{cases}\!\!\begin{array}{|l} 678 \\ \hline 135 \ldots 3 \\ \hline 19 \ldots 2 \\ \hline 2 \ldots 1 \\ \hline \end{array}\quad \left.\begin{array}{l} \\ 3 \\ 2 \end{array}\right\} 13 \qquad 9$$

Here we may regard the divisors as 35 and 9, and the remainders as 13 and 1.

∴ as before, the complete remainder
$=$ remainder $1 \times$ divisor $35 +$ remainder $13 = 48$.

From this we can deduce a rule for the complete remainder after division by factors.

The successive remainders 3, 2, 1 may be called partial remainders.

Rule. ***Multiply each partial remainder by the product of all the preceding divisors (except the one which produced it), and add the products, including the first remainder.***

It is important that the principle underlying this rule should be grasped, and, with a view to this, it is necessary to give learners a considerable drilling in the formation of the complete remainder where there are only 2 divisors, and afterwards in the extension to 3 or more.

EXAMPLES I. k. (*Early part Oral.*)

Divisor.

1. 3×7, remainders (in order) 2 and 4; find the complete remainder.
2. 5×7, „ „ 4 and 3; „ „
3. 5×8, „ „ 4 and 5; „ „
4. 9×4, „ „ 3 and 1; „ „
5. 6×7, „ „ 2 and 6; „ „

	Divisor.				
6.	11×9,	remainders (in order)	8 and 7;	find the complete remainder.	
7.	11×12,	,, ,,	2 and 0;	,,	,,
8.	$3 \times 5 \times 8$,	,, ,,	2, 4, 6;	,,	,,
9.	$7 \times 8 \times 9$,	,, ,,	5, 0, 1;	,,	,,
10.	$5 \times 6 \times 7$,	,, ,,	4, 3, 5;	,,	,,

Find the remainder in each of the following:

11. $624 \div 25$. **12.** $521 \div 25$. **13.** $1031 \div 25$. **14.** $962 \div 25$.
15. $1733 \div 25$. **16.** $1739 \div 25$. **17.** $2293 \div 125$. **18.** $9871 \div 125$.
19. $655 \div 25$. **20.** $1915 \div 25$. **21.** $13847 \div 125$. **22.** $20345 \div 125$.

Do the following divisions, testing each result:
[Work by factors when they are of help.]

23. $6266 \div 13$. **24.** $5071 \div 461$. **25.** $19605 \div 231$.
26. $36448 \div 136$. **27.** $18224 \div 268$. **28.** $7813 \div 601$.
29. $2964 \div 127$. **30.** $7847 \div 133$. **31.** $126025 \div 355$.
32. $95469 \div 789$. **33.** $949684 \div 972$. **34.** $1221 \div 123$.
35. $117849 \div 49$. **36.** $79876 \div 77$. **37.** $13441970 \div 567$.
38. $832985 \div 945$.

II. REDUCTION.

17. Reduction is the process of expressing numbers of a higher denomination in lower units and *vice versa*.

$$£3 = 720 \text{ pence},$$
$$7 \text{ shillings} = 84 \text{ pence}.$$

In the first case we are reducing £3 to pence, in the second we are reducing 7 shillings to pence.

$$£3.\ 7s. = 720 + 84 = 804 \text{ pence}.$$

Here we have reduced a sum of £3. 7*s.* to pence.

On the other hand, we might have to express 804 pence in higher denominations. The reduction would give us £3. 7*s.*

Reduction from a higher denomination to a lower involves multiplication; that from a lower to a higher involves division.

Oral work in questions of money is much aided by a knowledge of the following "pence-table," which ought to be committed to memory:

$12d. = 1s.$;	$20d. = 1s.\ 8d.$;	$24d. = 2s.$;	$30d. = 2s.\ 6d.$;
$36d. = 3s.$;	$40d. = 3s.\ 4d.$;	$48d. = 4s.$;	$50d. = 4s.\ 2d.$;
$60d. = 5s.$;	$70d. = 5s.\ 10d.$;	$72d. = 6s.$;	$80d. = 6s.\ 8d.$;
$84d. = 7s.$;	$90d. = 7s.\ 6d.$;	$96d. = 8s.$;	$100d. = 8s.\ 4d.$;
$108d. = 9s.$;	$110d. = 9s.\ 2d.$;	$120d. = 10s.$;	$130d. = 10s.\ 10d.$;
$132d. = 11s.$;	$140d. = 11s.\ 8d.$;	$144d. = 12s.$	

Plentiful Exercises in Oral Reduction should be given here.

EXAMPLE 1. **Express 45 pence in shillings and pence.**

Answer. $45d. = 5d.$ more than $40d. = 3s.\ 9d.$
or $45d. = 9d.$ more than $36d. = 3s.\ 9d.$
or $45d. = 3d.$ less than 4 shillings $= 3s.\ 9d.$

The pupil will soon get to adopt whichever of these methods gives him the result most easily and quickly.

An extension of the pence-table can be used without difficulty. For instance, it will soon become a matter of memory that $168d. = 14s.$, and that $200d. = 16s.\ 8d.$

Notice that £1 $= 240d. = 960$ farthings.

EXAMPLE 2. **Reduce 2000 farthings to £.**

1920 farthings = £2.

$\therefore$ 2000 farthings = £2 + $20d.$ = £2. $1s.\ 8d.$

EXAMPLE 3. **Reduce £3. 9s. 3d. to pence.**

£3. $9s.\ 3d. = 720d. + 111d. = 831d.$

Examples like these and the following may be done mentally.

EXAMPLE 4. **Find the cost of 510 halfpenny stamps.**

Cost $= 255d.$ = £1 + $15d.$ = £1. $1s.\ 3d.$

EXAMPLE 5. **Find the cost of 12 things at 5d. each.**

$5d. \times 12 = 5s.$

EXAMPLE 6. **Find the cost of 12 things at 3s. 2d. each.**

$38d. \times 12 = 38s.$ = £1. $18s.$

EXAMPLE 7. **Find the cost of 20 things at 7s. each.**

$7s. \times 20$ = £7.

EXAMPLES II. a. (*Oral.*)

At a penny a mile, what is the cost of a journey of

1. 33 miles?	**2.** 47 miles?	**3.** 99 miles?	**4.** 137 miles?
5. 53 miles?	**6.** 28 miles?	**7.** 82 miles?	**8.** 150 miles?
9. 170 miles?	**10.** 194 miles?	**11.** 243 miles?	**12.** 500 miles?

How much must be paid for

13. 50 twopenny stamps?
14. 107 halfpenny stamps?
15. 77 halfpenny stamps?
16. 560 penny stamps?

Find the cost of a roll of cloth at 1*d.* an inch of length, supposing the length to be

17. 3 ft. 11 in.
18. 12 ft. 7 in.
19. 159 in.
20. 252 in.
21. 71 in.
22. 114 in.
23. 197 in.

At 10 centimes per decimetre, find the value of

24. 3 m.
25. 28 m. 6 dm.
26. 42 m. 5 dm.
27. 700 cm.
28. 125 cm.
29. 27 m. 6 cm.

If the length of a man's pace is 30 inches, how many yards does he go in

30. 12 paces?
31. 54 paces?
32. 36 paces?
33. 60 paces?
34. 42 paces?
35. 120 paces?

Find the cost of

36.	12 things at 1*s.* 3*d.* each.	37.	12 things at 2*s.* 4*d.* each.
38.	,, ,, $4\frac{3}{4}d.$,,	39.	,, ,, $6\frac{1}{2}d.$,,
40.	,, ,, 1*s.* $2\frac{1}{2}d.$,,	41.	,, ,, 2*s.* $5\frac{3}{4}d.$,,
42.	,, ,, 10*s.* 5*d.* ,,	43.	,, ,, 12*s.* 6*d.* ,,
44.	,, ,, 9*s.* $8\frac{3}{4}d.$,,	45.	,, ,, $11\frac{3}{4}d.$,,
46.	,, ,, 14*s.* 7*d.* ,,	47.	,, ,, 17*s.* $8\frac{1}{2}d.$,,
48.	20 things at 15*s.* ,,	49.	20 things at 13*s.* ,,
50.	,, ,, £1. 2*s.* ,,	51.	,, ,, £1. 17*s.* ,,
52.	,, ,, £3. 19*s.* ,,	53.	,, ,, £7. 11*s.* ,,

18. Example 1. **Reduce £15. 7s. $9\frac{1}{4}$d. to farthings.**

	£. s. d.	
	15 . 7 . $9\frac{1}{4}$	
	20	
shillings	307	7*s.* added.
	12	
pence	3693	9*d.* added.
	4	
farthings	14773	1 farthing added.

Test the answer by reducing 14773 farthings to £. s. d.

4	14773
12	3693...1 farthing.
2,0	30,7...9 pence.
	15...7 shillings.
	£15. 7*s* $9\frac{1}{4}d.$

EXAMPLE 2. **Reduce 6985 lb. to tons, etc.**

$$\begin{array}{r|l}
28\begin{cases}4\\7\end{cases} & \begin{array}{l}6985\\ \hline 1746\ldots1\end{array} \\
4 & \ \ 249\ldots3 \quad \Big\}13 \\
2{,}0 & \ \ \ 6{,}2\ldots1 \\ \hline
 & \ \ \ \ 3\ldots2
\end{array}$$

3 tons 2 cwt. 1 qr. 13 lb.

Reduce the answer to pounds.

tons	cwt.	qr.	lbs.
3 .	2 .	1 .	13

$$\begin{array}{r}
3\ .\ 2\ .\ 1\ .\ 13\\
20\\ \hline
62\\
4\\ \hline
249\\
7\\ \hline
1743\\
4\\ \hline
6985
\end{array}$$

6985 lb.

EXAMPLE 3. **Reduce 5 half-crowns to shillings and sixpences.**

1 half-crown = 30 pence.

∴ 5 half-crowns = 150 pence = 12*s.* 6*d.*

Or thus, 1 half-crown = 5 sixpences.

∴ 5 half-crowns = 25 sixpences = 12*s.* 6*d.*

EXAMPLE 4. **Reduce 365 half-crowns to £. s. d.**

By multiplying by 30 we reduce to pence.

$$\begin{array}{r|l}
 & \ \ 365\\
 & \ \ \ \ 30\\ \hline
12 & 10950\\ \hline
2{,}0 & \ \ 91{,}2\ldots6
\end{array}$$

£45. 12*s.* 6*d.*

Better as follows :—

Divide by 8 to reduce the 365 half-crowns to £.

$$8\,\overline{)\,365}$$

45 and 5 half-crowns = £45. 12*s.* 6*d.*

EXAMPLES II. b. (*Oral.*)

Reduce

1. £1. 1*s.* 1*d.* to pence.
2. 1*s.* 8½*d.* to farthings.
3. 5*s.* 3*d.* to halfpence.
4. £2. 0*s.* 6*d.* to sixpences.
5. £1. 10*s.* to half-crowns.
6. 268 pence to £. *s.* *d.*
7. 56 farthings to shillings and pence.
8. 77 „ „ shillings and pence.
9. 961 farthings to £. *s.* *d.*
10. 17 half-crowns to £. *s.* *d.*
11. 1 yd. 1 ft. 1 in. to inches.
12. 2 yds. 2 ft. 7 in. to inches.
13. 1 mile to feet.
14. 13 ft. 4 in. to inches.
15. 87 in. to yds., ft. and in.
16. 2 sq. yds. 2 sq. ft. to sq. inches.
17. 1 ton to pounds.
18. 3 tons 5 cwt. to qrs.
19. 5 min. 8 sec. to seconds.
20. 73 pints to gallons, etc.

EXAMPLES II. c.

In doing examples :

(1) *Show up all the working, including the check.*

(2) *Avoid side sums.*

(3) *Give reasons whenever you can.*

(4) *Employ factors, if possible.*

(5) *Revise your work before proceeding to the next example.*

Reduce (verifying each result)

1. £8. 14*s.* 1*d.* to pence.
2. £14. 19*s.* 11*d.* to pence.
3. £18. 12*s.* 6*d.* to pence.
4. £14. 5*s.* $1\frac{1}{2}d.$ to farthings.
5. £15. 1*s.* $6\frac{1}{4}d.$ to farthings.
6. £30. 3*s.* $1\frac{1}{2}d.$ to farthings.
7. £1. 9*s.* $11\frac{3}{4}d.$ to farthings (1 farthing below £1. 10*s.*).
8. £1. 19*s.* $11\frac{1}{2}d.$ to farthings.
9. 14430 pence to £. *s. d.*
10. 5704 pence to £. *s. d.*
11. 17353 pence to £. *s. d.*
12. 17883 farthings to £. *s. d.*
13. 15302 farthings to £. *s. d.*
14. What is the cost of 1447 penny stamps ?
15. What is the cost of 735 halfpenny stamps ?

Reduce (verifying each result)

16. 73 half-crowns to £. *s. d.*
17. 383 half-crowns to £. *s. d.*
18. 250 florins to sixpences.
19. 725 sixpences to half-crowns.
20. £3. 17*s.* 6*d.* to half-crowns.
21. £21. 7*s.* 6*d.* to half-crowns.
22. £43. 14*s.* to florins.
23. £7. 5*s.* 6*d.* to sixpences.
24. 153 half-crowns to sixpences.
25. 3 cwt. 2 qrs. 11 lb. to lb.
26. 3 tons 12 cwt. 3 lb. to lb.
27. 5 cwt. 3 qrs. 1 lb. to ounces.
28. 2 tons 17 cwt. 1 qr. 13 lb. to pounds.
29. 1 ton 7 cwt. 1 qr. to qrs.
30. 177 lb. to cwt., etc.
31. 563 qrs. to tons, etc.
32. 278912 oz. to tons, etc.
33. 10 miles 5 fur. to yards.
34. 15 yds. 2 ft. 5 in. to inches.
35. 780 chains to miles and furlongs.
36. 3 miles 7 chains to chains.
37. 1756 sq. in. to sq. yds., etc.
38. 1 sq. yd. 1 sq. ft. 30 sq. in. to sq. inches.
39. 14 sq. yds. 5 sq. ft. 68 sq. in. to sq. inches.
40. 14637 sq. yds. to acres and sq. yds.
41. 3 ac. 2 ro. 12 sq. po. to sq. poles.
42. 2345 sq. in. to sq. yds., etc.
43. 5760 acres to sq. miles.
44. England, Wales and Scotland contain 87827 sq. miles. How many acres do they contain ?
45. In 16940 yards how much more than 9 miles is there ?

Reduce (verifying each result)

46. 3 days 15 hrs. to minutes.
47. 4 days 3 hrs. 13 min. to seconds.
48. 606563 seconds to days, etc.
49. 7 qrs. 5 bus. to gallons.

50. 15 qrs. 5 bus. 3 gall. to gallons.
51. 2 qrs. 3 bus. 4 gall. 1 qt. to pints.
52. 2500 pts. to quarters, etc. 53. 457 pts. to bushels, etc.
54. 7 qrs. 2 bus. 6 gall. 1 qt. to pints.
55. 4276 qts. to quarters. 56. 7 weeks 2 days 13 hrs. to seconds.
57. 7654 stones to tons, etc. 58. 307 chains to miles, etc.
59. 58780 sq. yds. to acres, etc.
60. 52073 sheets of paper to reams, etc.
61. 30 reams 5 quires 7 sheets to sheets.
62. 12 reams 1 quire 2 sheets to sheets.
63. 29054 sheets to reams, etc. 64. 72° 2′ 24″ to seconds.
65. 137680″ to degrees, minutes and seconds.
66. 578606″ to degrees, etc. 67. 80° 21′ 42″ to seconds.
68. How many half-crowns can be given away from a sum of £13. 17*s.* 6*d.*?
69. How many halfpenny stamps can be bought for £3. 6*s.* 7½*d.*?
70. How many plots of half an acre can be made from 3 sq. miles?
71. Find the cost of 4577 ounces at 1*d.* an ounce.
72. How many ounces at 1*d.* an ounce can be got for £57. 4*s.* 3*d.*?
73. How many farthings are there in £321. 16*s.* 6*d.*?
74. Express 1363 farthings in £. *s.* *d.*
75. „ 102988 „ „
76. Reduce 380 half-crowns to sixpences.
77. „ 3 ml. 6 fur. 130 yds. to yards.
78. „ 3 tons 5 cwt. 3 qrs. to quarters.
79. „ 12712 lb. to tons, etc.
80. „ 105603 ft. to miles and yards.

Express

81. 51 francs 5 centimes in centimes.
82. 3075 centimes in francs and centimes.
83. 2 m. 3 dm. 4 cm. in centimetres.
84. 7 m. 6 dm. 5 mm. in millimetres.
85. 18 Kg. 4 g. in grammes. 86. 2 Kg. 3 Hg. 8 Dg. in grammes.
87. 6 Hg. 3 Dg. 5 g. 8 dg. in centigrammes.
88. 8 Dm. 7 dm. in centimetres. 89. 15 sq. m. in sq. centimetres.
90. 3 hectares 4 ares 5 centiares in sq. metres.
91. 153 cl. in litres, etc. 92. 2 Dl. 9 l. 5 cl. in centilitres.
93. 17695 mg. in grammes, etc. 94. 47895 cents in dollars, etc.
95. 367 half-crowns in halfpence. 96. 1063 sixpences in £. *s.* *d.*
97. 438 half-guineas in shillings. 98. 1 ton 4 cwt. 3 qrs. in ounces.

Express

99. 1131 lb. in cwt., etc.

100. 7 miles 7 fur. 100 yds. in feet.

101. 2 miles 1 fur. 4 chains in yds.

102. 5432 sq. in. in sq. yds., etc.

103. 72000 c. in. in c. yds., etc.

104. 5 c. yds. 20 c. ft. 60 c. in. in c. inches.

105. 1 day in seconds.

106. 783 pints in bushels, etc.

107. 2 right angles in seconds.

108. $101^\circ\ 15'$ in seconds.

109. The area of the United Kingdom is 77,683,084 acres. Express this in sq. miles and acres.

110. The area of Wales is 7,370 sq. miles. How many acres does it contain?

111. If the pendulum of a clock beats once in a second, how many times does it beat in 24 hours 3 minutes 10 seconds?

112. Find the cost of 1 mile of wire at 1*d.* per yard.

113. Find the total of the railway fares for the following journeys, assuming the cost to be 1*d.* a mile:

Swindon to Cardiff, 93 miles.
Paddington to Fishguard, 288 miles.
Euston to Inverness, 568 miles.

114. 3215 persons contribute a penny each. Find the total contribution.

115. A parish is valued at £37,350, and has to pay 1*d.* for every pound of its value to support its library. How much is the payment?

116. At a concert there were some seats at 2*s.* 6*d.* each, some at 1 shilling and the rest at 6*d.* each. The receipts were £23. 2*s.* 6*d.* from the half-crown places, £35. 3*s.* from the shilling places and £21. 3*s.* 6*d.* from the sixpenny ones. Find the numbers in the three classes.

III. COMPOUND ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

19. A QUANTITY is **compound** when it contains more than one denomination, when, for instance, it contains pounds, shillings and pence, or when it contains feet and inches.

Compound Addition is addition of compound quantities, and similar definitions hold for Compound Subtraction, Multiplication and Division.

EXAMPLE 1. **Find the sum of**

£.	s.	d.
8 .	9 .	2
15 .	2 .	11
37 .	14 .	8

The sum of the column of pence is 21, *i.e.* 1s. 9d. ;

∴ we put down 9 in the pence-column and carry 1 shilling. This shilling added to the next column gives a total of 26 shillings, *i.e.* £1. 6s. ;

∴ we put down 6 in the shillings-column and carry £1. It only remains to put down the total of the pounds with the £1 added.

The result is £61. 6s. 9d.

In Compound Subtraction no fresh principle is involved: it is sometimes necessary to use the plan of "equal additions," which is employed in Simple Subtraction and is often called in that connection "borrowing ten."

EXAMPLE 2.

	£.	s.	d.
From	34 .	4 .	8
take	26 .	14 .	9

Here, since the 8d. is less than the 9d. in the subtrahend, we convert it into 20d. by adding a shilling. We are justified in doing this if we add a shilling to the minuend and so make the 14s. into 15s.

We say then, "What must be added to 9d. to make 20d.?" 11d.

"What must be added to 15s. to make 24s.?" 9s.

"What must be added to £27 to make £34?" £7.

The required result is £7. 9s. 11d.

It will be recognised that we have made *equal additions* to the subtrahend and minuend.

	£. s. d.		£. s. d.
Thus	34 . 4 . 8	increased by 20s. and 12d. =	34 . 24 . 20
	26 . 14 . 9	increased by £1 and 1s. =	27 . 15 . 9
		Difference =	7 . 9 . 11

Where the minuend is an integral number of £, the method which is adopted in shops in giving change may be useful. Suppose, for example, that a purchaser gives a sovereign in payment of a bill of 13s. $7\frac{1}{2}$d. The shopman lays down $\frac{1}{2}$d. and then says 13s. 8d, he then lays down 4d. and says 14s., he finally lays down 6s. and says £1. What he has laid down is 6s. $4\frac{1}{2}$d.: and this is the remainder when 13s. $7\frac{1}{2}$d. is subtracted from £1. In a subtraction question the various statements would be made in exactly the same way, but instead of *laying down* the $\frac{1}{2}$d., the 4d. and the 6s., you would *write* them down in succession.

EXAMPLE 3. **Take £3. 16s. 3d. from £5.**

Write down 9*d.* and say £3. 17*s.*
Write down 3*s.* and say £4.
Write down £1 and say £5.
The required remainder is £1. 3*s.* 9*d.*

The same method might be applied to weights, measures, etc.

EXAMPLES III. a.

Test all Totals.

Tests. Add from top to bottom as well as from bottom to top, or omit one line and add it in afterwards.

Add together :

1. £3. 7*s.* $2\frac{1}{4}$*d.*, £5. 12*s.* 8*d.*, £10. 11*s.* $3\frac{1}{2}$*d.*, £1. 13*s.* $7\frac{3}{4}$*d.*

2. £4. 11*s.* 7*d.*, £6. 2*s.* $4\frac{3}{4}$*d.*, £1. 0*s.* 7*d.*, £11. 13*s.* $0\frac{1}{2}$*d.*

3. £3. 12*s.* $11\frac{1}{2}$*d.*, £2. 4*s.* 10*d.*, £3. 5*s.* $7\frac{3}{4}$*d.*, £1. 0*s.* 1*d.*

4. £21. 17*s.* 5*d.*, £32. 18*s.* 3*d.*, £45. 0*s.* 11*d.*, £22. 1*s.* 0*d.*

5. £2. 14*s.* $8\frac{3}{4}$*d.*, £9. 10*s.* $2\frac{1}{2}$*d.*, £6. 13*s.* 8*d.*, £4. 8*s.* $3\frac{1}{4}$*d.*

6. £6. 2*s.* $3\frac{1}{4}$*d.*, £6. 18*s.* $11\frac{1}{2}$*d.*, £37. 2*s.* 0*d.*, £1. 18*s.* $7\frac{3}{4}$*d.*, £1. 14*s.* $5\frac{1}{4}$*d.*

7. £6. 19*s.* $11\frac{3}{4}$*d.*, £8. 18*s.* $10\frac{1}{2}$*d.*, £4. 19*s.* 8*d.*, £8. 14*s.* $7\frac{3}{4}$*d.*, £9. 15*s.* 0*d.*

	£.	s.	d.		£.	s.	d.		£.	s.	d.	
8.	1500	10	0	**9.**	18394	8	6	**10.**	210	15	2	**11.**
	199	10	0		9197	4	3		6356	16	0	**12.**
	14444	19	7		10000	0	0		2933	13	9	**13.**
	10100	6	3		998	17	6		148	15	1	**14.**
	52055	6	7		15304	2	9		4255	15	11	**15.**

Questions 11, 12, 13, 14, 15. Find the sums of 1st, 2nd, 3rd, 4th, 5th lines of questions 8, 9, 10.

16. Find the total of all the sums of money mentioned in questions 8, 9, 10.

Add together :

	£.	s.	d.		£.	s.	d.		£.	s.	d.	
17.	11193	14	6	**18.**	9984	0	0	**19.**	113	19	9	**20.**
	194896	1	9		183738	2	1		146	4	7	**21.**
	211436	11	4		172794	0	3		117	0	1	**22.**
	16437	8	1		15580	1	4		51	6	8	**23.**
	15496	7	8		16201	10	5		37	8	5	**24.**
	85239	8	11		84051	0	0		664	17	1	**25.**
	90983	16	5		89715	11	3		1797	14	9	**26.**

Questions 20–26. Add horizontally the corresponding lines in questions 17, 18, 19.

27. Find the total of all the sums of money mentioned in questions 17, 18, 19.

Add up :

	£.	s.	d.
28.	5192	1	11
	43719	17	6
	51224	15	10
	11839	4	5
	450	15	11
	16419	14	0

	£.	s.	d.
29.	7267	8	2
	37121	14	4
	38822	2	8
	13079	8	9
	325	2	4
	19894	11	8

	£.	s.	d.
30.	114383	19	11
	18197	5	2
	19511	18	8
	40466	5	3
	148417	16	6

	£.	s.	d.
31.	115576	2	6
	17969	14	7
	20412	0	3
	39829	11	3
	137893	13	4

	£.	s.	d.
32.	62581	8	0
	7978	6	7
	10256	8	8
	2447	13	4
	9096	1	11
	1201	1	4
	4532	16	0
	3549	8	0
	18214	0	4
	2128	0	8

	£.	s.	d.
33.	66715	16	1
	7914	17	1
	11227	5	5
	3056	10	3
	9654	1	3
	1427	3	7
	2849	15	2
	5373	0	10
	18726	10	8
	1875	16	2

	£.	s.	d.
34.	546	3	6
	101	16	9
	712	7	1
	575	15	5
	660	12	5
	411	16	10
	923	18	8
	87	19	10
	742	1	1
	934	15	0
	773	8	10

	£.	s.	d.
35.	950	1	3
	728	15	5
	959	10	2
	482	1	0
	573	8	1
	950	2	1
	678	12	8
	544	11	6
	71	0	11
	549	7	0
	316	11	1
	17	14	4

	£.	s.	d.
36.	434465	15	4
	128846	9	7
	79584	6	2
	141448	8	9
	703354	8	4
	26019	18	6
	593499	10	0
	146123	12	10
	195987	17	10

	£.	s.	d.
37.	409755	1	8
	116510	7	11
	85285	14	1
	147135	9	4
	743060	3	6
	24124	3	10
	591111	9	7
	159463	19	6
	197643	17	8

	£.	s.	d.
38.	5718	2	5
	113	19	9
	146	4	7
	117	0	1
	1761	11	2
	51	6	8
	37	8	5
	664	17	1
	1797	14	9

	£.	s.	d.
39.	2198	7	3
	301	12	9
	11973	7	7
	1237	19	10
	1620	14	10
	3928	18	4
	4939	8	2
	11519	3	6
	419	11	9
	4782	4	11
	1383	5	2
	8911	0	9

Add up:

40.

£.	s.	d.
269354	10	9
26765	15	1
718529	14	2
958341	0	0
408938	1	1
7316	5	4
324366	10	0
256167	13	5
75542	6	7
719313	12	9
206262	13	3
617202	19	2

41.

£.	s.	d.
274834	7	9
33975	0	5
7129	11	4
15508	17	5
3974	6	10
8128	16	0
14660	16	5
7373	12	0
5592	19	7
1262	9	2

42.

tons	cwt.	qrs.	lb.
4	3	0	14
70	15	1	18
18	9	0	2
7	7	3	4
5	10	0	2
3	0	2	26

43.

tons	cwt.	qrs.	lb.
2	8	2	13
8	11	1	17
5	2	3	10
6	5	3	11
1	1	2	5

44.

tons	cwt.	qrs.	lb.
6	10	2	27
23	5	3	11
27	10	0	25
54	13	3	23
14	15	0	18

45.

yds.	ft.	in.
12	2	7
23	2	8
6	1	10
14	0	5
37	1	9

46.

qrs.	bus.	gall.	qts.	pts.
3	6	2	0	1
7	8	1	2	1
24	7	5	3	0
7	7	7	0	0
16	4	0	3	1

47.

ac.	ro.	sq. po.
23	2	17
4	3	13
7	1	29
8	3	10
17	1	8

48.

oz.	dwt.	gr.
3	17	20
2	9	19
	10	10
1	12	8
3	6	16

49.

cwt.	qrs.	lb.	oz.
14	3	10	12
23	1	22	6
65	2	3	14
40	0	25	10
3	2	18	9

50.

ac.	ro.	sq. po
111	1	15
37	2	34
8	0	21
19	2	4
20	3	10
1	1	21

51.

sq. yds.	sq. ft.	sq. in.
3	7	100
9	2	73
14	8	30
15	3	84
10	1	50
4	5	6

52.

days	hrs.	min.	sec.
2	1	34	50
4	10	25	42
10	19	40	33
13	17	8	12
9	8	7	6

53.

19°	12′	35″
26	40	30
58	20	45
71	44	32
10	15	30

Two railway companies work the following lengths of line. Find their totals in miles and chains.

54.

miles		chains
619	.	37
161	.	59
7	.	5
123	.	49
15	.	15
166	.	54

55.

miles		chains
1406	.	54
253	.	0
203	.	17
40	.	42
10	.	20
466	.	55

Add up

56.

francs		centimes
21	.	75
17	.	50
11	.	5
38	.	95
47	.	15

57.

\$.		c.
191	.	50
2017	.	20
38	.	44
12	.	11
71	.	90
2	.	15

58. 1 m. 8 cm. 3 mm. + 2 Dm. 3 m. 5 cm. + 7 m. 8 dm. 2 mm.

59. 4 Km. 7 Hm. 5 m. + 4 Hm. 8 Dm. 7 m. + 3 Km. 1 m.

60. 8 g. 3 dg. 4 cg. + 2 dg. 1 mg. + 9 dg. 7 cg. + 2 g. 3 dg. 4 mg.

61. 2 Kg. 5 Hg. 3 g. + 8 Hg. 4 Dg. 9 g. + 5 Kg. 2 g. + 1 Kg. 1 Dg. 8 g.

62. 17 Hectares 5 ares + 2 Hectares 27 ares 5 centiares + 89 ares 7 centiares.

EXAMPLES III. b.

Each result to be tested by Addition.

Find the difference between

1. £12. 4*s.* 0*d.* and £3. 15*s.* 8*d.* **2.** £92. 14*s.* 11*d.* and £16. 3*s.* 8*d.*

3. £171. 14*s.* 4*d.* and £83. 18*s.* 11*d.*

4. £173. 11*s.* 0*d.* and £93. 14*s.* 7*d.*

5. £73. 16*s.* 0*d.* and £68. 0*s.* 8*d.* **6.** £691. 4*s.* 10*d.* and £218. 18*s.* 4*d.*

7. £4413. 15*s.* 10*d.* and £5192. 1*s.* 11*d.*

8. £465. 8*s.* 2*d.* and £1309. 1*s.* 3*d.*

9. £12287. 16*s.* 9*d.* and £13647. 4*s.* 6*d.*

10. 4 bus. 2 pks. 1 gall. 1 qt. and 1 bus. 3 pks. 1 gall. 5 pts.

11. 10 qrs. 3 bus. 1 pk. and 1 qr. 5 bus. 1 pk. 1 gall.

12. 75 oz. 15 dwt. 21 gr. and 80 oz. 2 dwt. 7 gr.

13. 2 lb. 11 oz. 15 dwt. 11 gr. and 10 lb. 8 oz. 14 dwt.

14. 1000 tons and 873 tons 3 cwt. 2 qrs. 26 lb.

15. 23 ac. 3 ro. 14 sq. po. and 30 ac. 1 ro.

16. 90° and 11° 14′ 30″. **17.** 49° 27′ 44″ and 39° 50′ 49″.

18. 35 miles 18 chains and 20 miles 43 chains.

Find the difference between

19. 7 miles 3 fur. 100 yds. and 2 miles 7 fur. 144 yds.
20. 4 days and 3 days 7 hrs. 50 min. 6 sec.
21. 31 Km. and 22 Km. 8 m.
22. 9 g. 4 dg. 3 cg. and 5 g. 9 dg. 7 cg.

Compound Multiplication.

20. Multiplication by a number less than 13.

EXAMPLE 1.

	£.	s.	d.
	7 .	12 .	$4\frac{1}{4} \times 5$
			5
	38 .	1 .	$9\frac{1}{4}$

5 farthings $= 1d. + \frac{1}{4}d.$

$4d. \times 5 + 1d. = 21d. = 1s.\ 9d.$

Put down $9d.$ and carry 1 to the shillings.

$12s. \times 5 + 1s. = 61s. = £3.\ 1s.$

Put down $1s.$ and carry 3 to the £.

$£7 \times 5 + £3 = £38.$

Multiplication by a number which contains factors.

EXAMPLE 2.

£.	s.	d.	
10 .	7 .	$8\frac{1}{2} \times 28$	
		4	
41 .	10 .	10	= 4 times the multiplicand.
		7	
290 .	15 .	10	= 28 ,, ,,

General Case.

EXAMPLE 3.

	£.	s.	d.	
A	7 .	12 .	$4\frac{1}{4} \times 358$	
			10	
B	76 .	3 .	$6\frac{1}{2}$	= 10 times the multiplicand.
			10	
C	761 .	15 .	5	= 100 ,, ,,
			3	
C × 3	2285 .	6 .	3	= 300 ,, ,,
B × 5	380 .	17 .	$8\frac{1}{2}$	= 50 ,, ,,
A × 8	60 .	18 .	10	= 8 ,, ,,
	2727 .	2 .	$9\frac{1}{2}$	= 358 ,, ,,

Multiplication by a large number.

E.g. Multiply £5. 13*s.* 7*d.* *by* 4831.

It is best here to multiply the £. *s.* and *d.* separately and add the results.

	£.	s.	d.
$7d. \times 4831 = 4831d. \times 7 = 33817d. = 2818s.\ 1d. =$	140	18	1
$13s. \times 4831 = 48310 + 3 \times 4831 = 48310 + 14493 = 62803s. =$	3140	3	0
$£5 \times 4831 =$	24155	0	0
$\therefore$ £5. 13s. 7d. $\times 4831 =$	27436	1	1

(The 2818*s.* 1*d.* might be added to the 62803*s.* and the sum of these, viz. 65621*s.* 1*d.*, might then be reduced to £3281. 1*s.* 1*d.* This added to the £24155 would give £27436. 1*s.* 1*d.* as before; but there is no advantage in arranging the work like this.)

Cases where special methods may be employed.

To multiply by 99.

	£.	s.	d.	
	7	12	$4\frac{1}{4}$	$\times 99$
			10	
	76	3	$6\frac{1}{2}$	
			10	
	761	15	5	
Subtract	7	12	$4\frac{1}{4}$	
	754	3	$0\frac{3}{4}$	= 99 times the multiplicand.

Multiplication by 12, 20, 240, especially in oral work or mental calculations.

$$3d. \times 12 = 36d. = 3s.$$
$$7s. \times 20 = 140s. = £7.$$
$$240d. = £1.$$
$$9d. \times 240 = 240d. \times 9 = £9.$$

Thus it is seen that

multiplying by 12 converts pence into shillings,
„ 20 „ shillings into £,
„ 240 „ pence into £.

EXAMPLES. **Find the cost of 12 articles at 3s. 5d. each.**

3*s.* 5*d.* × 12 = 41*d.* × 12 = 41*s.* = £2. 1*s.*

Find the cost of 240 things at 7s. 8d. each.

7*s.* 8*d.* × 240 = 92*d.* × 240 = £92.

Multiply £4. 15s. by 20.

£4. 15*s.* × 20 = 95*s.* × 20 = £95.

In mental calculations or oral work in multiplication, besides the methods already mentioned, the following are applicable in some instances.

EXAMPLE 1. **£39. 10s. × 7** = (£40 − 10*s.*) × 7 = £40 × 7 − 10*s.* × 7
= £280 − £3. 10*s.* = £276. 10*s.*

EXAMPLE 2. **13 things at 5s. 8d. each** = 68*d.* × 12 + 68*d.* × 1
= £3. 8*s.* + 5*s.* 8*d.* = £3. 13*s.* 8*d.*

The fact that there are 8 half-crowns in £1 may be useful.

EXAMPLE 3. **12s. 6d. × 17** = 5 half-crowns × 17 = 85 half-crowns
= £10 + 5 half-crowns = £10. 12*s.* 6*d.*

EXAMPLE 4. **7s. 6d. × 27** = 3 half-crowns × 27 = 81 half-crowns
= £10. 2*s.* 6*d.*

EXAMPLE 5. **46 things at 3s. 9d.** = 23 things at 7*s.* 6*d.*
= 69 half-crowns = £8. 12*s.* 6*d.*

It is useful to be able to find easily the yearly income when the daily income is known.

EXAMPLE 6. **1s. 5d. a day = how much a year?**
1*s.* 5*d.* × 365 = 17*d.* × 240 + 17*d.* × 120 + 17*d.* × 5
= £17 + half of £17 + 85*d.*
= £25. 17*s.* 1*d.*

Rough Checks.

21. These are useful in detecting large errors.

EXAMPLE 1. **Find the value of 91 yards at 4s. 3d. a yard.**
The answer is £19. 6*s.* 9*d.*
A rough estimate would be got by taking 90 yards at 4 shillings.
The value of 90 at 4*s.* = 360*s.* = £18.

EXAMPLE 2. **Find the value of 233 yards at 3s. 9d. a yard.**
233 at 3*s.* 9*d.* = £43. 13*s.* 9*d.*
Rough check. 240 at 3*s.* 9*d.* = 45*d.* × 240 = £45.

EXAMPLES III. c. (*Oral.*)

Multiply

1. 4*s.* 11*d.* by 5.
2. 7*s.* 1*d.* by 11.
3. 3*s.* 5*d.* by 12.
4. £1. 2*s.* 2*d.* by 9.
5. £1. 9*s.* 6*d.* by 7.
6. £1. 7*s.* 3*d.* by 6.
7. £2. 12*s.* 6*d.* by 5.
8. £3. 2*s.* 6*d.* by 10. [Take the £3 separately.]
9. 10*s.* 7*d.* by 12.
10. £4. 8*s.* by 20.
11. £1. 3*s.* 6*d.* by 20.
12. 9*s.* 2*d.* by 240.
13. 19*s.* 10*d.* by 17.
14. 5*s.* 4*d.* by 13.
15. £2. 6*s.* by 21.
16. What is the yearly income at 4*d.* a day?
17. ,, ,, ,, 7*d.* ,,
18. ,, ,, ,, 10*d.* ,,
19. ,, ,, ,, 1*s.* 1*d.* ,,

20. Find the value of 76 things at 2*s*. 6*d*. each.
21. " " 26 " 3*s*. 9*d*. "
22. " " 160 " 1*s*. 3*d*. "
23. " " 8*s*. 6*d*. × 120.
24. Find the value of 9*s*. 7*d*. × 240.
25. " " 4*s*. 2*d*. × 360.
26. " " 1*s*. 4*d*. × 365.
27. " " 1*s*. 6*d*. × 365.
28. " " £1. 5*s*. 6*d*. × 20.
29. " " £2. 6*s*. 2*d*. × 20.
30. " " 17*d*. × 241.
31. " " 12 things at 7*s*. 3*d*. each.
32. " " 24 " 11*d*. "
33. " " 40 " 11*s*. 6*d*. "
34. " " 240 " 9*s*. 3*d*. "

EXAMPLES III. d.

Apply rough checks to avoid large errors.

To ensure accuracy of results prove by Division, or by multiplying in a different manner: e.g. *if a multiplication by 231 has been worked by factors in the order 11, 7, 3, use the multipliers in the order 3, 7, 11.*

Use factors when available.

Multiply

1. £4. 7*s*. 11*d*. by 7.
2. £2. 4*s*. 9*d*. by 11.
3. 1 ton 17 cwt. 1 qr. 7 lb. by 7.
4. £9. 9*s*. 3*d*. by 16.
5. £7. 18*s*. 11*d*. by 28.
6. 10 yds. 2 ft. 8 in. by 35.
7. £5. 12*s*. 7¾*d*. by 56.
8. £216. 19*s*. 3½*d*. by 8.
9. £3. 17*s*. 6*d*. by 91.
10. £11. 16*s*. 3*d*. by 75.
11. £8. 6*s*. 6¼*d*. by 26.
12. £82. 8*s*. 10*d*. by 14.
13. 5° 37′ 30″ by 32.
14. £2. 10*s*. 1*d*. by 43.
15. £95. 12*s*. 2½*d*. by 64.
16. £332. 16*s*. 7½*d*. by 24.
17. 5 ac. 0 ro. 7 sq. po. by 21.
18. £18. 5*s*. 9¼*d*. by 22.
19. 1 yd. 1 ft. 8 in. by 490.
20. 6 ac. 1 ro. 8 sq. po. by 11.
21. £9. 19*s*. 1½*d*. by 243.
22. £111. 5*s*. 6½*d*. by 23.
23. £1. 8*s*. 11*d*. by 1503.
24. 1 mile 55 chains by 9.
25. £1. 2*s*. 4½*d*. by 315.
26. £16. 16*s*. 7½*d*. by 1009.
27. 3 tons 5 cwt. 1 qr. 21 lb. by 35.
28. £58. 0*s*. 1½*d*. by 147.
29. 5 dm. 7 cm. 8 mm. by 7.
30. 3 l. 5 dl. 9 cl. by 33.
31. $1 92 c. by 28.
32. 3 fr. 65 c. by 19.
33. 8 g. 4 dg. 7 cg. by 13.
34. 1 Kg. 4 Hg. 4 Dg. by 17.
35. £2. 0*s*. 1*d*. by 1054.
36. £1. 4*s*. 6½*d*. by 1121.
37. £109. 10*s*. 2*d*. by 77.
38. £1. 1*s*. 1*d*. by 1762.
39. £1. 4*s*. 6½*d*. by 5432.
40. £1. 4*s*. 8*d*. by 1464.
41. 1 ac. 2 ro. 35 sq. po. by 66.
42. £2. 7*s*. 8½*d*. by 139.

Multiply

43. £11. 19*s.* 7*d.* by 231.

44. £105. 12*s.* 2*d.* by 99.

45. 1 ton 7 cwt. 2 qr. 10 lb. by 43.

46. 2 fur. 4 chains by 233.

47. A man spends 9*s.* 3*d.* a day. What does he save out of a yearly income of £250?

48. If 317 yards of cloth are bought at 3*s.* 6*d.* a yard and sold at 4*s.* 1*d.*, what is the profit?

49. In a terrace of 30 houses, each house has a frontage of 2 ch. 35 links. What is the whole frontage of the terrace?

50. What yearly income corresponds to £2. 12*s.* 3*d.* a day?

51. A man buys wire-rope at 2*s.* and sells it at 3*s.* $7\frac{1}{2}$*d.* a yard. Find his profit on a quarter of a mile.

52. A passenger has 37 Kg. of luggage and is charged at the rate of 25 centimes for a Kg. What change does he get out of 10 francs in paying for the luggage?

53. Find the value of 1 oz. of gold, if 1 dwt. is worth 3*s.* $10\frac{3}{4}$*d.*

54. If a yard of stuff cost 3*s.* 5*d.*, what is the cost of 40 pieces, each containing 15 yards?

55. 7 men received 5*s.* each, 11 women 2*s.* 6*d.* each and 16 children 1*s.* 3*d.* each. Find the total amount distributed. [*Work in half-crowns.*]

56. Find the cost of 31 pairs of boots at £1. 5*s.* 6*d.* a pair.

57. Find the wages of 33 men for 6 days at 3*s.* 8*d.* a day.

58. A sum of money is composed of equal numbers of sovereigns, crowns, florins, sixpences, and pence. There are 95 coins altogether. What is the sum of money?

59. Goods are bought at $5\frac{3}{4}$*d.* a lb., and the cost of carriage is 2*d.* a lb. What is the gain on 1 cwt. sold for £5?

60. Find the weight of 1000 paving-stones which weigh 27 lb. 13 oz. each.

61. Find the length of cloth in 25 bales, each containing 14 m. 3 dm. 6 cm.

62. What is the total weight of 32 parcels, each containing 7 Kg. 3 Hg. 2 Dg. 5 g.?

63. Give a rough estimate of the number of miles in 128 kilometres by taking a kilometre to be 5 furlongs.

64. A bowl holds 2 l. 1 dl. 1 cl. Find the capacity of 74 such bowls.

65. From a box containing 90 Kg. there are made up 200 parcels, each weighing 2 Dg. 3 g. 4 dg. Find the weight remaining.

66. Find the number of hectares, etc., in 42 plots of ground containing 33 ares 50 centiares each.

67. What is the total length of 34 coils of wire, each measuring 1 Hm. 7 Dm. 5 dm.?

68. Find the cost of 6 Dm. 2 m. at 5 fr. 25 c. per metre.

69. Out of 700 francs I pay for 46 Kg. at 14 fr. 70 c. per Kg. What change do I get?

70. Find the weight of a train of 9 trucks, each weighing 4 tons 4 cwt. 17 lb.

71. Find the area which divides up into 15 allotments, each containing 2 roods 32 sq. poles.

72. Multiply 8 miles 4 fur. 9 chains by 32. Express it also in miles and chains.

73. Find the cost of 81 Kg. at 33 marks 75 pf. per Kg.

74. By means of multiplication add together £3. 2*s.* 1*d.*, £6. 4*s.* 2*d.*, £9. 6*s.* 3*d.*, £12. 8*s.* 4*d.*, £15. 10*s.* 5*d.*, £18. 12*s.* 6*d.*

Compound Division.

22. Under this head are included two distinct operations.

The dividend is a concrete quantity; but the divisor may be either an abstract number or a concrete quantity of the same sort as the dividend.

The question may be, *Divide* £19. 6*s.* 4*d.* *by* 38, in which case the answer is the concrete quantity 10*s.* 2*d.*; or the question may be, *Divide* £19. 6*s.* 4*d.* *by* 3*s.* 2*d.*, and the answer to this is the abstract number 122.

The former question (Divide £19. 6*s.* 4*d.* by 38) is equivalent to asking "If £19. 6*s.* 4*d.* be divided into 38 equal parts, what is each part?" This process may be called **Partition.**

The latter question (Divide £19. 6*s.* 4*d.* by 3*s.* 2*d.*) is equivalent to saying "How many times is a sum of 3*s.* 2*d.* contained in £19. 6*s.* 4*d.*?" The number of times is 122. This process may be named **Quotition.** (*Quot* = how many.)

(1) **Concrete ÷ abstract = concrete.**

(2) **Concrete ÷ concrete of the same sort = abstract.**

23. Short Division.

(*a*) When the divisor is less than 13.

£25. 4*s.* 6*d.* ÷ 11.

$$\begin{array}{r|l} & \text{£.} \quad s. \quad d. \\ 11 & 25\,.\,4\,.\;\;6 \\ \hline & \underline{\;\;2\,.\,5\,.\,10\;\;} \end{array} \quad \text{Remainder } 4d.$$

Explanation. Treating the £25 as the dividend, we get a quotient £2 and remainder £3. The £3. 4*s.* = 64*s.* 64*s.* ÷ 11 will give quotient 5*s.* and remainder 9*s.* The 9*s.* 6*d.* = 114*d.* 114*d.* ÷ 11 will give quotient 10*d.* and remainder 4*d.*

(*b*) When the divisor consists of factors less than 13.

Divide £233. 17*s*. 9*d*. *by* 99.

$$\begin{array}{r|l} 9 & 233\ .\ 17\ .\ 9 \\ \hline 11 & 25\ .\ 19\ .\ 9 \\ \hline & \quad 2\ .\ \ 7\ .\ 3 \end{array}$$

If there were any remainders, the complete remainder would be found as in Simple Division by factors.

Suppose the question to be: *Divide* £234. 4*s*. 3*d*. *by* 99.

$$\begin{array}{r|l} 9 & 234\ .\ 4\ .\ 3 \\ \hline 11 & 26\ .\ 0\ .\ 5 \quad \text{Remainder } 6 \\ \hline & \ \ 2\ .\ 7\ .\ 3 \quad \quad ,,\quad\ \ 8 \end{array} \Bigg\} \text{ pence.}$$

The complete remainder $= 8 \times 9 + 6 = 78$ pence.

24. Long Division.

(*c*) £6784. 0*s*. $2\frac{3}{4}d \div 311$.

```
          £.   s.  d.
    311 ) 6784 . 0 . 2¾ ( £21
          622
          ---
           564
           311
           ---
           253
            20
          -----
    311 ) 5060 ( 16s.
           311
           ---
           1950
           1866
           ----
             84
             12
          -----
    311 ) 1010 ( 3d.
            933
            ---
             77
              4
            ---
    311 )   311 ( 1q.
            311
            ---
```

Quotient, £21. 16*s*. $3\frac{1}{4}d$.

The reasons for the steps are made clearer by putting the quotients where they are; but when the process is understood the quotient may be arranged in one line as below.

$$\begin{array}{rl}
 & \text{£.} \quad s. \quad d. \\
 & \;\; 21 \,.\, 16 \,.\, 3\tfrac{1}{4} \\ \cline{2-2}
311 \,) & 6784 \,.\;\; 0 \,.\, 2\tfrac{3}{4} \\
 & \underline{622} \\
 & \;\;564 \\
 & \;\;\underline{311} \\
 & \;\;253 \\
 & \;\;\;\;\;\underline{20} \\
 & \;\;5060 \\
 & \;\;311 \\ \cline{2-2}
 & \;\;1950 \\
 & \;\;\underline{1866} \\
 & \;\;\;\;\;84 \\
 & \;\;\;\;\;\underline{12} \\
 & \;\;1010 \\
 & \;\;\;\underline{933} \\
 & \;\;\;\;\;77 \\
 & \;\;\;\;\;\;\;\underline{4} \\
 & \;\;\;311 \\
 & \;\;\;\underline{311}
\end{array}$$

25 Quotition.

(*d*) *Divide* 39 yds. 1 ft. 5 in. *by* 5 yds. 1 ft. 11 in.

The dividend and divisor must be reduced to one and the same denomination.

The dividend reduced to inches is 1421.

The divisor ,, ,, 203.

$\therefore$ the quotient is 7.

A sum of £28. 10s. 2½d. consists of sovereigns, half-sovereigns, half-crowns, shillings and halfpence in equal numbers. Find the number of each coin.

Make a heap consisting of 1 sovereign, 1 half-sovereign, 1 half-crown, 1 shilling and 1 halfpenny. The value is £1. 13*s*. 6½*d*.

It is necessary to find out how many such heaps are contained in the £28. 10s. $2\frac{1}{2}d.$; in other words, we must divide £28. 10s. $2\frac{1}{2}d.$ by £1. 13s. $6\frac{1}{2}d.$

The dividend = 13685 halfpence.
The divisor = 805 halfpence.

$$\begin{array}{r} 17 \\ 805\,\overline{)\,13685} \\ \underline{805} \\ 5635 \\ \underline{5635} \end{array} \quad \textit{Ans.}\ 17.$$

In questions where the divisor is a concrete quantity, it is necessary to reduce it and the dividend to the same denomination, and the working is sometimes much simplified by a judicious choice of that denomination.

Example 1. **Divide £13. 2s. 6d. by £1. 17s. 6d.**

Worked in pence it would be unnecessarily heavy. Expressed in the form "Divide £13 and 1 half-crown by £1 and 7 half-crowns" it becomes 105 half-crowns ÷ 15 half-crowns, *i.e.* 7.

Example 2. **Divide £20 between A and B so that A may get £3. 5s. 10d. more than B.**

The £3. 5s. 10*d.* must be set aside for A, and the remainder, viz. £16. 14s. 2*d.*, must be equally shared by A and B.

£16. 14s. 2*d.* ÷ 2 = £8. 7s. 1*d.*

Thus B gets £8. 7s. 1*d.*

A gets £8. 7s. 1*d.* + £3. 5s. 10*d.*, *i.e.* £11. 12s. 11*d.*

A's share and B's should be added together to test the correctness of the work.

£.	s.	d.
11	12	11
8	7	1
20	0	0

Example 3. **Divide £36. 15s. between A, B and C so that A may have twice as much as B, and B may have twice as much as C.**

If we give 1 share to C, then B must have 2 shares each of the same value as C's share, and A must have 4 such shares.

The total number of shares = 4 + 2 + 1 = 7.

Each share = £36. 15s. ÷ 7 = £5. 5s.

		£.	s.
C's portion	= 1 share =	5	5
B's ,,	= 2 shares =	10	10
A's ,,	= 4 shares =	21	0
	Total	36	15

EXAMPLES III. e. (*Oral.*)

1. £1 ÷ 3. **2.** £1 ÷ 3*d.* **3.** £1 ÷ 8. **4.** £3 ÷ 2*s.* 6*d.*
5. £1 ÷ 3*s.* 4*d.* **6.** 4 mls. ÷ 1 fur. **7.** £7 ÷ 240. **8.** £10. 6*s.* 8*d.* ÷ 5.
9. 2 cwt. 2 qrs. ÷ 7. **10.** £17 ÷ 20. **11.** £1. 13*s.* ÷ 12.

12. How much a day is £730 a year?

13. How many angles of 45° are there in 360°?

14. Three persons share equally the change out of £1 on paying a bill of 11*s.* 9*d.* How much does each get?

15. A sum of 1 guinea is composed of equal numbers of florins, shillings and sixpences. How many are there of each?

16. £1 is spent in railway tickets at 1*s.* 3*d.* How many are bought?

17. Divide £29. 17*s.* 6*d.* by 30. (£30 minus 2*s.* 6*d.*)

18. Divide £35. 14*s.* by 36. (£36 minus 6*s.*)

19. How many times is 1*s.* 8*d.* contained in 6*s.* 8*d.*?

20. " " 2*s.* 6*d.* " 17*s.* 6*d.*?

21. " " 1*s.* 1*d.* " 14*s.* 1*d.*?

22. How many lengths of 22 yds. are there in 2 furlongs?

23. How many hurdles each 8 ft. 6 in. long are required for a fence 170 yds. long?

24. If 6 oz. of tobacco cost 3*s.* 9*d.*, what is the cost of 1 oz.?

25. How many metres of ribbon at 75 c. per metre can be bought for 6 fr.?

26. If 15 lb. of coffee cost 16*s.* 3*d.*, what is the cost of 1 lb.?

27. A wheel is 11 ft. in circumference. How many revolutions does it make in going a furlong?

28. If a motor car consumes 1 gallon of petrol every 21 miles, how much does it consume in 336 miles?

29. What is the cost of 1 cwt. at £4. 10*s.* per ton?

30. What is the cost of 1 lb. at £11. 4*s.* per cwt.?

31. 56 equal weights make up 1 ton. What is the weight of each?

32. A train is travelling 45 miles per hour. How many yards does it travel in a minute?

33. If a man is paid £15. 8*s.* for 14 weeks, what are his weekly wages?

34. Amongst how many must £5. 17*s.* be divided that each may get 9 shillings?

35. If I pay £26. 15*s.* for a ton, what do I pay for 1 cwt.?

36. Divide £9. 3*s.* 6*d.* between two persons so that one may have twice as much as the other.

37. Divide 12*s.* 8*d.* between two persons, giving one 3 times as much as the other.

38. I distribute 20 francs, giving 1 fr. 25 c. to each person. To how many do I give?

39. A sum of £6 consists of an equal number of crowns, florins and sixpences. How many are there of each sort?

EXAMPLES III. f.

Test results by multiplication.

Work the following by Short Division when possible:

1. £38. 12*s.* 11*d.* ÷ 5.
2. £64. 1*s.* 9*d.* ÷ 9.
3. £23. 8*s.* 5*d.* ÷ 7.
4. £402. 6*s.* 6*d.* ÷ 11.
5. £51. 8*s.* 5*d.* ÷ 28.
6. £151. 8*s.* ÷ 16.
7. £7987. 19*s.* ÷ 24.
8. £10455. 4*s.* 6*d.* ÷ 99.
9. £2305. 3*s.* 9*d.* ÷ 231.
10. 5 miles 5 chains. ÷ 3
11. 278 yds. ÷ 36.
12. £99. 15*s.* $9\frac{1}{4}$*d.* ÷ 13.
13. £8432. 2*s.* 10*d.* ÷ 77.
14. £331. 11*s.* $5\frac{1}{2}$*d.* ÷ 139.
15. £5545. 8*s.* 1*d.* ÷ 29.
16. £17563. 16*s.* $6\frac{3}{4}$*d.* ÷ 411.
17. £41820. 18*s.* ÷ 792.
18. 145 tons 8 cwt. 2 qrs. 16 lb. ÷ 8.
19. 31 acres 2 roods ÷ 5.
20. 20 tons 15 cwt. 2 qrs. 12 lb. ÷ 27.
21. 29 g. 1 dg. 6 cg. ÷ 12.
22. 184 m. 3 dm. 6 cm. 8 mm. ÷ 23.
23. Three persons share equally a yearly income of £428. 17*s.* 6*d.* How much has each daily?
24. In the year 1866 the population of the United Kingdom was approximately 30 millions, and the value of foreign food imported was 51 million £. How much was that per head?
25. £686. 15*s.* 5*d.* ÷ 347.
26. 37 acres 3 ro. 10 sq. po. ÷ 22.
27. If an oz. of gold be worth £3. 17*s.* 6*d.*, what is the value of 1 dwt.?
28. £18594. 13*s.* 4*d.* ÷ 256.
29. A sum of £85187. 10*s.* has to be divided equally amongst 120 persons. What is the share of each?
30. 69 miles 7 fur. 2 chains ÷ 466.

Divide

31. 288 yds. 2 ft. 5 in. by 7 yds. 2 ft. 5 in.
32. £1805. 12*s.* by £1. 4*s.* 8*d.*
33. £698. 5*s.* $7\frac{1}{2}$*d.* by £5 13*s* $6\frac{1}{2}$*d.*
34. £352. 12*s.* 6*d.* by £3. 17*s.* 6*d.*
35. £2173. 1*s.* 9*d.* by £4. 6*s.* 9*d.*
36. 63 ac. by 12 ac. 2 ro. 16 sq. po.
37. £107. 13*s.* 7*d.* by £2. 10*s.* 1*d.*
38. 281 yds. by 15 yds. 1 ft. 10 in.
39. 57 tons 1 qr. 7 lb. by 1 ton 12 cwt. 2 qrs. 9 lb.
40. 86 miles 5 chains by 1 mile 55 chains.
41. £1270. 3*s.* by £3. 6*s.* 6*d.*
42. 46 m. 2 dm. 5 cm. 2 mm. by 1 m. 4 dm. 9 cm. 2 mm.

43. 530 fr. 45 c. by 5 fr. 15 c.

44. 75 g. 4 cg. by 2 g. 3 dg. 4 cg. 5 mg.

45. 150 marks 8 pf. by 4 marks 69 pf.

46. 75 dollars 20 cents by 4 dollars 70 cents.

47. 5 Km. 6 Hm. 3 Dm. 1 m. by 9 Hm. 3 Dm. 8 m. 5 dm.

48. If a glacier move 1 ft. 3 in. in a day, how long does it take to go 180 yds.?

49. How far does it go in a year?

50. A sum of £8207. 5*s.* 4*d.* ÷ £73. 5*s.* 7*d.* = 112. Test this by Short Division.

51. How many revolutions does a wheel whose circumference is 13 ft. make in passing over 1 mile 2 fur. 82 ft.?

52. A man saves £162. 7*s.* 6*d.* out of a yearly income of £500; what does he spend daily?

53. A sum of £5. 8*s.* 6*d.* consists of equal numbers of half-sovereigns, crowns and sixpences. How many are there of each sort of coin?

54. Divide £66. 13*s.* 6*d.* between 2 men, giving to one 6 times as much as to the other.

55. A man taking tickets for a party on a journey paid £9. 5*s.*, the fare for each person being 4*s.* $7\frac{1}{2}d.$ The party numbered 37. How many unnecessary tickets did he take?

56. A sum of £14. 13*s.* 3*d.* consists of half-sovereigns, half-crowns, and threepenny pieces in equal numbers. How many are there of each sort of coin?

57. How many parcels each containing 51 francs 35 c. make up together 821 fr. 60 c.?

58. Divide 84 m. 7 dm. 3 cm. by 37.

59. How many sums of \$10. 75¢. are contained in \$731?

60. If 15 lb. cost £2. 1*s.* $10\frac{1}{2}d.$, what is the cost of 1 lb.; and what weight can be bought for £12. 11*s.* 3*d.*?

61. Divide £30. 11*s.* between 2 men so that one shall have £3. 5*s.* more than the other.

62. If 1 bushel of malt cost 5*s.* 10*d.*, how much can I buy for £27. 2*s.* 6*d.*?

63. If 3 cwt. 1 qr. 7 lb. cost £71. 2*s.* 2*d.*, what is the cost of 1 lb., and how much will £10. 3*s.* 2*d.* buy?

64. If 1 lb. Troy of standard gold be coined into $46\frac{1}{2}$ sovereigns, what is the value of 1 oz., it being understood that a gold coin contains its full value?

65. 20 Troy pounds of standard gold are coined into 934 sovereigns and 1 half-sovereign. What is the value of 1 oz.?

IV. REVISION PAPERS.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

IV. a.

1. Simplify $52 \div 13 + 7(23+17) - 15 \times 6 + 8$.

2. The distance from Paddington to Swindon is $77\frac{1}{4}$ miles, from Swindon to Bristol $41\frac{1}{4}$; find the fare from Paddington to Bristol, *via* Swindon, assuming the cost to be 1*d.* a mile.

3. Find the total of the following account:

£.	s.	d.
15	11	7
16	7	3
20	4	5
	10	6
32	11	1
41	6	8

4. Our exports in 1883 amounted to £305,437,070, and in 1902 they were £349,238,274. How much was the increase?

5. Multiply £7. 3*s.* $4\frac{1}{4}$*d.* by 108.

6. A sum of 780 dollars 50 cents is divided amongst 25 men. What does each get?

7. How many pieces 7 ft. 3 in. in length can be cut from a ball of string containing 110 ft.? What length remains?

IV. b.

1. Fill up the missing balance in this account.

	£.	s.	d.
	49	3	4
	46	7	8
	2	7	1
	2	19	3
	2	12	10
	1	15	0
	1	10	0
Balance	*	*	*
Total	155	12	9

2. If the divisor be 125, the quotient 16, remainder 8, what is the dividend?

3. A school numbers 630, including 45 masters. How many boys are there to each master?

4. Simplify $21 \times 3 + 7 - 5^2 - 84 \div 12$.

5. In a book there are 330 pages; each page contains 36 lines and a line contains on an average 52 letters. How many letters are there in the book?

6. Multiply £1. 10*s.* 8*d.* by 65.

7. Divide 7 Km. 2 Hm. 3 m. by 125.

IV. c.

1. Divide the product of 7175 and 4027 by 28189.

2. If £1 = 4 dollars 85 cents, express £711 in dollars and cents.

3. A man spends £413. 13*s.* 4*d.* in a year of 365 days. What is his average daily expenditure?

4. A motor-car takes 15 seconds to go 1 Hm. What is its speed in Km. per hour?

5. If I pay a cab-driver a penny per minute and he drives 4 miles in half an hour, at what rate do I pay him per mile?

6. If 2 tons 1 qr. 6 lb. be made up into 32 equal parcels, what is the weight of each parcel?

7. Find the rent of 135 acres at £2. 7*s.* 8*d.* an acre.

IV. d.

1. Divide sixty millions eight hundred and forty-two thousands five hundred and thirty-five by five hundred and seven; and express the answer in words.

2. Multiply 15840 by 456.

3. Divide 8276181 by 723.

4. Reduce 735934 ounces to tons, etc.

5. Find the cost of 796 pints at 12*s.* 7*d.* per gallon.

6. Make out the following bill, giving its total: 14 lb. of pork at $6\frac{3}{4}d.$ per lb., $10\frac{1}{2}$ lb. of beef at $10\frac{1}{2}d.$ per lb., 9 lb. of sausages at $8\frac{1}{2}d.$ per lb.

7. The water-rate for a garden is 6*d.* per quarter for each ten sq. poles. What is the whole charge for a year for a garden of 1 acre 3 roods?

IV. e.

1. Express in centimetres the difference between 7 Dm. 3 m. 6 dm. 4 cm. and 9 Dm. 8 cm.

2. Simplify $48 - (7+4) \times 3 - 5 - 15 \div 5$.

3. Multiply 424 by 3303. Test by casting out elevens.

4. The dividend being 731, the remainder 2 and quotient 81, what is the divisor?

Reduce - Pattern

5. 1183 Kg. of coal have to be moved by means of a barrow holding 81 Kg. How many journeys are required, and what weight is carried on the last journey?

6. A ton of goods costing £312. 5*s.* 6*d.* is sold for £424. 5*s.* 6*d.* Find the profit on each lb.

7. Divide 873388 by 108, using short division. Explain how the remainder is found.

IV. f.

1. Divide three hundred and fifty million one thousand three hundred and two by seven hundred and nine.

2. How many farthings are there in £159. 17*s.* $11\frac{3}{4}$*d.*?

3. Multiply £4. 15*s.* $6\frac{3}{4}$*d.* by 905.

4. Find the total cost of 3 dozen pocket handkerchiefs at 9*d.* each, 29 collars at $9\frac{1}{2}$*d.* each and 13 neckties at 1*s.* $11\frac{3}{4}$*d.* each.

5. Find the cost of 170 miles of telegraph wire at £14. 16*s.* 8*d.* per mile.

6. By means of the cost of a furlong, find that of 3 furlongs at the same price per mile.

7. A man spends on the average 4*s.* $9\frac{1}{2}$*d.* a day. How much does he spend in a year of 365 days.
[Find his expenditure in (240+120+5) days.]

IV. g.

1. Find the cost of 20,000 halfpenny stamps.

2. Supply the missing line in this account:

	£.	s.	d.
	4	14	6
		19	6
	4	14	9
	3	13	7
	1	18	6
	*	*	*
Total	25	13	5

3. Multiply 7 tons 10 cwt. 3 qrs. 24 lb. 11 oz. by 24.

4. A sum of £98. 0*s.* 9*d.* consists of an equal number of sovereigns, shillings and pence. How many are there of each coin?

5. If a ton of coal cost £1. 4*s.* 7*d.*, how many tons can be bought for £8. 12*s.* 1*d.*?

6. A sum of £217. 7*s.* 7*d.* is to be divided into 80 equal shares. What must be added to it to make a share an integral number of pence? Find the value of a share then.

7. A house and furniture are worth £4876. 11*s.* 7*d.*; the house is worth 6 times the furniture. What is the furniture worth?

IV. h.

1. What is the dividend when the divisor is 131, quotient 703 and remainder 52?

2. Find the value of $15^2 \times (17-13)+1-8^2$.

3. A train goes 45 miles an hour. How many feet per second is that?

4. From a distance of 7 miles 17 chains, distances of 12 chains 11 yards each are marked off. How many of these are there? What distance remains?

5. Find the value of $(54100-226)\times 177 \div 1593$.

6. How many pounds of tea at 1*s.* $10\frac{1}{2}$*d.* per lb. can be bought for £2. 3*s.* 6*d.*? What sum is left over?

7. Between two places 3 miles 5 furlongs apart there are 11 telegraph wires. What is the total weight of wire between them, if 22 yds. of wire weigh 20 lb.?

IV. k.

1. Reduce 6 ac. 2 ro. 17 sq. poles to sq. poles.

2. Divide 120544 by 792, using factors.

3. The circumference of a wheel is 7 ft. 4 inches. How many miles does it go in 4320 revolutions?

4. Find the whole weight of a train consisting of an engine and 7 carriages, the engine weighing 38 tons 3 cwt. 2 qrs. 11 lb. and each carriage 12 tons 1 cwt. 3 qrs. 7 lb.

5. Divide £720. 11*s.* 3*d.* by £2. 4*s.* $5\frac{3}{4}$*d.*

6. How much butter at 2 fr. 25 c. a kilogram would cost 11 fr. 25 c.?

7. A ship requires the stores mentioned below, and can obtain them at the price named by calling at port A or B. Calculate the total cost at each port, and the saving effected by buying in the port which is on the whole cheaper.

	Quantities.	Prices in A.	Prices in B.
Butter -	$22\frac{1}{2}$ cwt.	8*d.* per lb.	75*s.* per cwt.
Flour -	40 sacks.	28*s.* per sack.	28*s.* per sack.
Fruit -	50 cwt.	$1\frac{1}{2}$*d.* per lb.	$1\frac{3}{4}$*d.* per lb.

IV. l.

1. Simplify $111 \div (4\times 7-3+60\div 5)$.

2. Supply the missing line in this account:

	£.	s.	d.
	1164	5	2
	1313	1	4
	*	*	*
Total	2512	0	9

3. What is the dividend if the quotient be 11, remainder 13 and divisor 17?

4. How many tons of coal can be bought for £108. 11*s.* 6*d.* if 2 tons cost £2. 3*s.*?

5. Three persons share equally a yearly income of £469. 18*s.* 9*d.* How much has each daily?

6. An area of 347 ac. 1 ro. 20 sq. po. is divided into 14 equal lots. What is the size of each lot?

How much land would have to be added to the whole to make each lot contain an exact number of acres?

7. Out of a sum of money 17 men received 3*s.* $6\frac{1}{2}$*d.* each, and there was 5*s.* 6*d.* left. What was the original sum?

IV. m.

1. If a man's pulse beats 68 times in a minute, how many beats does it give in 3 hours $42\frac{1}{2}$ minutes?

2.
$$\begin{array}{r} 2769 \\ \underline{1342} \\ 2769 \\ 8207 \\ 11076 \\ \underline{5538} \\ \underline{3705998} \end{array}$$

Test this multiplication by casting out the nines.

If there is an error, detect by the process in what line it occurs.

3. Reduce 16200 ft. to miles and yards.

4. How many times can 2769 be subtracted from 11076, and what is the remainder after the last subtraction?

5. The 1st class fare from London to Geneva being £5. 13*s.* 7*d.* and that for a return ticket being £9. 1*s.* 7*d.*, how much is saved by a party of 6 if they take return tickets instead of single tickets each way?

6. How many such travellers would there have to be to effect a saving of £45. 11*s.* 8*d.*?

7. Find the value of 1000 tons at £6. 11*s.* $7\frac{1}{2}$*d.* a ton.

IV. n.

1. The divisor being 83, the quotient 144 and remainder 37, what is the dividend?

2. How many articles costing 7*s.* 2*d.* each can be bought for £43. 14*s.* 4*d.*?

3. If the dividend be £233. 14*s.* 7*d.*, the remainder 5 pence and the quotient £3. 5*s.* 10*d.*, what is the divisor?

4. What is the yearly income of a man who has 1*s.* 5*d.* a day?

5. Find the change out of a £5 note on buying five and a half railway tickets for a journey of 136 miles, the fare being a penny a mile.

6. 1453 telegraph poles are required for 66 miles, one at each end of this distance. How far are they apart? (Only 1452 intervals.)

7. Multiply 3 g. 4 dg. 6 cg. by 500.

IV. o.

1. Reduce 3 tons 2 cwt. 7 oz. to ounces.

2. A man's housekeeping expenses amount on an average to £1. 15*s.* $3\frac{1}{2}$*d.* a day. What do they come to for the year 1909?

3. How many seconds are there between 3 p.m. on Monday and 6.30 p.m. on the following Wednesday?

4. Gold wire of a certain thickness is worth 1 fr. 56 c. per centimetre. What should be the cost of 2 metres 9 cm. of this wire?

5. A man has £108. 13*s.* 4*d.* Find how many things, each costing £5. 4*s.* $2\frac{1}{2}$*d.*, he can buy, and how much will remain over.

6. Find the total value of 11 lb. of tea at 1*s.* $6\frac{1}{2}$*d.* per lb., 10 boxes of candles at $7\frac{1}{2}$*d.* per box, 10 lb. of bacon at 8*d.* per lb. and 12 lb. of cheese at $9\frac{3}{4}$*d.* per lb.

7. A boy is required to multiply £13. 3*s.* 7*d.* by 105. He multiplies by the factors 3, 5, 7 in order. In this way he gets 3 products. Explain how many times £13. 3*s.* 7*d.* the sum of these products will be. Using addition and subtraction only, show how to find £13. 3*s.* 7*d.* × 93 from the products in the boy's sum.

IV. p.

1.

44		28	80	12
16	48		32	64
		52	84	36
40	72	4		88
92			8	60

In this square each of the columns and each of the rows should have the same sum.

Fill up the missing numbers.

2. In a minute a man takes 110 steps of 33 inches. How many miles and yards does he go in an hour?

3. A man, having £153. 3*s.* 6*d.* in his bank, draws out £15. 10*s.* and £8. 4*s.* 8*d.*, pays in £29. 1*s.* 2*d.* and draws out £75. 10*s.* 4*d.* What has he left in?

4. Find the cost of school accommodation for 2750 children at £3. 17*s.* 1*d.* for each.

5. A man bought cloth at 4*s.* 9*d.* a yard for £23. 5*s.* 6*d.* How many yards did he buy?

6. A grocer buys a ton of sugar for £18. 10*s* and has to pay 15*s.* 6*d.* for carriage. How much profit does he make by selling at $2\frac{1}{2}d.$ a lb.?

7. How many telegraph poles 56 yards apart are there in a distance of 1 mile 1040 yds., there being one at each end?

IV. q.

1. Express 5 Kg. 3 g. 4 cg. in milligrams.

2. A motor-car is travelling at the rate of 32 Km. 4 Hm. per hour; find the number of metres it covers in a second.

3. The workmen employed in a certain factory earn on the average 5*s.* $2\frac{1}{2}d.$ each per day, and 234 men are employed. Find the total sum paid in wages for a week of 6 days.

4. What sum of money multiplied by 213 will give £815. 3*s.* $4\frac{1}{2}d.$?

5. Find the cost of 104 ac. 3 ro. at £10. 14*s.* a rood.

6. A man with a household of 9 consumes 185000 gallons of water in a year. His neighbour with a household of 4 consumes 38000 gallons. Each pays £3. 4*s.* a year for his supply. Calculate for each household the daily average consumption per head (to the nearest gallon), and the cost per 1000 gallons (to the nearest penny).

7. Find the total cost of 19 lb. of nails at $4\frac{1}{2}d.$ per lb., 300 yards of rope at 7*d.* per dozen yards, 17 iron rods at 1*s.* $3\frac{1}{2}d.$ each and 28 hammers at $10\frac{1}{2}d.$ each.

IV. r.

1. What number must be divided by 872 to produce 367 without remainder?

2. Reduce 1 mile 5 furlongs 103 yards to feet.

3. Find the value of 233 things at £8. 19*s.* 3*d.* each.

4. A farm containing 164 ac. 2 ro. 5 sq. po. is divided into 25 equal allotments. Find the rent of each allotment at 3*d.* a sq. pole.

5. How many sums of 3*s.* $8\frac{1}{2}d.$ amount to £100. 13*s.* $7\frac{1}{2}d.$?

6. The wages of 5 men for 6 weeks being £14. 15*s.*, find the wages of 1 man for 1 week, and hence the wages of 4 men for 3 weeks.

7. A grocer buys tea at £8 a cwt. He packs it in 14 lb. canisters and sells these at £1. 4*s.* 6*d.* each. The canisters cost him 7*d.* each and their delivery 1*s.* $4\frac{1}{2}d.$ each. Find his profit on each cwt.

V. FACTORS, PRIMES, MULTIPLES.

26. AN exact divisor of a number, *i.e.* one that divides it without remainder, is called a **measure** or **factor** of that number. It is sometimes called a **divisor.**

Thus 7 is a measure or factor of 35.

A **multiple** of a number is one that contains it an exact number of times.

Thus 55 is a multiple of 11, since it contains 11 five times.

A number consisting of factors is called a **composite** number; a number which contains no factor greater than 1 is called a **prime**. 2, 3, 17, 43 are instances of primes.

When we express the number 273 in the form $13 \times 7 \times 3$, we are said to have **decomposed** or **resolved it into factors.**

There is no difficulty in making a list of all the primes up to a moderate limit of size. Suppose you wish to find all the primes below 100. First write down in order all the numbers from 1 to 99. Beginning after the 2, underline every second number counting from it, viz. 4, 6, 8, etc. Beginning after the 3, underline every 3rd number counting from it. You have thus marked all the multiples of 2 and all those of 3.

It is not necessary to underline the multiples of 4; for they are all multiples of 2, and have therefore already been marked.

Underline every 5th number after 5, viz. 10, 15, 20, etc.

The multiples of 6 have already been marked as multiples of 2 and also as multiples of 3.

Proceed with 7 as with 2, 3 and 5.

8 and 9 need not be considered; for their multiples have already been marked as multiples of 2 or 3.

There is no need to go farther than this: for if any number less than 100 has a factor greater than 10 it has also a factor less than 10. For instance, 91 has a factor 13 and a factor 7.

∴ it is not necessary to go as far as the multiples of 13 in order to underline 91: for 91 would already be underlined as a multiple of 7.

Some of the numbers will be found to be underlined more than once, but the only ones with which we are concerned are

those which are entirely unmarked. These are all the primes less than 100.

A table so formed, by striking out all the composite numbers, is known as the "Sieve of Eratosthenes."

The primes below 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Tests of Divisibility.

27. As a help towards resolving a number into factors it is necessary to know how to tell whether a number is divisible by 2, 3, 4, 5, etc., without remainder.

A number is exactly divisible

by 2, if it is even;
by 3, if the sum of its digits is divisible by 3;
by 4, if the number formed by the last two digits is divisible by 4;
by 5, if it ends with 5 or 0;
by 6, if it is even and divisible by 3;
by 8, if the number formed by the last three digits is divisible by 8;
by 9, if the sum of its digits is divisible by 9;
by 11, if the sum of the digits in the odd places and the sum of those in the even places have a difference zero or a multiple of 11.

For divisibility by 7, 13, or a higher number, the best way is to try by actual division.

Multiplication of Numbers in the Index-form.

28. Powers of a number are multiplied together by adding their indices.

[Note that, when no index is expressed, the index 1 is understood: *e.g.* 3^1 is the same as 3.]

The rule of adding the indices together is clearly seen to be true by examining an example.

$$5^4 \text{ means } 5 \times 5 \times 5 \times 5.$$

$$5^3 \text{ means } 5 \times 5 \times 5.$$

$$\therefore\ 5^4 \times 5^3 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7.$$

Again $$3^4 = 3 \times 3 \times 3 \times 3.$$
$$\therefore\ 3^4 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5.$$

Here we add the index 4 to the index 1, since $3 = 3^1$.

EXAMPLE. **Express the product 343×49 in index-form.**

$$343 \times 49 = 7^3 \times 7^2 = 7^5.$$

NOTE.—A full stop is sometimes used instead of the sign of multiplication, *e.g.* $3.5.7$ instead of $3 \times 5 \times 7$; but it must not be employed where there is any risk of confusion with a decimal point.

The method of resolving into factors may be illustrated by examples. It is most useful to resolve into *prime* factors.

EXAMPLE 1. **Resolve 315 into prime factors.**

It is divisible by 9. Divide by 9. The quotient $35 = 5 \times 7$.

$$\therefore\ 315 = 9 \times 5 \times 7 = 3 \times 3 \times 5 \times 7 = 3^2 \times 5 \times 7.$$

EXAMPLE 2. **Resolve 5148.**

5148 is divisible by 11, by 9 and by 4.

$$\begin{array}{r|l} 11 & 5148 \\ \hline 9 & 468 \\ \hline 4 & 52 \\ \hline & 13 \end{array} \qquad \begin{aligned} 5148 &= 4 \times 9 \times 11 \times 13 \\ &= 2^2 \times 3^2 \times 11 \times 13. \end{aligned}$$

If a number is not divisible by 3, 5 or 11, which facts can be readily ascertained at sight, try 7, 13 or 17 by actual division.

It is useful to be able to recognise some products of certain primes, *e.g.* $111 = 3 \times 37$, $221 = 13 \times 17$,

$$1001 = 11 \times 91 = 11 \times 7 \times 13.$$

Through these we should be able to decompose such a number as 111111; for $111111 = 111 \times 1001 = 3 \times 37 \times 11 \times 7 \times 13$.

EXAMPLE 3. **Decompose 142857.**

This is evidently divisible by 9. Also it is divisible by 11; for

$$(7+8+4)-(5+2+1) = 11.$$

$$\begin{array}{r|l} 11 & 142857 \\ \hline 9 & 12987 \\ \hline 13 & 1443 \\ \hline 3 & 111 \\ \hline & 37 \end{array} \qquad \begin{aligned} \therefore\ 142857 &= 3^2 \times 3 \times 11 \times 13 \times 37 \\ &= 3^3 \times 11 \times 13 \times 37. \end{aligned}$$

EXAMPLE 4. **Examine whether 211 is a prime or not.**

211 is less than 15^2. $\therefore$ when divided by a number above 15, it will give a quotient less than 15. Thus if it contains a factor above 15 it must also contain one below 15.

$\therefore$ in testing whether it does contain factors it is not necessary to go above 15: for, as we have seen, if it has no factor below 15, it certainly has none above 15.

We can see that 2, 3, 5, 7, 11 will none of them divide 211 exactly, nor will any of the composite numbers, such as 14; $\therefore$ only 13 remains to be tried.

13 proves not to be a factor. $\therefore$ 211 is a prime.

29. Factors may be used for turning a number into a complete square, and in some cases for finding square root: and similarly for cubes and cube roots. This will be made clear by examples.

It should be noted that the *even* powers of a number are squares.

For instance, $5^4 = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 25^2$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 8 \times 8 = 8^2.$$

If the index is a multiple of 3, we have a cube.

Thus $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times 4 \times 4 = 4^3.$

What factor would make 2^3 a square? The factor 2.

For $2^3 \times 2 = 2 \times 2 \times 2 \times 2 = 2^4.$

What factor would make $3^5 \times 7^3$ a square?

$3^5 \times 7^3$ would become a square if it were multiplied by 3×7; for it would then be $3^6 \times 7^4$.

Square Roots.

30. We have seen that 5^4 is the square of 5^2.

$$\therefore\ 5^2 = \sqrt{5^4}.$$

Also $2^6 = 2^3 \times 2^3 =$ the square of 2^3.

$$\therefore\ 2^3 = \sqrt{2^6}.$$

Cube Roots.

Since $5^6 = 5^2 \times 5^2 \times 5^2$, it follows that $5^2 = \sqrt[3]{5^6}$.

Similarly $7^9 = 7^3 \times 7^3 \times 7^3$; $\therefore\ 7^3 = \sqrt[3]{7^9}$.

EXAMPLE 1. **What factor will make 1575 a square?**

$$\begin{array}{r|l} 9 & 1575 \\ \hline 7 & 175 \\ \hline & 25 \end{array} \qquad 1575 = 3^2 \times 5^2 \times 7.$$

Multiplication by 7 will make it $3^2 \times 5^2 \times 7^2$.
It then becomes the square of $3 \times 5 \times 7$, *i.e.* of 105.

EXAMPLE 2. **Find $\sqrt{17424}$ by factors.**

$$\begin{array}{r|l} 4 & 17424 \\ \hline 4 & 4356 \\ \hline 9 & 1089 \\ \hline & 121 \end{array} \qquad \sqrt{17424}=\sqrt{4^2\times 3^2\times 11^2}=4\times 3\times 11=132.$$

EXAMPLE 3. **Find $\sqrt{12544}$.**

$$\sqrt{12544}=\sqrt{2^8\times 7^2}=2^4\times 7=16\times 7=112.$$

EXAMPLE 4. **What factor will make 686 a cube?**

$$686=2\times 343=2\times 7^3.$$

The factor 4 is required; for $2^3=8$.

EXAMPLE 5. **Find by factors $\sqrt[3]{3375}$.**

$$\begin{array}{r|l} 5 & 3375 \\ \hline 5 & 675 \\ \hline 5 & 135 \\ \hline & 27 \end{array} \qquad \sqrt[3]{3375}=\sqrt[3]{5^3\times 3^3}=5\times 3=15.$$

Recapitulation.

31. Points to be remembered in resolving a number into factors.

(1) ***Find out by observation whether it is divisible by 2, 3, 4, 5, 9, or 11.***

(2) ***By actual division find out whether it is divisible by 7, 13, 17 or higher primes if necessary.***

(3) After dividing by several factors we may come to a quotient about which we are in doubt as to whether it is a prime or not; *e.g.* in resolving 6657,

$$\begin{array}{r|l} 3 & 6657 \\ \hline 7 & 2219 \\ \hline & 317 \end{array}$$

the quotient 317 has to be tested.

In testing this we need not use any divisor above 17, for 317 is less than 18^2.

317 is not divisible by 3, or 11, or 7, or 13, or 17;

$\therefore$ it is a prime.

EXAMPLES V. a. (*Oral.*)

Give the prime factors of the following numbers:

1. 30.	**2.** 77.	**3.** 45.	**4.** 72.	**5.** 84.
6. 91.	**7.** 95.	**8.** 132.	**9.** 242.	**10.** 51.
11. 75.	**12.** 126.	**13.** 114.	**14.** 135.	**15.** 143.

Mention all the primes between

16. 40 and 50. **17.** 60 and 70. **18.** 90 and 100. **19.** 100 and 110.

Replace the index x by the correct number in the following:

20. $3 \times 3 \times 3 \times 3 = 3^x$. **21.** $2^3 \times 2^2 = 2^x$.

22. $5^3 \times 5 = 5^x$. **23.** $5 \times 25 = 5^x$.

Which of the following are divisible by 9, by 11, or by both?

24. 2002. **25.** 428571. **26.** 386514.

27. 735658. **28.** 62613. **29.** 571428.

Which of the following are divisible by 8?

30. 30584. **31.** 92392. **32.** 74352. **33.** 162834.

Supply missing digits to make the following divisible by 9:

34. 43*71. **35.** 2534*7. **36.** 6*827. **37.** 203*1.

What are the factors of the following?

38. 91. **39.** 57. **40.** 65. **41.** 111. **42.** 105.

43. 85. **44.** 119. **45.** 95. **46.** 187. **47.** 143.

48. 231. **49.** 147. **50.** 1001. **51.** 221.

EXAMPLES V. b.

Decompose into prime factors, using the index-form where a factor is repeated:

1. 147. **2.** 315. **3.** 440. **4.** 287. **5.** 1250.

6. 891. **7.** 1456. **8.** 650. **9.** 1089. **10.** 1458.

11. 442. **12.** 1050. **13.** 6750. **14.** 4851. **15.** 3465.

16. 7623. **17.** 2431. **18.** 8840. **19.** 11011. **20.** 9324.

21. 1443. **22.** 7293. **23.** 7777. **24.** 10395.

25. 15288. **26.** 152152. **27.** 12584. **28.** 511104.

29. 30105. **30.** 33495. **31.** 17360. **32.** 285714.

What factors turn the following into complete squares?

33. 32. **34.** 128. **35.** 125. **36.** 468.

37. 1573. **38.** 9792. **39.** 14175. **40.** 72576.

Find by factors the square root of

41. 441. **42.** 784. **43.** 484. **44.** 576. **45.** 1089.

46. 6561. **47.** 3025. **48.** 2401. **49.** 11025.

50. 11664. **51.** 5929. **52.** 9604. **53.** 1521.

54. 2704. **55.** 15876. **56.** 148225. **57.** 69696.

Find what factor will convert the following into cubes:

58. 864. **59.** 4500. **60.** 1372. **61.** 33075.

VI. HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE.

32. NUMBERS are said to be **prime to each other** when they contain no common factor. Thus 15 and 28, though not primes, are prime to each other; for no factor of one of them is a factor of the other.

Consider the two numbers 3465 and 6699.

$$3465 = 3 \times 3 \times 5 \times 7 \times 11,$$
$$6699 = 3 \times 7 \times 11 \times 29.$$

3 is a common factor, so is 7, and so is 11.

21, 33, and 77 are also common factors.

But suppose that we want to know what is the **highest** number which is a common factor of 3465 and 6699.

The 3, the 7 and the 11 must *all* be included as factors.

$\therefore$ the **Highest Common Factor** is $3 \times 7 \times 11$, *i.e.* 231.

Denote this by the initials H.C.F.

The name **Greatest Common Measure** (G.C.M.) is sometimes used for it, and sometimes the name **Highest Common Divisor** (H.C.D.).

When a factor is repeated any number of times in both of the numbers whose H.C.F. is required, care must be used to take the factor the correct number of times in the H.C.F.

EXAMPLE 1. **Suppose the given numbers, expressed in factors, are**

$$2^3 \times 5^3 \times 7 \times 11 \text{ and } 2^4 \times 5^2 \times 11 \times 13,$$

i.e. $2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 11$ and $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 13$.

The factor 2 occurs 3 times in both, the 5 occurs twice in both and the 11 once in both. $\therefore$ the H.C.F. is $2^3 \times 5^2 \times 11$.

If one of the given numbers does not easily decompose, the factors of the other may perhaps suggest its factors.

EXAMPLE 2. **Find the H.C.F. of 1247 and 2871.**

1247 does not easily split into factors, but 2871 is evidently divisible by both 9 and 11.

$$2871 = 9 \times 319 = 3^2 \times 11 \times 29.$$

1247 is not divisible by 3 or 11. $\therefore$ we try 29 as divisor.

$$1247 \div 29 = 43. \quad \therefore \ 1247 = 43 \times 29.$$

The H.C.F. is 29.

33. If the numbers whose H.C.F. is required are not easily resolved into factors, the *Long Method* should be used.

This method, of which we give an example, depends upon the fact that in any example of Division, the remainder, if any, contains all the factors common to the divisor and dividend.

EXAMPLE. **Find the H.C.F. of 703 and 851.**

Divide 851 by 703. Remainder 148.
Use 148 for divisor and 703 for dividend. Remainder 111.
Use 111 for divisor and 148 for dividend. Remainder 37.
Use 37 for divisor and 111 for dividend. Remainder 0.
When we come upon a zero remainder, the H.C.F. is found: it is the last divisor.

In this case the H.C.F. is 37.

```
703 ) 851 ( 1
      703
      ---
      148 ) 703 ( 4
            592
            ---
            111 ) 148 ( 1
                  111
                  ---
                   37 ) 111 ( 3
                        111
                        ---
```

Better form of arrangement of work.

Quotient.			Quotient.
4	703	851	1
	592	703	
3	111	148	1
	111	111	
		37	

In the second arrangement the remainder 148 is used as divisor and the 703 as dividend *without displacing either of them*. The quotient 4 in this case is put on the left (since the divisor is on the right), but the quotients are of no importance.

EXAMPLES VI. a. (*Oral.*)

Find the H.C.F. of

1. 12, 15.	**2.** 12, 16.	**3.** 27, 33.	**4.** 49, 35.
5. 45, 54.	**6.** 51, 57.	**7.** 72, 96.	**8.** 98, 105.
9. 63, 147.	**10.** 125, 150.	**11.** 74, 111.	**12.** 65, 91.
13. 210, 154.	**14.** 100, 625.	**15.** 147, 343.	**16.** 96, 128.
17. 85, 119.	**18.** 95, 105.	**19.** 95, 114.	**20.** 51, 119.

In the following leave the H.C.F. expressed in factors (with indices where required):

21. 2×5^2, $2^2 \times 5^3$.

22. $3 \times 5^2 \times 7^3$, $5 \times 7^2 \times 11$.

23. $3^2 \times 7^4 \times 11$, $3 \times 7^2 \times 11^2$.

24. $2^3 \times 5^2 \times 13$, $2 \times 5^3 \times 13$.

25. $5 \times 13 \times 7$, $5^2 \times 7 \times 13^2$.

26. $2 \times 3 \times 7 \times 29$, $2 \times 5 \times 29$.

EXAMPLES VI. b.

Find by factors the H.C.F. of

1. 2310, 1650.	**2.** 3465, 2574.	**3.** 1395, 3100.
4. 2205, 11340.	**5.** 1470, 2058.	**6.** 44856, 14685.

7. 1792, 2048.
8. 468, 1266.
9. 2472, 3264.
10. 9438, 2275.
11. 1573, 689.
12. 2646, 72576.
13. 13860, 12285.
14. 4662, 5476.

Find by factors or otherwise the H.C.F. of

15. 36801, 34075.
16. 4067, 5146.
17. 7581, 2023.
18. 28742, 10265.
19. 428571, 714285.
20. 31185, 33638.
21. 15293, 19769.
22. 307692, 230769.
23. 99, 187, 143.
24. 273, 455, 1001.
25. 315, 504, 945.
26. 299, 345, 391.

Lowest Common Multiple (L.C.M.).

34. The L.C.M. of two or more numbers is the lowest number which is a multiple of each of them.

To find the L.C.M. of numbers express them in prime factors and write down the L.C.M. from observation.

EXAMPLE. **Find the L.C.M. of 80, 125, 150, 120.**

$$80 = 2 \times 2 \times 2 \times 2 \times 5.$$
$$125 = 5 \times 5 \times 5.$$
$$150 = 2 \times 3 \times 5 \times 5.$$
$$120 = 2 \times 2 \times 2 \times 3 \times 5.$$

The factor 2 occurs 4 times in one of the numbers, and no greater number of times in any of the others.

$\therefore$ 2 must occur 4 times in the L.C.M.

5 occurs 3 times in 125, and no oftener in any of the others.

$\therefore$ 5 must occur 3 times in the L.C.M.

The factor 3 does not occur more than once in any, and there are no other prime factors.

$$\therefore \text{ the L.C.M.} = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 3$$
$$= 6000.$$

EXAMPLE. Using the index-form we have in like manner the L.C.M. of $5^2 \times 7$, $2 \times 3^2 \times 7^3$, and $3 \times 7^2 \times 11$ is $2 \times 3^2 \times 5^2 \times 7^3 \times 11$.

35. When the numbers do not easily resolve into factors we may use the fact that

the L.C.M. of two numbers = their product ÷ their H.C.F.

The truth of this statement may be seen from an example.

Required the L.C.M. of 215 and 903.

$$215 = 43 \times 5; \quad 903 = 43 \times 3 \times 7.$$

Here the H.C.F. is 43.

$$\text{The L.C.M.} = 43 \times 5 \times 3 \times 7 = \frac{43 \times 5 \times 43 \times 3 \times 7}{43} = \frac{215 \times 903}{43}$$

$$= \text{the product of the numbers} \div \text{their H.C.F.}$$

After thus finding the L.C.M. of two numbers we may take this L.C.M. with a third number, and find the L.C.M. of this pair by the same rule; and so on if there are more numbers still.

Better than this, however, is the plan of employing the H.C.F. to get the factors of numbers which are not easily decomposed.

EXAMPLE. **Find the L.C.M. of 6509, 713, 2547.**

2547 is divisible by 9, and by finding the H.C.F. of the first two we get 23 as a factor of each.

By division we find the other prime factors.

Thus
$$6509 = 23 \times 283,$$
$$713 = 23 \times 31,$$
$$2547 = 3^2 \times 283,$$
$$\text{L.C.M.} = 3^2 \times 23 \times 31 \times 283.$$

Problems in H.C.F. and L.C.M.

36. EXAMPLE 1. **From a vat 42 pints have to be taken out and put into one vessel and 54 pints into another. What is the capacity of the largest measure which can be used?**

We want the largest number of pints which is contained an exact number of times in 42 pints and an exact number of times in 54 pints: *i.e.* we want the H.C.F. of 42 and 54.

$\therefore$ the measure must hold 6 pints.

EXAMPLE 2. **Three bells toll at intervals of 8, 9, 10 seconds respectively. If they start together, when will they next sound together?**

The first bell tolls at times which are the multiples of 8; and similarly for the others.

$\therefore$ they toll simultaneously at times which, measured from the start, are common multiples of 8, 9, 10 seconds.

$\therefore$ the first of these times is the L.C.M. of 8, 9, 10.

i.e. at the end of 360 seconds they sound together for the first time after the start.

EXAMPLE 3. **What is the greatest number which, when used as a divisor of 75 and 109, leaves a remainder 7 in each case?**

If we subtract 7 from 75 and from 109, we make both the divisions exact.

$\therefore$ we require the largest exact divisor of 68 and 102.

The required number is their H.C.F. 34.

EXAMPLES VI. c.

Some of these may be taken orally.

Find the L.C.M. of

1. $2 \times 3^2, 3 \times 5$. **2.** $2^4 \times 5, 2 \times 5^2$. **3.** $2^3 \times 3^2 \times 5, 2^2 \times 3 \times 5^2$.

4. 10, 15. **5.** 14, 21. **6.** 34, 51.

7. 16, 18, 24. **8.** 12, 15, 18. **9.** 26, 39, 52.

10. 74, 111. **11.** 2, 3, 4, 5, 6. **12.** 5, 6, 7, 8, 9.
13. 15, 16, 18, 20. **14.** 75, 100, 125. **15.** 12, 16, 21, 28.
16. 9, 12, 14, 105. **17.** 24, 36, 40, 54. **18.** 26, 28, 91, 104.
19. 40, 45, 48, 60, 72. **20.** 21, 56, 35, 80, 42.
21. 14, 25, 35, 49, 77. **22.** 91, 119, 221.

Find the L.C.M. of the following (leaving the result in factors):

23. $3^2 \times 5 \times 7$, $2 \times 3 \times 5^2$. **24.** 32, 34, 85, 98, 136.
25. 132, 165, 220, 231. **26.** 65, 121, 1001, 1331, 3535.
27. 555, 1221, 2035. **28.** 143, 49, 5929, 8281.
29. 28742, 10265. **30.** 666, 1295, 2331.
31. 3640, 17472. **32.** 1885, 6916, 7917.
33. 6279, 6578, 7007. **34.** 629, 3108, 3626.

Problems in H.C.F. and L.C.M.

EXAMPLES VI. d.

[*A good training for a class may be provided by setting them to invent problems in* H.C.F. *and* L.C.M.]

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

1. Find the least number giving remainder 7 when divided by 9, 12, 16.

2. Find the greatest number giving remainder 5 when used as a divisor of 77 and 104.

3. Find the greatest number which gives remainder 7 when used as divisor of 121, and 10 when used as divisor of 181.

4. In a straight rosebed the roses of the front row are 6 ft. apart, and those of the second row are 8 ft. apart. At one end they are exactly one behind the other. How far off does this first occur again?

5. Two men walk together, one taking 98, the other 112 steps in a minute. They start simultaneously with the same foot. After how many steps are they again "in step"?

6. Three bells begin tolling together, and toll at intervals of 21, 24, 28 seconds respectively. When do they first toll together again?

7. 34, 51 and a third number have H.C.F. 17 and L.C.M. 510. What values can the third number have?

8. Three lorries do respectively 15, 18 and 20 miles an hour. Each takes an exact number of hours over the same journey. What is the shortest journey for which this can be true?

9. From three casks 40, 35 and 55 pints are respectively to be drawn. What is the capacity of the measure which will do this most quickly?

10. A wheel 9 ft. in circumference, having a mark on its tyre, runs along a track divided by lines into distances of 33 ft. The mark on the wheel starts on the ground at the first line. Which is the next line where it will be on the ground?

7

VII. AREAS, WALLS OF A ROOM, VOLUMES.

37. A **parallelogram** is a four-sided figure whose opposite sides are parallel.

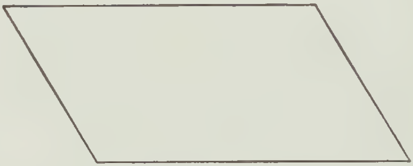


FIG. 1.

A straight line joining opposite angular points of a quadrilateral is called a **diagonal.**

A **rectangle** is a parallelogram which has all its angles right angles.

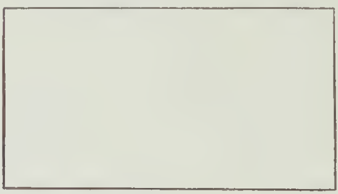


FIG. 2.

N.B.—It will be seen that if one of the angles of a parallelogram is a right angle, all the other angles are right angles.

Area of a Rectangle.

Let ABCD represent a rectangle, such that AB = 7 ft. and AD = 5 ft.

Let AB be divided into seven equal parts, so that each part represents 1 ft., and at each point of division draw a parallel to AD or BC.

Also let AD be divided into five equal parts, so that each part represents 1 ft., and draw parallels to AB or DC at each point of division.

We have now divided the whole rectangle into five rows, each consisting of seven small rectangles, and each of these small rectangles is 1 ft. long and 1 ft. wide, *i.e.* each small rectangle represents 1 square foot.

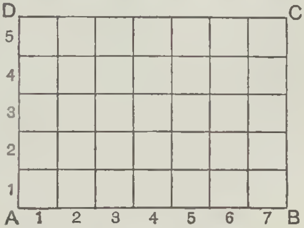


FIG. 3.

Therefore the area of the whole rectangle is equal to these 35 (7×5) small squares, *i.e.* 35 sq. ft.

In the same way, if we had a rectangle a feet in length and b ft. in width, we could divide it into b rows each consisting of a sq. ft.

Therefore the area of such a rectangle $= a \times b$ sq. ft. or ab sq. ft.

Similarly if a rectangle has sides x inches and y inches long, its area can be shown to be xy sq. in.

We may state the above result as follows:

The number of units of length in the length, multiplied by the number in the breadth, will give the number expressing the area of a rectangle.

38. It is frequently said that 7 ft. × 3 ft. = 21 sq. ft. This is not a strictly right way of expressing it; for a concrete quantity like 3 ft. cannot be used as a multiplier. Strictly one would say, 'The area of a rectangle measuring 7 ft. by 3 ft.

= 7 sq. ft. × 3 = 21 sq. ft.'

The loose expression 7 ft. × 3 ft. is used to mean the same as this.

The sides of a rectangle must be measured in the same units. Thus if one side of a rectangle is 2 ft. long, and an adjacent side 3 inches long, its area is (3×24) sq. inches, for 2 feet = 24 inches.

A square is a rectangle whose sides are equal.

∴ the area of a square having a 6 inch side $= 6 \times 6 = 36$ sq. inches.

Also, in general terms, the area of a square whose side is of length $a = a \times a = a^2$.

Note carefully that an area of 2 sq. inches is quite a different thing from a 2 inch square.

An area of 2 sq. in. is shown below, and it consists of two squares, each 1 sq. in. in area.

FIG. 4.

A 2 inch square is a square, each of whose sides is 2 in. long. Therefore its area $= 2 \times 2 = 4$ sq. in.

EXAMPLE 1. **Find the area of a rectangle 4 ft. 3 in. long and 2 ft. 6 in. wide.**

4 ft. 3 in. $=$ 51 in.

2 ft. 6 in. $=$ 30 in.

$\therefore$ the area required $= 51 \times 30 = 1530$ sq. in.
$= 10$ sq. ft. 90 sq. in.

EXAMPLE 2. **A surveyor finds a plot of ground is 3 chains 40 links in length by 1 chain 20 links in breadth. Find its area in square chains and square links.**

3 chains 40 links $=$ 340 links,

1 chain 20 links $=$ 120 links.

$\therefore$ the area $= 340 \times 120 = 40800$ sq. links
$= 4$ sq. chains 800 sq. links.

EXAMPLE 3. **A rectangular lawn 60 yards by 40 yards is surrounded by a path 2 yards wide. Find the area of the path.**

The common-sense way of doing questions of this sort is to find the total area up to the outside edge of the path, and to subtract the area of the lawn.

Here the measurements from outside edge to outside edge are

64 yds. and 44 yds.

$\therefore$ the total area $= (64 \times 44)$ sq. yds. $= 2816$ sq. yds.

The area of the lawn $= (60 \times 40)$ sq. yds. $= 2400$ sq. yds.

The area of the path $=$ the difference of these
$= 416$ sq. yds.

39. In the same way we can deal with

(1) the area of the margin of a printed page, by subtracting the area of the printed part from that of the whole page;

(2) the area of the margin of floor round a carpet;

(3) the area of a picture-frame.

EXAMPLE. **The width of a frame is 3 inches, and its external measurements are 3 ft. and 2 ft. 6 in. Find the area of the frame.**

The internal dimensions are 2 ft. 6 in. and 2 ft.

$$\therefore \text{ area of frame} = (36 \times 30 - 30 \times 24) \text{ sq. in.}$$
$$= 360 \text{ sq. in.}$$

Area of the Walls of a Room.

40. If a learner is told that, to find the area of the walls of a room, he must add together the length and breadth of the room and multiply the result by twice the height, he is apt to think that this is quite different from the rule for area which he has been in the habit of applying, viz. multiply the length by the breadth.

It is much simpler for him to have the one principle, and there is no need for any other.

The measurement all the way round any figure, *i.e.* the sum of the lengths of its sides, is called its **perimeter.**

Now, if the walls of a room could be arranged to form one straight wall (as below), their whole surface would be a rectangle having for dimensions the perimeter and the height of the room.

Cut out a strip of paper like the annexed figure, and bend it to form a model of the walls of the room.

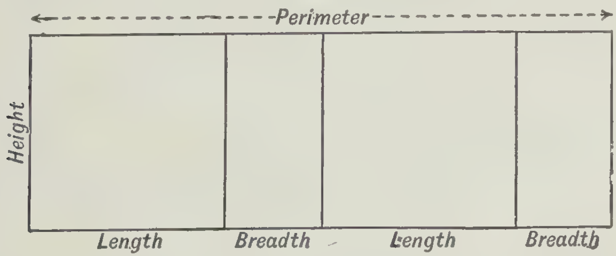


FIG. 5.

Thus ***the area of the walls of a room***
= perimeter of the room × the height.

EXAMPLE 1. **Find the area of the walls of a room whose length is 24 ft., breadth 15 ft., height 11 ft.**

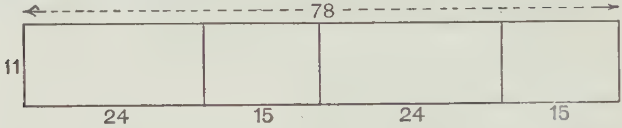


FIG. 6.

$$\text{Area of walls} = \text{perimeter} \times \text{height} = (24 + 15 + 24 + 15) \times 11$$
$$= (78 \times 11) \text{ sq. ft.} = 858 \text{ sq. ft.}$$

EXAMPLE 2. **Find the cost of painting the outside of a box with a lid, the external length, breadth and depth being 4 ft., 3 ft. and 2 ft. respectively and the charge 2d. per sq. ft.**

$$\text{The area of the walls} = \text{perimeter} \times \text{depth} = (4 + 3 + 4 + 3) \times 2$$
$$= 28 \text{ sq. ft.}$$

$$\text{The area of top and bottom} = (4 \times 3) \times 2 = 24 \text{ sq. ft.}$$

$$\therefore \text{ the area to be painted} = 52 \text{ sq. ft.}$$

$$\text{Cost} = 104d. = 8s.\ 8d.$$

41. The area of a rectangle = length × breadth ;

∴ the length of a rectangle = area ÷ breadth,

and the breadth of a rectangle = area ÷ length.

If the area is expressed in **sq. feet**, the breadth or length must be expressed in **feet**, and until this is done the division cannot be performed.

EXAMPLE 1. **A two-acre field is 40 yds. wide; find its length.**

$$2 \text{ acres} = 2 \times 4840 \text{ sq. yds.};$$

$$\therefore \text{ the length} = \frac{2 \times 4840}{40} = 2 \times 121 = 242 \text{ yds.}$$
$$= 11 \text{ chains.}$$

EXAMPLE 2. **A builder has to floor a room 20 ft. long and 15 ft. 9 in. wide with boarding 9 inches wide. How many yards of boarding will he require?**

$$\text{The area of boarding} = \text{the area of the floor} = 240 \times 189 \text{ sq. in.}$$

$$\text{Length of board required} = \text{area} \div \text{width}$$
$$= (240 \times 189 \div 9) \text{ inches}$$
$$= 240 \times 21 \text{ inches}$$
$$= 420 \text{ ft.} = 140 \text{ yds.}$$

42. Important. In doing examples, the work should be checked in some way. Even a rough check is valuable.

E.g. The area of a room 11 ft. 3 in. by 12 ft. 6 in. is somewhat greater than 132 sq. ft., for $11 \times 12 = 132$.

Division can be checked by multiplication.

Multiplication ,, ,, division.

After an example is worked out, it should always be revised thoroughly (not merely read through) before another is attempted.

EXAMPLES. VII. a. (*Oral.*)

What are the areas of the following rectangles?

1. 3 in. by 7 in. **2.** 9 ft. by 5 ft. **3.** 3 yds. by 9 yds.

4. 1 ft. by 21 ft. **5.** 2 ft. by 6 in., (1) in sq. in., (2) in sq. ft.

6. 3 ft. by 4 in., (1) in sq. in., (2) in sq. ft.

7. 3 yds. by 2 ft. in sq. ft. **8.** 1 ft. by 7 yds. in sq. ft.

9. 1 m. by 7 dm. **10.** 3 m. by 9 dm. **11.** 1 m. by 5 cm.

12. 3 m. by 8 cm. **13.** 7 dm. by 9 cm. **14.** 5 cm. by 7 mm.

15. 2 m. 3 dm. by 5 dm. **16.** 2 Km. by 17 m.

17. 2 chains by 13 chains. **18.** 1 mile by 5 chains.

19. A rectangular area of 72 sq. ft. is 8 ft. wide. What is its length?

20. ,, ,, 5 sq. yds. ,, 5 ft. ,, ,, ,,

21. ,, ,, 14 sq. yds. ,, 7 ft. ,, ,, ,,

22. ,, ,, 5 acres ,, 10 chs. long. What is its width?

23. ,, ,, 3 sq. m. ,, 12 dm. wide. ,, length?

24. ,, ,, 2 sq. m. ,, 8 cm. ,, ,, ,,

Find the perimeters of the following rooms:

25. Length 12 ft., breadth 9 ft. 6 in.

26. ,, 5 m. 6 dm., ,, 4 m.

27. ,, 7 m. 5 dm., ,, 5 m. 6 dm.

28. ,, 19 ft. 2 in., ,, 10 ft. 11 in.

What are the dimensions of the carpet in the following rooms?

29. Floor 15 ft. by 11 ft.; border 2 ft. wide all round.

30. Floor 18 ft. 6 in. by 14 ft. 3 in.; border 1 ft. 6 in. wide all round.

Find the area of the walls of each of the following rooms:

31. Length 14 ft., breadth 11 ft., height 8 ft.

32. ,, 15 ft., ,, 12 ft., ,, 10 ft.

33. ,, 5 m., ,, 4 m., ,, 3 m.

34. Find the length of carpet 2 feet wide required for a rectangular area of 24 sq. yds.

EXAMPLES VII. b.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

Find the area of each of the following rectangles:

1. 20 ft. by 12 ft.
2. 13 ft. by 7 ft.
3. 2 ft. 3 in. by 1 ft. 6 in.
4. 4 yds. 2 ft. by 5 yds. 1 ft.
5. 9 yds. by 6 yds. 2 ft.
6. 13 ft. 4 in. by 4 ft. 6 in.
7. 7 ft. 2 in. by 4 ft. 1 in.
8. 3 yds. 2 ft. 3 in. by 2 yds. 2 ft.

Find the area of the floor of each of the following rooms:

9. Length 28 ft., breadth 13 ft.
10. " 17 ft. 6 in., " 11 ft. 5 in.
11. " 16 ft. 9 in., " 12 ft. 6 in.
12. " 5 m. 5 dm., " 4 m.

13. How many acres are there in a rectangular field 242 yds. by 200 yds.?

14. How many acres in one 15 chains by 14 chains?

15. The floor of a room has a length of 31 ft. and area 37 sq. yds. 8 sq. ft. What is its breadth?

16. Find the area of a rectangle 20 m. 5 dm. by 16 m.
17. " " 4 m. 8 dm. by 2 m. 5 dm.
18. " " 40 Dm. by 7 Dm. 5 m.

Give the result in Hectares.

19. A rectangular field containing 6 Hectares has a length of 625 m. What is the length of the fence round it?

20. A floor 23 feet by 17 feet has a stained border 1 ft. 6 in. wide. What is the area of the border?

21. A running-path round a rectangular ground has a width of 20 ft. The dimensions of the field inside the path are 100 yards and 80 yards. Find the area of the path and the length of its inner edge.

Find the area of the walls of the following rooms:

22. Height 10 ft., length 18 ft., breadth 13 ft.
23. " 9 ft., " 16 ft., " 12 ft.
24. " 12 ft., " 19 ft. 6 in., " 14 ft. 6 in
25. " 11 ft., " 18 ft. 3 in., " 13 ft. 3 in
26. " 10 ft., " 16 ft. 4 in., " 12 ft. 2 in.

27. The walls of a room measure 84 sq. yds. Find the cost of painting them at 1*s.* 2*d.* a sq. yd.

28. The floor of a room 7 m. by 5 m. has to be covered all over with a carpet 7 dm. wide. What length of carpet does it take?

29. A floor measuring 12 m. 6 dm. by 10 m. is made of boards 3 dm. 5 cm. in width. Find the length of boarding used.

30. Find the cost of painting the four sides and the bottom of a tank 12 feet long, 5 feet wide and 4 feet deep at 4*d.* a sq. ft.

Volumes.

43. A solid figure which has six faces, each face being a square, is called a **cube**. If each edge were a foot, each face would be a square foot, and the solid would be a cubic foot.

When we speak of the **volume** or **solid content** of a body, we mean the number of cubic units which it contains, whether those units be cubic inches, cubic feet, cubic millimetres or other measures.

If each of the six faces be a rectangle of any shape, the figure is called a **rectangular solid** or **rectangular parallelepiped**; but it is desirable to have a less cumbrous name for an object of such common occurrence. The word **cuboid** seems to supply what is wanted.

44. ***The volume of a cuboid is found by multiplying together the numbers expressing (in the same unit) its length, breadth and depth.***

Let ABCD$abcd$ be a cuboid, such that AB $= p$, B$b = q$ and AD $= r$ units of length.

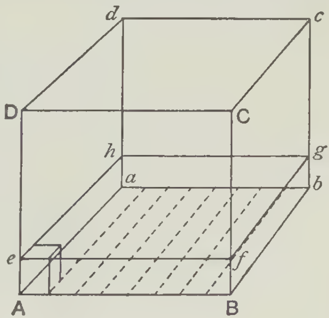


FIG. 7.

Draw a plane $efgh$ parallel to the base Ab, at unit distance from it.

The area ABba consists of $p \times q$ units of area.

$\therefore$ we see from the diagram that the vol. of the layer A$g = p \times q$ units of volume.

Also the whole volume consists of r such layers.

$\therefore$ the whole vol. $= p \times q \times r$ units of volume.

If a cuboid has a length of 7 inches, breadth 5 inches and depth 4 inches, its volume is $7 \times 5 \times 4$ cubic inches, *i.e.* 140 c. in.;

or, as it may be expressed, a cuboid measuring 7 by 5 by 4 inches contains 140 cubic inches.

45. In a cuboid $V = lwh$, where l, w, h are the length, width and height. $\therefore\ h = V \div lw$, and $wh = V \div l$.

In words, *the volume divided by the area of the base will give the height*, where we may consider the base to be any one of the faces, and the height to be the perpendicular distance between that and the opposite face.

Also, *the area of any face may be found by dividing the volume by the distance of that face from the opposite one.*

Surface of Cube or Cuboid.

46. *The surface of a cube* $= 6x^2$, where x = the length of an edge.

The cuboid has 6 faces, 2 of them measuring l by w,
2 ,, ,, w by h,
2 ,, ,, h by l.

Thus the area of the surface of a cuboid $= 2lw + 2wh + 2hl$
$= 2(lw + wh + hl)$.

The area of the walls of a room $= 2hl + 2hw = 2(l + w)h$ = *perimeter* × *height.*

Example. **Given that a rectangular block of wood contains 2 c. ft. 1419 c. inches, and that its base measures 2 ft. 1 in. by 1 ft. 3 in., find its height.**

2 c. ft. 1419 c. inches = 4875 c. inches.

$$\text{Height} = \frac{\text{volume}}{\text{area of base}} = \frac{4875}{25 \times 15} = 13 \text{ inches.}$$

47. The following method will give the volume of wood composing a rectangular box.

Find the volume of the whole box, including the wood, and from this subtract the volume contained inside the box.

Example. **Find the volume of wood $\frac{1}{2}$ inch thick composing a closed box whose internal dimensions are 3 ft. by 2 ft. by $1\frac{1}{2}$ ft.**

Here the external length = 36 inches + $\frac{1}{2}$ inch at each end
= 37 inches,
external width = 25 ,,
external depth = 19 ,,
Volume = 37 × 25 × 19 = 17575 c. inches.
Internal volume = 36 × 24 × 18 = 15552 c. inches.
Volume of wood = difference = 2023 c. inches.

48. Apply checks to test accuracy in all cases.

In some cases this can be done by slightly altering the dimensions given, in some by working the question by a different method.

In the example above we find

the external surface × thickness = 2103 c. inches

and the internal surface × thickness = 1944 c. inches.

The true volume ought to lie between these two results, as we have found.

This is an instance of one form of rough check.

EXAMPLES VII. c.

1. A cube has an edge of 7 ft. What is its volume?

2. " " " 1 ft. 1 in. " "

3. " " " 4 cm. " "

4. A cuboid measures 1 m. by 5 dm. by 8 cm. What is its volume?

5. What is the depth of a box containing 3 c. ft., if the length and breadth are 2 ft. 8 in. and 1 ft. 6 in.?

6. How many boxes 3 dm. by 2 dm. by 1 dm. can be contained in a case measuring internally 1 m. 5 dm. by 1 m. by 5 dm.?

7. In a case 4 ft. by 2 ft. by 1 ft., how many boxes, each 4 in. by 2 in. by 2 in., can be put?

8. A room 20 ft. long and 12 ft. high contains 4560 c. ft. Find its width.

9. Find also the area of the walls.

10. How many bricks 9 in. by 4 in. by 3 in. are required for a wall 10 yds. long, 8 ft. high and 1 ft. 6 in. thick?

VIII. FRACTIONS.

49. If we divide a thing into 2 equal parts, each part is one half (written $\frac{1}{2}$) of the whole; if into 3 equal parts, each part is one-third (written $\frac{1}{3}$) of the whole.

The thing which is divided may be called the **unit**.

If we divide the unit into 10 equal parts, each part is one-tenth of it; and if we take 7 of these parts we get seven-tenths (written $\frac{7}{10}$) of the unit.

In each of these cases we have a **fraction** of the whole.

A **fraction** is one or more of several equal parts into which a whole is divided.

In any fraction the number written below the line is called the **denominator** and records the number of equal parts into which the whole is divided, while the number above the line is called the **numerator** and records how many of those parts are taken.

The numerator and denominator of a fraction may be called its **members.**

What is the meaning of $\frac{12}{12}$? It means dividing a thing into 12 equal parts and taking all of them. This is, of course, the same as taking the whole of that one thing.

$$\text{Thus } \tfrac{12}{12}=1.$$

[A whole number, *i.e.* one which can be expressed without a fraction, is called an **integer.**]

50. $\frac{1}{20}$ of £1 = 1 shilling. $\frac{20}{20}$ of £1 = 20 shillings = £1.

$\frac{10}{20}$ of £1 = 10 shillings = $\frac{1}{2}$ of £1.

$$\therefore \tfrac{10}{20}=\tfrac{1}{2}.$$

Again, $\frac{5}{20}$ of £1 = 5 shillings = $\frac{1}{4}$ of £1.

$$\therefore \tfrac{5}{20}=\tfrac{1}{4}.$$

If we were to divide 1 foot into 12 equal parts, each part would be 1 inch.

$\frac{3}{12}$ of a foot would be 3 inches, which is the length we should get by dividing the foot into 4 equal parts and taking one of them.

Thus $\frac{3}{12}$ of a foot = 3 inches = $\frac{1}{4}$ of a foot.

$$\therefore \tfrac{3}{12}=\tfrac{1}{4}.$$

This may be illustrated graphically.

FIG. 8.

Here AB represents a length divided into 12 equal parts, AC being one of these.

AB may also be regarded as being divided into 4 equal parts, of which AD is one.

integer = number which have no fractions

denominator = [illegible]

It matters not whether we take the 3 equal parts AC, CE, ED and add them together, or take the part AD which they make up.

Thus $\frac{3}{12}=\frac{1}{4}$.

Another illustration is afforded by areas ruled into squares.

In the figure, ABCD is one column out of the three columns which make up the whole area.

$\therefore$ ABCD $=\frac{1}{3}$ of the whole.

But ABCD contains 4 squares out of the 12 which make up the whole.

$\therefore$ ABCD $=\frac{4}{12}$ of the whole.

$\therefore \frac{4}{12}=\frac{1}{3}$.

FIG. 9.

51. From reasoning of this sort it becomes evident that ***a fraction is not altered in value by multiplying numerator and denominator by any number, nor is it altered by dividing numerator and denominator by any number, provided the number used is the same for numerator and denominator.***

From Fig. 8 we see that

$$\frac{3}{12}+\frac{4}{12}=\text{AD}+\text{DF}=\text{AF}=\frac{7}{12},$$

just as 3 lb. + 4 lb. = 7 lb.

Fractions with the same denominator can be added together by putting the sum of their numerators for numerator and leaving the denominator unchanged.

Similarly subtraction of fractions with the same denominator is performed by subtracting one numerator from the other and leaving the denominator unchanged.

Fractions of Concrete Quantities.

52. EXAMPLE. **Find the value of $\frac{5}{11}$ of £3. 12s. 5d.**

Here we require to divide £3. 12*s*. 5*d*. into 11 equal parts and to take 5 of them, *i.e.* to divide by 11 and multiply the quotient by 5.

£3. 12*s*. 5*d*. ÷ 11 = 6*s*. 7*d*.

6*s*. 7*d*. × 5 = £1. 12*s*. 11*d*.

The same result would be got if we first multiplied by 5 and then divided by 11.

	£.	s.	d.
	3 .	12 .	5
			5
11	18 .	2 .	1
	1 .	12 .	11

Further examples will be found after multiplication and division of fractions.

EXAMPLES VIII. a. (*Oral* 1–75.)

1. What is $\frac{1}{4}$ of a £?
2. What is $\frac{1}{4}$ of a shilling?
3. „ „ yard?
4. What is $\frac{1}{3}$ of a florin?
5. What is $\frac{1}{3}$ of a foot?
6. „ „ half-crown?
7. What is $\frac{3}{4}$ of a £?
8. What is $\frac{3}{4}$ of a shilling?
9. „ „ ton?
10. „ „ yard?

Fig. 10.

AB *is divided into 12 equal parts.*

11. In Fig. 10 what fraction of AB is AK?
12. „ „ „ AD?
13. „ „ „ AG?
14. „ „ „ AH?
15. what fraction of AL is AK?
16. „ „ „ AD?
17. „ „ „ AE?
18. what fraction of AK is AG?
19. „ „ „ AH?
20. Express the fraction $\frac{AE}{AB}$ with denominator 12, and also with denominator 4. What do you thus prove?

Fig. 11.

AB *is divided into 24 equal parts.*

21. In Fig. 11, at what points is AB divided into fourths?
22. „ „ „ „ thirds?
23. „ „ „ „ sixths?
24. What fraction of AB is one of the smallest divisions?
25. How many twenty-fourths are there in AY?
26. „ „ thirds „ „
27. „ „ sixths „ „
28. From the figure express $\frac{1}{3}$ with denominator 24.
29. „ „ „ $\frac{2}{3}$ „ „
30. „ „ „ $\frac{1}{4}$ „ „

31. From the figure express $\frac{3}{4}$ with denominator 24.
32. " " " $\frac{1}{6}$ " "
33. " " " $\frac{5}{6}$ " "
34. What fraction of the whole AB is AQ?
35. " " " QY?
36. " " " AY?
37. By means of the figure, find the value of $\frac{1}{2}+\frac{1}{6}$.
38. By taking XQ from AQ, find the value of $\frac{1}{2}-\frac{1}{6}$.
39. Find the value of $\frac{1}{3}+\frac{1}{6}$.
40. In Fig. 12, by counting squares, find what fraction the area EHKD is of APQD.
41. What fraction of the area ABCD is the area EFCD?
42. What fraction of the area EFCD is the area EHKD?
43. What fraction of the area ABCD is the area EHKD?
44. What fraction of one column, such as AL, is half the area EHKD?
45. Hence find what fraction EHKD is of ABCD.
46. What fraction of APQD is AGHE?
47. Prove from a figure that $\frac{4}{10}=\frac{2}{5}$.
48. " " $\frac{5}{20}=\frac{1}{4}$.
49. " " $\frac{8}{12}=\frac{2}{3}$.
50. " " $\frac{6}{30}=\frac{1}{5}$.

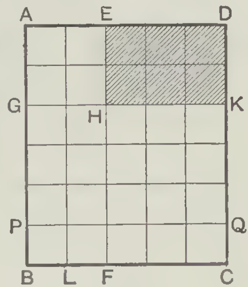


Fig. 12.

Give the value of

51. $\frac{1}{2}$ of 5*s*.
52. $\frac{1}{4}$ of 5*s*.
53. $\frac{1}{3}$ of £1.
54. $\frac{1}{5}$ of £2. 10*s*.
55. $\frac{2}{5}$ of £1.
56. $\frac{3}{10}$ of £1. 10*s*.
57. $\frac{7}{12}$ of 1 foot.
58. $\frac{5}{6}$ of £1.
59. $\frac{3}{28}$ of 1 cwt.
60. $\frac{2}{3}$ of 7*s*. 6*d*.
61. $\frac{2}{5}$ of 12*s*. 6*d*.
62. $\frac{2}{3}$ of £1.
63. $\frac{1}{4}$ of 6*s*. 8*d*.
64. $\frac{1}{8}$ of £3.
65. $\frac{2}{3}$ of a guinea.
66. $\frac{2}{13}$ of £1. 6*s*.
67. $\frac{5}{9}$ of £1. 16*s*.
68. $\frac{1}{5}$ of 12*s*. 6*d*.
69. $\frac{3}{5}$ of 13*s*. 4*d*.
70. $\frac{1}{12}$ of 7*s*.
71. $\frac{5}{12}$ of 7*s*.
72. $\frac{5}{9}$ of 2*s*. 3*d*.
73. $\frac{3}{7}$ of 2*s*. 11*d*.
74. $\frac{1}{11}$ of a mile, in yards.
75. $\frac{1}{4}$ of an acre, in sq. yds.

Find the value of

76. $\frac{1}{7}$ of £4. 13*s*. 4*d*.
77. $\frac{4}{15}$ of £1. 17*s*. 6*d*.
78. $\frac{4}{13}$ of 4 cwt. 7 lb.
79. $\frac{5}{12}$ of 7*s*. 6*d*. + $\frac{3}{4}$ of 5*s*.
80. $\frac{5}{17}$ of 10 cwt. 19 lb.
81. $\frac{4}{5}$ of £3. 12*s*. 6*d*.
82. $\frac{3}{7}$ of 6 yds. 1 ft. 3 in.

Reduction of a Fraction to its Lowest Terms.

53. A fraction may be simplified by removing any factor which is common to numerator and denominator.

Thus $\frac{17}{34}$ is a disguised form of $\frac{1}{2}$.

For $$\frac{17}{34}=\frac{1\times 17}{2\times 17}=\frac{1}{2}.$$

To reduce a fraction to its lowest terms we must remove *all* common factors from numerator and denominator. This may be done in several steps by removing the common factors as they are discovered or in one step **by removing from numerator and denominator their H.C.F.**

EXAMPLE. **Reduce $\frac{2730}{5355}$ to its lowest terms.**

$$\frac{2730}{5355}=\frac{546\times \cancel{5}}{1071\times \cancel{5}}=\frac{182\times \cancel{3}}{357\times \cancel{3}}=\frac{26\times \cancel{7}}{51\times \cancel{7}}=\frac{26}{51};$$

or thus: the H.C.F. $=105$. $\frac{2730}{5355}=\frac{26\times 105}{51\times 105}=\frac{26}{51}$.

In any case, it is well to notice the way in which the cancelling, *i.e.* the removal of common factors, should **not** be done.

Wrong way of cancelling.

$$\frac{\begin{array}{c}26\\ \cancel{182}\\ \cancel{546}\\ \cancel{2730}\end{array}}{\begin{array}{c}\cancel{5355}\\ \cancel{1071}\\ \cancel{357}\\ 51\end{array}}$$

Errors are more easily made in this way, and are more difficult to detect.

EXAMPLES VIII. b. (*Oral* 1–31.)

Reduce to lowest terms:

1. $\frac{5}{10}$. 2. $\frac{4}{6}$. 3. $\frac{6}{9}$. 4. $\frac{3}{6}$. 5. $\frac{4}{12}$. 6. $\frac{8}{10}$.
7. $\frac{6}{8}$. 8. $\frac{9}{12}$. 9. $\frac{12}{18}$. 10. $\frac{10}{15}$. 11. $\frac{14}{35}$. 12. $\frac{15}{25}$.
13. $\frac{2\times 3}{3\times 5}$. 14. $\frac{11}{33}$. 15. $\frac{2\times 7}{35}$. 16. $\frac{15}{40}$. 17. $\frac{13\times 2}{13\times 3}$.
18. $\frac{7\times 11}{13\times 11}$. 19. $\frac{a\times x}{b\times x}$. 20. $\frac{a\times c}{a\times d}$. 21. $\frac{200}{300}$. 22. $\frac{27}{36}$.
23. $\frac{49}{56}$. 24. $\frac{64}{72}$. 25. $\frac{17}{85}$. 26. $\frac{3^2\times 5}{3\times 5^2}$. 27. $\frac{2\times 3\times 5^2}{2^2\times 3\times 5^2}$.
28. $\frac{52}{65}$. 29. $\frac{72}{96}$. 30. $\frac{40}{56}$. 31. $\frac{91}{98}$.

[*Revise and check your work before proceeding to the next example.*]

Reduce to lowest terms:

32. $\frac{315}{729}$.	33. $\frac{247}{323}$.	34. $\frac{550}{726}$.	35. $\frac{304}{665}$.	36. $\frac{598}{1058}$.
37. $\frac{625}{1700}$.	38. $\frac{630}{936}$.	39. $\frac{1407}{2211}$.	40. $\frac{185}{333}$.	41. $\frac{2405}{2639}$.
42. $\frac{2052}{4617}$.	43. $\frac{6409}{7395}$.	44. $\frac{2128}{4655}$.	45. $\frac{8019}{10692}$.	46. $\frac{714285}{999999}$.

54. A **proper fraction** has its numerator less than its denominator.

A fraction whose numerator is greater than its denominator is called an **improper fraction.**

Meaning of an Improper Fraction,

e.g. $\frac{13}{5}$.

$\frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = 1 + 1 + \frac{3}{5} = 2 + \frac{3}{5}$ (written $2\frac{3}{5}$, called 2 and $\frac{3}{5}$).

When we say $\frac{13}{5}$ of a certain unit, we mean twice the unit and $\frac{3}{5}$ of the unit besides. In the form $2\frac{3}{5}$ it is known as a **mixed number.**

To turn an Improper Fraction into a Mixed Number,

e.g. $\frac{25}{7}$.

$$\frac{25}{7} = \frac{7}{7} + \frac{7}{7} + \frac{7}{7} + \frac{4}{7} = 1 + 1 + 1 + \frac{4}{7} = 3\frac{4}{7}.$$

∴ an improper fraction may be reduced to a mixed number by dividing the numerator by the denominator. The quotient is put down as an integer; the remainder forms the numerator of a new fraction, whose denominator is the same as before.

To turn a Mixed Number into an Improper Fraction.

$$3\frac{5}{8} = 1 + 1 + 1 + \frac{5}{8} = \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{5}{8} = \frac{29}{8}$$

$$= \frac{\text{integer} \times \text{den}^{\text{r}} + \text{num}^{\text{r}}}{\text{den}^{\text{r}}}.$$

EXAMPLE. **Turn $5\frac{11}{14}$ into an improper fraction.**

The required improper fraction $= \frac{5 \times 14 + 11}{14} = \frac{81}{14}$.

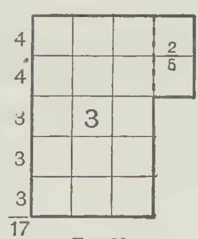


FIG. 13.

Graphic Illustration.

55. The mixed number $3\frac{2}{5}$ may be represented by a figure containing 3 columns and an incomplete one.

The complete columns contain 5 squares each, and the incomplete one contains 2 of such squares.

The figure thus represents $3\frac{2}{5}$ of a column.

But the figure also contains 17 squares, *i.e.* 17 fifths of a column. $\therefore \quad 3\frac{2}{5} = \frac{17}{5}$.

The numbers on the left of the diagram show how many squares there are in the various rows.

EXAMPLES VIII. c. (*Oral* 1–25 and 32–47.)

1. What integer is represented by $\frac{21}{7}$?
2. „ „ „ $\frac{45}{9}$?
3. „ „ „ $\frac{44}{11}$?
4. Express 1 as a fraction with denominator 5.
5. „ 2 „ „ „ 5.
6. „ 2 „ „ „ 7.
7. „ 3 „ „ „ 11.
8. „ 5 „ „ numerator 60.
9. „ 6 „ „ „ 42.
10. „ 4 „ „ „ 28.
11. What fractions with denominator 5 lie between 0 and 1?
12. What is the least fraction exceeding 1 which has denominator 15?
13. How many sevenths make 1?
14. What are 14 sevenths equivalent to?
15. Express 10 sevenths in another form.
16. Between what consecutive integers does $\frac{17}{3}$ lie?
17. „ „ „ $\frac{25}{8}$ „

Reduce the following improper fractions to mixed numbers, simplifying if possible:

18. $\frac{7}{3}$.	**19.** $\frac{7}{4}$.	**20.** $\frac{11}{3}$.	**21.** $\frac{12}{5}$.	**22.** $\frac{23}{7}$.
23. $\frac{27}{8}$.	**24.** $\frac{23}{13}$.	**25.** $\frac{120}{9}$.	**26.** $\frac{58}{15}$.	**27.** $\frac{96}{17}$.
28. $\frac{285}{21}$.	**29.** $\frac{112}{91}$.	**30.** $\frac{355}{113}$.	**31.** $\frac{148}{111}$.	

Reduce the following mixed numbers to improper fractions:

32. $1\frac{2}{3}$.	**33.** $1\frac{3}{4}$.	**34.** $1\frac{5}{7}$.	**35.** $1\frac{11}{34}$.	**36.** $2\frac{3}{4}$.
37. $2\frac{5}{6}$.	**38.** $3\frac{1}{2}$.	**39.** $3\frac{1}{3}$.	**40.** $10\frac{3}{8}$.	**41.** $10\frac{5}{9}$.
42. $3\frac{1}{8}$.	**43.** $4\frac{2}{9}$.	**44.** $7\frac{5}{12}$.	**45.** $8\frac{10}{11}$.	**46.** $12\frac{5}{12}$.
47. $100\frac{17}{19}$.	**48.** $5\frac{111}{130}$.	**49.** $6\frac{13}{72}$.	**50.** $11\frac{19}{95}$.	**51.** $114\frac{3}{5}$.
52. $99\frac{7}{8}$.	**53.** $9\frac{31}{401}$.			

Comparison, Addition and Subtraction of Fractions.

56. A fraction may, without any alteration of value, be changed in form by multiplying its numerator and denominator by any number, provided that the same number is used for both.

This enables us to compare the size of fractions, and to add or subtract them.

For addition or subtraction it is necessary that the fractions should have the same denominator. We can add together 5 thirteenths and 3 thirteenths just as we can add together 5 pence and 3 pence.

Comparison of fractions can also be effected by causing the fractions to have the same denominator.

EXAMPLE. **Compare $\frac{3}{5}$ and $\frac{5}{8}$.**

$$\frac{3}{5}=\frac{3\times 8}{5\times 8}=\frac{24}{40},$$
$$\frac{5}{8}=\frac{5\times 5}{8\times 5}=\frac{25}{40}.$$

25 fortieths must be greater than 24 fortieths.

$$\therefore\ \frac{5}{8}>\frac{3}{5}.$$

57. *Add together $\frac{3}{5}$ and $\frac{5}{8}$.*

$$\frac{3}{5}+\frac{5}{8}=\frac{24}{40}+\frac{25}{40}=\frac{24+25}{40}=\frac{49}{40}=1\frac{9}{40}.$$

Find the difference between $\frac{3}{5}$ and $\frac{5}{8}$.

[The difference between two numbers is expressed by putting the sign $\sim$ between them, indicating that the smaller is to be taken from the larger.]

$$\frac{3}{5}\sim\frac{5}{8}=\frac{24\sim 25}{40}=\frac{1}{40}.$$

58. In comparing, adding, or subtracting fractions we make them have a common denominator. This must be a common multiple of the denominators.

In the preceding example we might have made that common denominator 400, but it would have been unnecessarily large.

To save trouble we make it as small as we can; that is to say, we use the L.C.M. of the denominators.

EXAMPLE 1. **Express $\frac{3}{7}$ with a denominator 63.**

7 must be multiplied by 9 to make it 63.

$\therefore$ 3 must also be multiplied by 9.

Thus, $$\frac{3}{7}=\frac{3\times 9}{7\times 9}=\frac{27}{63}.$$

The mental process gone through, to find the new numerator, is

$$63\div 7=9,\quad 3\times 9=27.$$

NOTE.—In comparing fractions we make use of the obvious fact that, if the denominators are the same, the greatest fraction is the one which has the greatest numerator.

On the other hand, if we reduced fractions to a common *numerator*, the greatest fraction would be the one with the least denominator.

The same facts may be stated as follows: A fraction is increased by increasing its numerator and leaving its denominator unchanged; and a fraction is diminished by increasing its denominator and leaving its numerator unchanged.

EXAMPLE 2. **Find the sum of $\frac{3}{8}$, $\frac{13}{15}$, $\frac{1}{24}$, $\frac{7}{30}$.**

The least common denominator = the L.C.M. of 8, 15, 24, 30 = 120.

$$120 \div 8 = 15, \quad 120 \div 15 = 8, \quad 120 \div 24 = 5, \quad 120 \div 30 = 4.$$

$$\begin{aligned} \tfrac{3}{8}+\tfrac{13}{15}+\tfrac{1}{24}+\tfrac{7}{30} &= \tfrac{3\times 15}{120}+\tfrac{13\times 8}{120}+\tfrac{1\times 5}{120}+\tfrac{7\times 4}{120} \\ &= \tfrac{45}{120}+\tfrac{104}{120}+\tfrac{5}{120}+\tfrac{28}{120} \\ &= \tfrac{45+104+5+28}{120} \\ &= \tfrac{182}{120}=\tfrac{91}{60}=1\tfrac{31}{60}. \end{aligned}$$

The working may be condensed as follows:

$$\begin{aligned} \tfrac{3}{8}+\tfrac{13}{15}+\tfrac{1}{24}+\tfrac{7}{30} &= \tfrac{45+104+5+28}{120}=\tfrac{182}{120} \\ &= \tfrac{91}{60}=1\tfrac{31}{60}. \end{aligned}$$

Addition and Subtraction of Mixed Numbers.

59. EXAMPLE 1.

$$\begin{aligned} 13\tfrac{1}{2}+15\tfrac{2}{3} &= 13+15+\tfrac{1}{2}+\tfrac{2}{3}=28+\tfrac{3+4}{6} \\ &= 28+\tfrac{7}{6}=28+1\tfrac{1}{6}=29\tfrac{1}{6}. \end{aligned}$$

EXAMPLE 2. **Subtract $13\frac{1}{2}$ from $15\frac{2}{3}$,**

The result will not be altered if we take 13 from both the minuend and subtrahend.

Thus
$$\begin{aligned} 15\tfrac{2}{3}-13\tfrac{1}{2} &= 2\tfrac{2}{3}-\tfrac{1}{2}=2+\tfrac{2}{3}-\tfrac{1}{2} \\ &= 2+\tfrac{4-3}{6}=2\tfrac{1}{6}. \end{aligned}$$

EXAMPLE 3. **Subtract $90\frac{14}{15}$ from $178\frac{7}{24}$.**

$$178\tfrac{7}{24}-90\tfrac{14}{15}=88\tfrac{7}{24}-\tfrac{14}{15}.$$

Here the $\frac{14}{15} > \frac{7}{24}$;

$\therefore$ we take $87+1\frac{7}{24}$ instead of $88\frac{7}{24}$ in order to get a fraction greater than $\frac{14}{15}$.

$$87+\tfrac{31}{24}-\tfrac{14}{15}=87+\tfrac{31\times 5-14\times 8}{120}=87\tfrac{43}{120}.$$

60. It is sometimes convenient to change the form of some of the fractions in a question involving addition or subtraction. Thus $\frac{19}{20}$ may be put in the form $1-\frac{1}{20}$.

EXAMPLE. **Simplify $\frac{17}{18}+\frac{23}{24}$.**

$$\begin{aligned} \tfrac{17}{18}+\tfrac{23}{24} &= 1-\tfrac{1}{18}+1-\tfrac{1}{24}=2-\left(\tfrac{1}{18}+\tfrac{1}{24}\right) \\ &= 2-\tfrac{7}{72}=1\tfrac{65}{72}. \end{aligned}$$

61. ***It is important to be able readily to subtract a fraction from 1 or from any whole number.***

If we have to take $\frac{2}{7}$ from 1, we know that, since $1=\frac{7}{7}$, the remainder must be $\frac{5}{7}$.

$10-\frac{3}{11}=9+1-\frac{3}{11}=9\frac{8}{11}$, the intermediate step being omitted after a time.

Oral examples similar to VIII. d. 29–32 given to beginners will soon produce a profitable readiness in dealing with some questions in fractions.

62. If from £1 we had to pay three separate accounts of 4*s*. 5*d*., 7*s*. 1*d*. and 8*s*., we should add the 4*s*. 5*d*., the 7*s*. 1*d*. and the 8*s*. together and deduct the sum, 19*s*. 6*d*., from the £1, if we wished to know how much change would be left. But the actual operations would be as follows:

From £1 pay away 4*s*. 5*d*.;
from the remainder pay 7*s*. 1*d*.;
from the remainder pay 8*s*.

The former operation would be written

$$£1-(4s.\ 5d.+7s.\ 1d.+8s.).$$

The latter operation would be written

$$£1-4s.\ 5d.-7s.\ 1d.-8s.$$

The result would be 6*d*., whichever way we did it.

Stated generally, if a, b, c, d denote numbers or quantities of the same sort, the principle is that

$$a-b-c-d=a-(b+c+d).$$

This is of great use in the simplification of some questions in combined addition and subtraction.

In such a question as "Simplify $\frac{3}{8}+1\frac{3}{4}-\frac{1}{2}-\frac{3}{16}+2\frac{1}{6}-3\frac{1}{12}$," we find the sum of the **negative** quantities, viz. those preceded by a *minus*, and take this sum from the sum of the others, which are the **positive** quantities.

Thus,
$$\begin{aligned}&\tfrac{3}{8}+1\tfrac{3}{4}-\tfrac{1}{2}-\tfrac{3}{16}+2\tfrac{1}{6}-3\tfrac{1}{12}\\&=\left(\tfrac{3}{8}+1\tfrac{3}{4}+2\tfrac{1}{6}\right)-\left(\tfrac{1}{2}+\tfrac{3}{16}+3\tfrac{1}{12}\right)\\&=3+\tfrac{9+18+4}{24}-\left(3+\tfrac{24+9+4}{48}\right)\\&=\tfrac{31}{24}-\tfrac{37}{48}=\tfrac{62-37}{48}=\tfrac{25}{48}.\end{aligned}$$

EXAMPLES VIII. d. *(Oral.)*

1. How many quarters = one half?
2. How many sixths = one third?
3. How many twelfths = one quarter?
4. How many eighths = one half?
5. How many fifteenths = one third?
6. Express one fifth in twentieths.
7. Express three-quarters in twelfths.
8. Express two-fifths in tenths.
9. Express three-eighths in twenty-fourths.
10. Add together three-eighths and five-eighths.
11. Take five-elevenths from unity.

Which is the greater

12. $\frac{2}{3}$ or $\frac{3}{4}$? 13. $\frac{2}{5}$ or $\frac{3}{7}$? 14. $\frac{2}{3}$ or $\frac{5}{8}$? 15. $\frac{5}{6}$ or $\frac{7}{10}$?

Add together

16. $\frac{1}{2}$ and $\frac{1}{4}$. 17. $\frac{1}{3}$ and $\frac{1}{2}$. 18. $\frac{1}{3}$ and $\frac{1}{4}$. 19. $\frac{2}{3}$ and $\frac{1}{2}$.
20. $\frac{1}{3}$ and $\frac{1}{6}$. 21. $\frac{2}{3}$ and $\frac{1}{6}$. 22. $\frac{2}{3}$ and $\frac{5}{6}$. 23. $\frac{3}{4}$ and $\frac{2}{5}$.
24. $\frac{1}{12}$ and $\frac{1}{18}$. 25. $\frac{3}{10}$ and $\frac{5}{12}$.

Find the value of

26. $1-\frac{5}{12}$. 27. $1-\frac{7}{11}$. 28. $\frac{3}{4}-\frac{2}{3}$. 29. $17-1\frac{3}{10}$.
30. $23-2\frac{4}{11}$. 31. $29-6\frac{5}{13}$. 32. $10\frac{1}{4}-3\frac{1}{8}$.
33. $14\frac{3}{4}-6\frac{3}{8}$. 34. $7\frac{2}{3}+1\frac{1}{6}$. 35. $\frac{1}{3}+\frac{1}{4}-\frac{5}{12}$.
36. $\frac{1}{3}+\frac{3}{4}+\frac{5}{6}$. 37. $1\frac{1}{2}+2\frac{2}{3}-3\frac{1}{6}$. 38. $\frac{3}{10}+\frac{7}{100}$.
39. $\frac{7}{10}-\frac{9}{100}$. 40. $\frac{1}{10}+\frac{3}{100}+\frac{7}{1000}$.

EXAMPLES VIII. e.

COMPARISON, ADDITION AND SUBTRACTION.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

1. Express $\frac{4}{5}$ with a denominator 60.
2. ,, $\frac{5}{18}$,, 90.
3. ,, $\frac{2}{37}$,, 148.
4. ,, $\frac{10}{19}$,, 114.
5. ,, $\frac{3}{91}$,, 1001.
6. ,, $\frac{1}{7}$,, 999999.

Arrange in ascending order of magnitude

7. $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}$. 8. $\frac{2}{3}, \frac{3}{5}, \frac{7}{10}$. 9. $\frac{11}{15}, \frac{5}{7}, \frac{8}{11}, \frac{11}{21}$.

10. $\frac{5}{6}, \frac{12}{17}, \frac{7}{9}, \frac{25}{34}$. 11. $\frac{3}{5}, \frac{4}{7}, \frac{6}{11}, \frac{17}{35}$.

Add together

12. $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, 1$. 13. $\frac{3}{8}, \frac{4}{7}$. 14. $\frac{1}{3}, \frac{1}{2}, \frac{5}{6}$. 15. $\frac{2}{3}, \frac{3}{4}, \frac{5}{7}$.

16. $\frac{3}{4}, \frac{5}{6}, \frac{7}{12}, \frac{1}{3}$. 17. $\frac{3}{17}, \frac{3}{34}, \frac{7}{51}$. 18. $1\frac{7}{20}, \frac{21}{640}$. 19. $1\frac{1}{2}, \frac{3}{4}, 2\frac{5}{6}, \frac{7}{8}$.

Simplify

20. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$. 21. $3\frac{1}{2}+\frac{13}{6}+\frac{2}{3}+\frac{1}{4}+\frac{1}{12}$.

22. $1\frac{1}{3}+2\frac{1}{4}+3\frac{1}{8}+4\frac{1}{24}$. 23. $\frac{1}{4}+\frac{1}{6}+\frac{1}{3}+\frac{5}{24}+\frac{109}{144}$.

24. $1\frac{1}{10}+1\frac{1}{35}+\frac{13}{14}$. 25. $1\frac{1}{2}+2\frac{4}{5}+1\frac{14}{25}$.

26. $29\frac{2}{3}+5\frac{1}{6}+10\frac{5}{9}+6\frac{7}{30}$. 27. $4\frac{2}{5}+5\frac{5}{6}+6\frac{7}{8}$.

28. $3\frac{5}{12}+3\frac{7}{30}+\frac{33}{16}+\frac{49}{48}$. 29. $2-\frac{1}{6}$. 30. $6-\frac{3}{5}$. 31. $7-\frac{17}{42}$.

32. $10-\frac{31}{36}$. 33. $\frac{13}{30}-\frac{4}{15}$. 34. $1\frac{3}{10}-\frac{4}{5}$. 35. $3\frac{5}{12}-2\frac{1}{6}$.

36. $6\frac{1}{52}-2\frac{1}{13}$. 37. $2\frac{3}{20}-1\frac{7}{25}$. 38. $6\frac{5}{7}-2\frac{2}{3}$. 39. $4\frac{2}{3}-\frac{5}{7}$.

40. $17\frac{4}{15}-10\frac{5}{24}$. 41. $8\frac{1}{6}-\frac{13}{8}$. 42. $\frac{123}{20}-\frac{17}{5}$. 43. $100-\frac{1}{100}$

44. $1\frac{3}{8}+2\frac{7}{15}-(\frac{3}{4}+\frac{11}{60})$. 45. $1\frac{10}{17}+1\frac{3}{85}-1\frac{5}{34}-\frac{27}{51}$. 46. $1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}$.

47. $\frac{13}{16}-\frac{5}{24}-\frac{1}{2}$. 48. $8\frac{1}{7}-\frac{5}{8}-\frac{1}{6}+\frac{5}{14}$. 49. $\frac{3}{14}-\frac{4}{7}+2-\frac{1}{8}$.

50. $\frac{6}{385}+\frac{8}{11}-\frac{4}{5}-\frac{2}{35}+\frac{1}{7}$. 51. $2+\frac{14}{21}+2-\frac{8}{15}+\frac{49}{10}+3-\frac{1}{6}$.

52. $\frac{12}{15}+4-\frac{7}{14}+\frac{35}{20}$. 53. $2\frac{2}{3}+1\frac{7}{15}-\frac{4}{5}+4\frac{9}{10}-3\frac{1}{2}+2\frac{5}{6}-1\frac{3}{4}$.

54. $10\frac{1}{3}+3\frac{1}{5}+5-\frac{1}{7}-\frac{4}{15}$.

55. What must be added to $\frac{19}{21}$ to make it $1\frac{4}{7}$?

56. What must be added to $\frac{5}{9}$ to make it equal to $\frac{2}{3}+\frac{2}{5}$?

57. A man gave away $\frac{1}{4}$ of his estate to one person, $\frac{1}{8}$ to another, $\frac{1}{16}$ to another and $\frac{1}{32}$ to another. What fraction of it remained to him?

58. From $\frac{2}{3}$ of £1 take $\frac{1}{15}$ of £1, and give the result as a fraction of a £.

Arrange in descending order of magnitude:

59. $\frac{5}{7}, \frac{18}{25}, \frac{7}{10}$. 60. $\frac{17}{20}, \frac{4}{5}, \frac{13}{16}, \frac{3}{4}$. 61. $\frac{31}{39}, \frac{23}{26}, \frac{9}{13}$.

62. Find the values of $\frac{1}{3}$ of £1, $\frac{1}{4}$ of £1, $\frac{1}{6}$ of £1, and find their sum.

63. Find the value of $(\frac{1}{3}+\frac{1}{4}+\frac{1}{6})$ of £1.

64. 1*s.* 8*d.* is $\frac{1}{12}$ of £1; and 7*s.* 6*d.* is $\frac{3}{8}$ of £1. Find by addition what part 9*s.* 2*d.* is of £1.

MULTIPLICATION AND DIVISION OF FRACTIONS.

A Fraction of a Fraction and Multiplication of Fractions.

63. *Consider the meaning of* $\frac{3}{5}$ *of* $\frac{7}{11}$ *of* 18*s*. 4*d*.

$$\tfrac{7}{11} \text{ of } 18s.\ 4d. = (\tfrac{1}{11} \text{ of } 18s.\ 4d.) \times 7 = 20d. \times 7 = 140d.$$

$\therefore$ $\frac{3}{5}$ of $\frac{7}{11}$ of 18*s*. 4*d*. $= \frac{3}{5}$ of 140*d*. $=$ 84*d*.

But $\frac{21}{55}$ of 18*s*. 4*d*. $= \frac{21}{55}$ of 220*d*. $=$ 4*d*. $\times$ 21 $=$ 84*d*.

So $\frac{3}{5}$ of $\frac{7}{11} = \frac{21}{55}$.

Graphic illustration of a Fraction of a Fraction.

64. In Fig. 14 AEFD contains 3 columns, while the whole ABCD contains 8 of the same.

$\therefore$ AEFD $= \frac{3}{8}$ of the whole area.

AEHK contains 5 rows out of the 7 which make up AEFD.

$\therefore$ AEHK $= \frac{5}{7}$ of $\frac{3}{8}$ of the whole.

But AEHK contains 15 squares, and the whole contains 56 squares.

$\therefore$ AEHK $= \frac{15}{56}$ of the whole.

$\therefore$ $\frac{5}{7}$ of $\frac{3}{8} = \frac{15}{56}$.

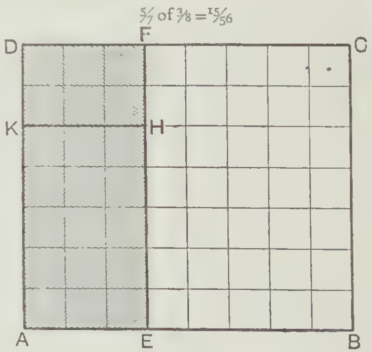


Fig. 14.

Hence we see that the single fraction which is equal to $\frac{5}{7}$ of $\frac{3}{8}$ is the fraction

whose num^r is the product of the num^rs 5 and 3,
and „ denom^r „ „ denom^rs 7 and 8.

Examples. $\frac{2}{3}$ of $\frac{5}{6} = \frac{2 \times 5}{3 \times 6} = \frac{5}{9}$.

$$1\tfrac{2}{3} \text{ of } \tfrac{7}{10} = \tfrac{5}{3} \text{ of } \tfrac{7}{10} = \tfrac{5 \times 7}{3 \times 10} = \tfrac{7}{6} = 1\tfrac{1}{6}.$$

$$2\tfrac{2}{3} \text{ of } 3\tfrac{1}{8} = \tfrac{8}{3} \text{ of } \tfrac{25}{8} = \tfrac{8 \times 25}{3 \times 8} = \tfrac{25}{3} = 8\tfrac{1}{3}.$$

Multiplication of a Fraction by an Integer.

65. (i) $\frac{7}{11} \times 3 = \frac{7}{11} + \frac{7}{11} + \frac{7}{11} = \frac{7+7+7}{11} = \frac{21}{11}$.

(ii) $\frac{7}{20} \times 4 = \frac{7}{20} + \frac{7}{20} + \frac{7}{20} + \frac{7}{20} = \frac{28}{20} = \frac{7}{5}$ (since numerator and denominator may be divided by 4 without altering the value of the fraction).

From this we see that ***multiplication by a whole number may be performed either by multiplying the numerator and leaving the denominator unchanged as in (i), or by dividing the denominator and leaving the numerator unchanged as in (ii).***

A mixed number may be treated either by reducing it to an improper fraction or without doing so.

Thus $$5\tfrac{2}{3} \times 7 = \tfrac{17}{3} \times 7 = \tfrac{119}{3} = 39\tfrac{2}{3};$$
or $$5\tfrac{2}{3} \times 7 = 35 + \tfrac{14}{3} = 35 + 4\tfrac{2}{3} = 39\tfrac{2}{3}.$$

Division of a Fraction by an Integer.

66. (*a*) $\frac{10}{17} \div 5$.

10 shillings $\div 5 = 2$ shillings.

Similarly 10 seventeenths $\div 5 = 2$ seventeenths.

i.e. $\frac{10}{17} \div 5 = \frac{2}{17}$.

(*b*) $\frac{3}{8} \div 5$.

We know that $\frac{3}{8} = \frac{15}{40}$.

$\therefore \quad \frac{3}{8} \div 5 = \frac{15}{40} \div 5 = \frac{3}{40}$.

$\therefore$ ***to divide by an integer we may either divide the numerator as in** (a)**, or multiply the denominator as in** (b).*

A mixed number should be first reduced to an improper fraction. $3\frac{5}{17} \div 11 = \frac{56}{17} \div 11 = \frac{56}{187}$.

When the divisor has factors, we may use these separately.

Thus $$\tfrac{3}{5} \div 6 = \tfrac{3}{5} \div (3 \times 2) = \tfrac{1}{5} \div 2 = \tfrac{1}{10}.$$

67. *Multiply* $\frac{7}{11}$ *by* $\frac{3}{5}$.

According to the definition of multiplication, 13×8 is the sum of 8 quantities when each of these quantities is 13.

$\therefore$ the multiplier must be a whole number.

By the following reasoning we can give a meaning to multiplication by a fraction, *e.g.* $\frac{7}{11} \times \frac{3}{5}$.

If we multiply by 3, we get $\frac{21}{11}$.

But we have used a multiplier 5 times as great as it ought to be;

$\therefore$ to get the correct result we must divide by 5.

$$\frac{21}{11} \div 5 = \frac{21}{55}.$$

Thus $$\frac{7}{11} \times \frac{3}{5} = \frac{21}{55}.$$

$\frac{3}{5} \times \frac{7}{11}$ would give the same result.

The rule deduced from this is, ***Take the product of the numerators for numerator and the product of the denominators for denominator.***

Also $\frac{7}{11} \times \frac{3}{5} \times \frac{3}{4} = \frac{21}{55} \times \frac{3}{4} = \frac{63}{220}$

$$= \frac{\text{product of numerators}}{\text{product of denominators}};$$

and similarly for more than 3 fractions.

NOTE.—It will be observed that $\frac{3}{5}$ of $\frac{7}{11}$ is the same as $\frac{7}{11} \times \frac{3}{5}$ or $\frac{3}{5} \times \frac{7}{11}$.

68. Mixed numbers are to be reduced to improper fractions before the rule for multiplication is applied.

Thus $$7\tfrac{2}{3} \times 5\tfrac{1}{2} = \frac{23}{3} \times \frac{11}{2} = \frac{253}{6} = 42\tfrac{1}{6}.$$

69. Cancelling. The work is often simplified by the removal of factors which are common to numerator and denominator;

e.g. $$\frac{10}{13} \times \frac{39}{75} \times \frac{7}{12} = \frac{\cancel{5} \times \cancel{2} \times \cancel{13} \times \cancel{3} \times 7}{\cancel{13} \times \cancel{5} \times 5 \times \cancel{3} \times 6 \times \cancel{2}} = \frac{7}{30}.$$

When all the factors of numerator and denominator cancel, the result is 1; for the numerator and denominator were equal before the cancelling.

EXAMPLES VIII. f. (*Oral.*)

Simplify

1. $\frac{3}{8} \times 4.$	**2.** $\frac{21}{10} \times 5.$	**3.** $1\frac{3}{5} \times 3.$	**4.** $2\frac{1}{20} \times 2.$
5. $\frac{3}{28} \times 7.$	**6.** $\frac{7}{10} \times 6$	**7.** $8\frac{5}{7} \times 5.$	**8.** $1\frac{4}{15} \times 3.$
9. $7\frac{3}{50} \times 10.$	**10.** $4\frac{7}{15} \times 12.$	**11.** $\frac{3}{4} \div 5.$	**12.** $1\frac{3}{5} \div 8.$
13. $7\frac{1}{7} \div 10.$	**14.** $\frac{3}{20} \div 6.$	**15.** $8\frac{3}{4} \div 7.$	**16.** $9\frac{1}{11} \div 20.$
17. $2\frac{1}{6} \div 39.$	**18.** $7\frac{1}{3} \div 33.$		

[*Give explanations of the steps in the following questions.*]

19. $\frac{1}{3}$ of an estate had to be divided equally among 5 persons. What share of the whole estate did each get?

20. $3\frac{3}{5}$ of a ton was divided amongst 9 men. What fraction of a ton did each man get? How many cwt.? Check the latter answer by using hundredweights at starting.

21. $\frac{1}{4}$ of an estate belonging to A is equally divided between 3 men. One man's share is given back to A. What fraction of the estate remains to A?

EXAMPLES VIII. g.

Simplify

1. $\frac{5}{7}\times\frac{3}{4}$.
2. $\frac{4}{9}\times\frac{3}{8}$.
3. $\frac{12}{25}\times\frac{10}{27}$.
4. $\frac{7}{39}\times\frac{26}{31}$.
5. $\frac{35}{36}\times\frac{27}{28}$.
6. $1\frac{1}{13}\times 3\frac{1}{7}$.
7. $\frac{76}{135}\times\frac{9}{19}$.
8. $\frac{2}{3}$ of $\frac{1}{4}$ of 3.
9. $\frac{3}{7}\times\frac{13}{15}\times\frac{49}{65}$.
10. $\frac{3}{7}$ of $2\frac{1}{9}$ of $2\frac{4}{19}$.
11. $\frac{2}{3}$ of $\frac{9}{10}$ of $\frac{35}{51}\times\frac{17}{21}$.
12. $2\frac{1}{17}\times 1\frac{3}{7}\times 1\frac{1}{33}\times 2\frac{3}{4}\times\frac{41}{125}$.
13. $\frac{4}{5}$ of $\frac{49}{75}$ of $\frac{16}{91}$ of $\frac{125}{148}$ of $\frac{111}{112}$.
14. $1\frac{4}{9}\times\frac{3}{8}\times 1\frac{3}{4}\times\frac{4}{21}\times\frac{2}{5}\times\frac{10}{13}$.
15. $\frac{12}{13}\times 2\frac{2}{3}\times 1\frac{1}{25}\times 1\frac{11}{64}$.
16. $\frac{2}{7}$ of $\frac{69}{95}$ of $\frac{49}{58}$ of $\frac{114}{161}$.
17. $\frac{7}{3}$ of $11\frac{1}{9}\times 4\frac{3}{100}\times\frac{27}{1001}$.
18. $3\frac{1}{4}\times(3-\frac{11}{13})\times\frac{5}{7}\times(1-\frac{11}{15})$.
19. $8\frac{1}{8}\times(1-\frac{1}{91})\times\frac{14}{27}$.
20. $(5-\frac{5}{18})(1+\frac{10}{17})(1-\frac{13}{15})$.
21. $(\frac{1}{2}-\frac{1}{9})\times 1\frac{1}{10}\times(2-\frac{1}{14})\times 2\frac{1}{2}\times 2\frac{2}{7}$.
22. $(\frac{3}{7})^2\times(\frac{19}{9})^2\times(\frac{42}{19})^2$.

70. The **reciprocal** of a fraction is the fraction inverted, *i.e.* with numerator and denominator interchanged.

The reciprocal of $\frac{7}{9}$ is $\frac{9}{7}$.

The name may be applied to an integer.

The reciprocal of 16 is the reciprocal of $\frac{16}{1}$, *i.e.* $\frac{1}{16}$.

Division of Fractions.

71. *Divide* $\frac{11}{13}$ *by* $\frac{3}{5}$. The meaning of this is, "By what must we multiply $\frac{3}{5}$ to make it $\frac{11}{13}$?"

Multiplying by $\frac{5}{3}$ will make it 1, and a further multiplication by $\frac{11}{13}$ will make it $\frac{11}{13}$.

The required answer then is $\frac{11}{13}\times\frac{5}{3}$.

Here the divisor $\frac{3}{5}$ has been inverted and the sign of division replaced by that of multiplication. Similarly for other fractions.

Hence the rule, ***"To divide by a fraction, invert the divisor and multiply,"*** or ***"To divide by a fraction, multiply by its reciprocal."***

72. It is necessary to bear in mind the convention that the signs $\times$, "of" where equivalent to $\times$, and $\div$ bind quantities more closely together than $+$ and $-$.

Thus the expression $\frac{1}{2}$ of $\frac{1}{3}+\frac{7}{10}\div\frac{3}{5}+\frac{9}{17}\times\frac{5}{8}$ has the same meaning as $(\frac{1}{2}$ of $\frac{1}{3})+(\frac{7}{10}\div\frac{3}{5})+(\frac{9}{17}\times\frac{5}{8})$.

In such a case as $\frac{3}{5}\div\frac{7}{8}$ of $\frac{3}{11}$ it is understood that $\frac{7}{8}$ of $\frac{3}{11}$ is one quantity and might as well be enclosed in brackets; whereas if it were $\frac{3}{5}\div\frac{7}{8}\times\frac{3}{11}$, the meaning would be the same as if it were $(\frac{3}{5}\div\frac{7}{8})\times\frac{3}{11}$.

Thus $\frac{3}{5}\div\frac{7}{8}$ of $\frac{3}{11}=\frac{3}{5}\div(\frac{7\times3}{8\times11})=\frac{3}{5}\times\frac{8\times11}{7\times3}=\frac{88}{35}=2\frac{18}{35}$.

Whereas $\frac{3}{5}\div\frac{7}{8}\times\frac{3}{11}=\frac{3}{5}\times\frac{8}{7}\times\frac{3}{11}=\frac{72}{385}$.

EXAMPLES VIII. h.

[*Revise your work before proceeding to the next Example.*]

Simplify:

1. $\frac{5}{12}\div\frac{3}{10}$.
2. $\frac{3}{14}\div\frac{5}{7}$.
3. $\frac{1}{8}\div1\frac{5}{16}$.
4. $1\frac{1}{5}\div\frac{2}{15}$.
5. $\frac{5}{18}\div3\frac{7}{12}$.
6. $1\frac{2}{5}\div8\frac{3}{5}$.
7. $7\frac{1}{2}\div2\frac{7}{9}$.
8. $5\frac{1}{10}\times\frac{4}{9}\div28\frac{1}{3}$.
9. $\frac{1}{4}$ of $2\frac{2}{7}\div\frac{3}{5}$.
10. $(2-\frac{13}{8})\div\frac{27}{56}$.
11. $(\frac{1}{3}-\frac{1}{17})\div\frac{105}{136}$.
12. $\frac{5}{13}$ of $(1-\frac{2}{11})\div\frac{100}{1001}$.
13. $(1\frac{1}{4}+2\frac{3}{4}+19\frac{2}{5})\div3\frac{1}{4}$.
14. $(3\frac{1}{2}+5\frac{5}{6})\times\frac{3}{14}\div(\frac{3}{7}\times1\frac{5}{8})$.
15. $17\div(2-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16})$.
16. $(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32})\div2\frac{2}{15}$.
17. $2\frac{13}{25}\times(\frac{1}{2}+\frac{4}{7})\div\frac{27}{40}$.
18. $3\frac{1}{18}\times(2-\frac{5}{11})\div5\frac{5}{16}$.
19. $(2\frac{3}{5}+\frac{3}{10})\div(1\frac{1}{2}+1\frac{1}{4}+\frac{7}{8})$.
20. $(\frac{2}{7}+2\frac{8}{63}+1\frac{11}{21})\div(\frac{3}{5}+1\frac{1}{6}-\frac{11}{15})$.
21. $\frac{1}{13}\times(\frac{1}{5}+\frac{1}{6}+\frac{1}{60}+\frac{13}{42}-\frac{1}{45})\div(\frac{3}{22}$ of $2\frac{5}{14}-\frac{1}{4})$.
22. $5-\frac{15}{8}\div\frac{27}{56}$.
23. $(5-\frac{15}{8})\div\frac{27}{56}$.
24. $\frac{5}{13}$ of $2-\frac{3}{11}$.
25. $\frac{5}{13}$ of $(2-\frac{3}{11})$.
26. $5\frac{2}{3}\div2-\frac{1}{2}$.
27. $5\frac{2}{3}\div(2-\frac{1}{2})$.
28. $(\frac{1}{2}+\frac{1}{3})\div(\frac{1}{2}+\frac{1}{4})$.
29. $\frac{1}{2}+\frac{1}{3}\div\frac{1}{2}+\frac{1}{4}$.
30. $\frac{1}{2}+\frac{1}{3}\div(\frac{1}{2}+\frac{1}{4})$.
31. $(2\frac{5}{6}+7\frac{1}{16})\div(\frac{1}{5}+3\frac{1}{6}+2\frac{1}{3})$.
32. $2\frac{5}{6}+7\frac{1}{16}\div\frac{1}{5}+3\frac{1}{6}+2\frac{1}{3}$.
33. $2\frac{5}{6}+7\frac{1}{16}\div(\frac{3}{5}+3\frac{1}{6})+2\frac{1}{3}$.

Applications of Addition and Subtraction of Fractions. Problems on Time and Work.

73. EXAMPLE 1. **One man, A, can do a piece of work in 3 hours, another, B, can do the same piece in 4 hours; how much can the two working together do in an hour?**

A can do $\frac{1}{3}$ of the work in an hour,

B ,, $\frac{1}{4}$,, ,,

$\therefore$ the work done by A and B together in an hour $=\frac{1}{3}+\frac{1}{4}$

$=\frac{7}{12}$ of the whole.

How long do A and B together take for the whole work?

They do 7 twelfths in an hour.

$\therefore$ they do one twelfth in $\frac{1}{7}$ of an hour.

$\therefore$ they do $\frac{12}{12}$, *i.e.* the whole, in $\frac{12}{7}$ of an hour; *i.e.* in $1\frac{5}{7}$ hr.

Or thus. Suppose they together take x hours.

Then $\frac{1}{x}$ is the part they do together in 1 hour.

$$\therefore \frac{1}{x} = \text{the sum of the parts done by A and by B in 1 hour}$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}.$$

$$\therefore x = \frac{12}{7} = 1\frac{5}{7}.$$

$\therefore$ the time required $= 1\frac{5}{7}$ hr.

Example 2. **Two men can do a piece of work in 3 and 2 days respectively. How long do they take if they work together?**

The accompanying figure illustrates this.

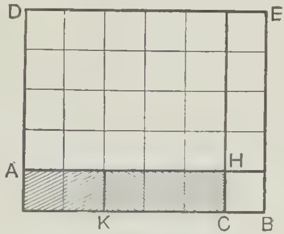


Fig. 15.

The whole work is represented by the rectangle AB.

The first man's daily work is represented by AK, *i.e.* $\frac{1}{3}$ of AB.

The second man's daily work is represented by KH, *i.e.* $\frac{1}{2}$ of AB.

Together they do AC in a day.

In 5 days they do CD. In 1 more they do CE.

$\therefore$ in 6 days they do DB, *i.e.* in 6 days they do the work 5 times over.

$\therefore$ the whole is done in $\frac{6}{5}$ of a day.

Example 3. **A can mow a field in 6 hours, A and B together can do it in 4 hours; how long would B alone take?**

A alone can mow $\frac{1}{6}$ of the field in an hour.

A and B together ,, $\frac{1}{4}$,, ,,

The part which B alone can mow in an hour $= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ of the field.

$\therefore$ B alone takes 12 hours to mow the whole field.

Example 4. **A bath can be filled by one tap in 20 minutes, by another in half an hour, and can be emptied by a waste-pipe in 15 minutes. If all the pipes are opened, how long will it take to fill?**

Every minute the 1st pipe puts in $\frac{1}{20}$ of what the bath can contain.

,, ,, 2nd ,, $\frac{1}{30}$,, ,, ,,

,, ,, 3rd pipe takes out $\frac{1}{15}$,, ,, ,,

$\therefore$ the fraction of the whole capacity put in every minute while all three pipes are open $= \frac{1}{20} + \frac{1}{30} - \frac{1}{15} = \frac{3+2-4}{60} = \frac{1}{60}$.

$\therefore$ the whole bath is filled in 60 minutes.

EXAMPLES VIII. k.

PROBLEMS ON TIME AND WORK. ILLUSTRATIONS OF THE USE OF FRACTIONS.

[*Questions* 1–17 *may be taken orally.*]

1. If a pipe fills a cistern in 3 hours:
what portion will it fill in one hour?
" " " 2 hours?
" " " half an hour?

2. If a pipe fills $\frac{1}{6}$ of a cistern in one hour:
how long will it take to fill the cistern?
" " " half the cistern?
" " " one-third of " ?

3. If one pipe fills $\frac{1}{2}$ of a cistern in an hour, and a second fills $\frac{1}{4}$ in an hour, how much will they fill in an hour when running together?

4. If a pipe fills $\frac{5}{6}$ of a cistern in one hour, how long will it take to fill the cistern?

5. If a pipe fills $\frac{1}{2}$ of a cistern in an hour, and a second empties $\frac{1}{4}$ in an hour, what fraction of the cistern will be filled in an hour when both pipes are open?

6. When a pipe has been open half an hour, $\frac{1}{8}$ of a cistern is filled. Another pipe is then opened, which, running alone, would empty $\frac{1}{12}$ of the cistern in half an hour; how much of the cistern is full one hour from the start?

7. The content of a cistern is used three times over each day: how many hours a day must a tap be kept running which will fill the cistern in 6 hours?

8. One pipe fills a cistern in 12 minutes, and another fills it in 6 minutes: how much will they fill in one minute when running together?

9. One pipe will fill a tank in $5\frac{1}{2}$ hours; how many pipes of the same size must be used to fill the tank in half an hour?

10. If a man does $\frac{1}{4}$ of a piece of work in one day, how long does he take to do the whole piece?

11. If a man does $\frac{2}{3}$ of a piece of work in a day, how long does he take to do the whole piece?

12. If a man does $\frac{1}{3}$ and a boy does $\frac{1}{4}$ of a piece of work in one day, how much will they do in a day when working together?

13. A can do a piece of work in 6 days; A and B working together can do it in 4 days. How much of the work can B do in a day?

14. A does a piece of work in 8 days, which A and B, working together, can do in $1\frac{1}{3}$ days. How much can B do in a day? How long will B take to do the work?

15. If one man can do a piece of work in 8 days, how many men must be employed to do it in 2 days?

16. If 3 men take 2 days to do a piece of work, how long will 2 men take to do it?

17. If 4 men do a piece of work in 9 days, how long will 3 men take to do it?

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Revise your work before proceeding to the next example.*

18. A can mow a field in three days, B can do it in four days. How much is mown at the end of 1 day if both work at it?

19. A can do a piece of work in 8 hours, A and B together in 5 hours. What part can B alone do in 1 hour?

20. A can mow a lawn in 30 minutes, B in 60 minutes. How long will they take if both work at it?

21. Two men, A and B, can separately do a piece of work in 8 hrs. and 12 hrs., and a boy, C, can do it in 24 hours. How long do they take if all work together?

22. Three men take respectively 15, 16, 18 hours over a piece of work. How much is done by them together in 1 hour?

23. A cistern is supplied by 3 taps requiring 10, 12, 15 minutes respectively to fill it. How long does it take to fill when all are open?

24. Three equal men working together can do a piece of work in 10 days. One man is replaced by a boy who does only half a man's work. How long do they then take over it?

25. Two taps fill a cistern separately in 3 hrs. and 5 hrs. If both are turned on together, what part do they fill in 1 hour?

26. If two taps took 44 and 77 minutes respectively to fill a cistern, how long would it take to fill with both running?

27. A man who can dig a garden in 33 hours works with a boy who would take 88 hours. What time do they take?

28. A piece of work which occupies two men for 35 minutes could be done by one of them in an hour. How long would the other take by himself?

29. A, B and C together take 4 hours over a piece of work, A and C 6 hours, B and C 8 hours over the same amount of work. How long would each take by himself?

30. Two men together take 9 hours over that which would occupy one of them 21 hours. How long would the other take?

31. Two pipes take 5 and 10 minutes separately to fill a tank; and a waste-pipe would empty it in a quarter of an hour. Find the time taken to fill it with all open together.

32. It is found that a bath can be filled by two taps running together in 21 minutes; but, when the waste pipe is open as well as these, the whole bath is emptied in 28 minutes. How long would the emptying take if the two supply taps were closed?

Complex Fractions.

74. Principle involved: ***The value of a fraction is not altered by multiplying its two members by the same number.***

In any example, after simplifying the numerator and denominator, we get a fraction such as $\dfrac{\frac{7}{15}}{\frac{31}{18}}$.

To reduce this we multiply numerator and denominator by a number which will make each of them an integer. 90, the L.C.M. of 15 and 18, will be the most convenient.

$$\frac{\frac{7}{15}\times 90}{\frac{31}{18}\times 90}=\frac{\frac{7}{15}\times 15\times 6}{\frac{31}{18}\times 18\times 5}=\frac{42}{155}.$$

EXAMPLE. **Simplify** $\dfrac{2\frac{1}{4}-\frac{1}{3}\text{ of }3\frac{2}{3}}{\frac{2}{5}\text{ of }1\frac{2}{3}+\frac{13}{72}}$.

$$\text{The fraction}=\frac{2\frac{1}{4}-\frac{11}{9}}{\frac{2}{3}+\frac{13}{72}}=\frac{\frac{9}{4}-\frac{11}{9}}{\frac{2}{3}+\frac{13}{72}}=\frac{\frac{37}{36}}{\frac{61}{72}}$$

$$=\frac{\frac{37}{36}\times 72}{\frac{61}{72}\times 72}=\frac{74}{61}=1\tfrac{13}{61}.$$

75. Care must be taken to make the main line of a complex fraction quite distinct by drawing it thicker than the other lines.

An instance will show how neglect of this makes it impossible in some cases to decide what is the value of the fraction;

e.g. $$\frac{2}{\frac{3}{7}}=\frac{2\times 7}{\frac{3}{7}\times 7}=\frac{14}{3}=4\tfrac{2}{3}.$$

$$\frac{\frac{2}{3}}{7}=\frac{\frac{2}{3}\times 3}{7\times 3}=\frac{2}{21}.$$

No meaning can be assigned to $\frac{\frac{2}{3}}{7}$ where there is nothing to show which is the main line of the fraction.

We have seen that complex fractions can be simplified by the application of the principle of multiplying numerator and denominator by a common quantity;

e.g. $$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{\frac{a}{b}\times bd}{\frac{c}{d}\times bd}=\frac{ad}{bc}.$$

The same result would be obtained by dividing $\frac{a}{b}$ by $\frac{c}{d}$;

$$e.g.\quad \frac{\frac{3}{8}}{\frac{5}{7}} = \frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \times \frac{7}{5} = \frac{21}{40}.$$

General Principles in Fractions.

76. Reducing to a common denominator, for comparison, addition or subtraction, is the same in principle as the process of adding or subtracting any concrete quantities: ***they must be of the same denomination before they can be added or subtracted.***

(1) Just as $5d. + \frac{1}{3}s. = 5d. + 4d. = 9d.$,

so $\frac{5}{12} + \frac{1}{3} = 5$ twelfths $+ 4$ twelfths $= 9$ twelfths.

The treatment of mixed numbers, by reduction to improper fractions, is the same as ***reducing to a common denominator and then adding.***

(2) Just as $2s. + \frac{5}{12}s. = 24d. + 5d. = 29d.$,

so $2\frac{5}{12} = 24$ twelfths $+ 5$ twelfths $= 29$ twelfths.

Simplification of complex fractions, after the numerator and denominator have been reduced by addition, multiplication, etc., is an application of the principle that ***the numerator and denominator of a fraction may be multiplied by any (the same) number without alteration of the value of the fraction.***

The same principle (with division used instead of multiplication) leads at once to cancelling and to reduction of a fraction to lowest terms.

In such practical questions on addition and subtraction as Time and Work Problems the principle is that the ***time must be the same before the work done by different agents can be added together;*** *e.g.* we must add together the amounts of work done by them in 1 hour, or perhaps, in another question, the work done in 1 day.

Fractions of Concrete Quantities. No doubt as to a possible way of finding $\frac{5}{12}$ of 3 cwt. 1 qr. 8 lb. can be felt by even a beginner when he knows the fundamental meaning of a fraction: for he knows that 'finding $\frac{5}{12}$' means dividing into 12 equal parts and taking 5 of these (*i.e* 5 times one of these twelfths).

EXAMPLES VIII. 1. (1–16 *Oral.*)

Simplify

1. $\dfrac{\frac{1}{4}}{\frac{1}{2}}$. 2. $\dfrac{\frac{3}{8}}{\frac{5}{8}}$. 3. $\dfrac{1\frac{2}{5}}{3\frac{1}{5}}$. 4. $\dfrac{\frac{2}{7}}{\frac{6}{7}}$. 5. $\dfrac{\frac{1}{6}}{\frac{2}{3}}$. 6. $\dfrac{1\frac{1}{6}}{3\frac{1}{3}}$.

7. $\dfrac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}}$. 8. $\dfrac{\frac{1}{6}}{\frac{1}{2}+\frac{1}{3}}$. 9. $\dfrac{3\frac{1}{2}}{14}$. 10. $\dfrac{5\frac{1}{2}}{33}$. 11. $\dfrac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}$.

12. $\dfrac{3\frac{1}{2}}{1-\frac{1}{2}}$. 13. $\dfrac{1\frac{1}{3}}{1\frac{7}{9}}$. 14. $\dfrac{2\frac{7}{8}}{8\frac{5}{8}}$. 15. $\dfrac{1-\frac{1}{3}}{1-\frac{1}{6}}$. 16. $\dfrac{\frac{3}{8}}{1\frac{5}{16}}$.

[*Revise each example before proceeding to the next.*]

17. $\dfrac{1\frac{5}{22}}{2-\frac{4}{11}}$. 18. $\dfrac{3-\frac{1}{17}}{3\frac{1}{8}}$. 19. $\dfrac{\frac{8}{9}\text{ of }1\frac{1}{2}\text{ of }\frac{12}{13}}{\frac{1}{4}-\frac{1}{52}}$. 20. $\dfrac{\frac{2}{11}+\frac{1}{3}}{6-\frac{1}{3}}$.

21. $\dfrac{2\frac{1}{2}+\frac{1}{6}}{3\frac{2}{3}-\frac{1}{6}}$. 22. $\dfrac{2\frac{1}{2}-1\frac{5}{6}}{1\frac{1}{2}+\frac{5}{6}}$. 23. $\dfrac{3\frac{1}{4}-1}{3\frac{1}{4}\times 3\frac{1}{4}-1}$.

24. $\dfrac{2\frac{1}{2}\times 2\frac{1}{2}\times 2\frac{1}{2}-1}{2\frac{1}{2}\times 2\frac{1}{2}-1}$. 25. $\dfrac{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$. 26. $\dfrac{3\frac{1}{2}+\frac{1}{4}}{\frac{3}{4}+\frac{3}{16}}$.

27. $\dfrac{\frac{1}{8}-\frac{1}{18}+\frac{1}{24}}{\frac{1}{8}+\frac{1}{18}+\frac{1}{24}}$. 28. $\dfrac{4}{7\frac{1}{16}}+\dfrac{1}{3\frac{2}{37}}$.

29. $\dfrac{3\frac{1}{3}\text{ of }\frac{2}{5}-\frac{1}{12}}{\frac{17}{18}-\frac{3}{4}\text{ of }1\frac{2}{9}}$. 30. $\dfrac{7\frac{1}{2}}{6\frac{3}{4}}-\dfrac{5\frac{1}{3}}{7\frac{1}{2}}$.

31. $\dfrac{2\frac{1}{3}+\frac{3}{4}}{3\frac{1}{4}+2\frac{5}{12}}+\dfrac{1}{3\frac{1}{11}}$. 32. $\dfrac{5\frac{1}{2}-3\frac{3}{8}}{1\frac{1}{2}+1\frac{1}{3}}+\dfrac{1\frac{1}{8}+2\frac{5}{6}}{3\frac{1}{3}-1\frac{3}{4}}$.

33. $\dfrac{23\frac{4}{15}\div 12}{7\frac{1}{6}+8\frac{2}{5}-9\frac{3}{4}}$. 34. $\dfrac{2\frac{1}{3}+\frac{3}{4}}{15\frac{1}{2}-1\frac{5}{8}}+1\frac{2}{3}\text{ of }2\frac{4}{15}$.

35. $\frac{4}{3}\text{ of }\dfrac{13\frac{2}{3}\div 1\frac{1}{2}}{1-\frac{1}{6}\text{ of }\frac{2}{3}\text{ of }\frac{4}{5}}$. 36. $\dfrac{3\frac{2}{5}+4\frac{1}{15}}{3\frac{2}{3}-2\frac{8}{9}}\div\dfrac{3\frac{1}{7}\text{ of }3\frac{1}{5}}{2\frac{3}{4}\text{ of }3\frac{3}{7}}$.

37. $1\frac{4}{7}-\frac{3}{4}\left(\frac{1}{15}+\frac{1}{5}\right)+\frac{3}{104}\div\dfrac{2\frac{3}{16}}{3-\frac{5}{6}}$.

38. If $a=2\frac{7}{15}+1\frac{9}{20}-3\frac{1}{24}$, and $b=(9\frac{1}{4}\times\frac{25}{36})\div(6\frac{1}{4}\times 1\frac{16}{21})$, simplify the fraction $\dfrac{a\sim b}{a}$.

Simplify

39. $\dfrac{8\frac{3}{4}}{5\frac{5}{8}}\times\dfrac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}}$. 40. $\dfrac{1\frac{2}{3}+1\frac{1}{4}-\frac{5}{12}}{1\frac{2}{3}-1\frac{1}{4}+\frac{5}{12}}\div\dfrac{(1\frac{2}{3}+1\frac{1}{4})\times\frac{5}{12}}{1\frac{2}{3}\times(1\frac{1}{4}-\frac{5}{12})}$.

41. $\dfrac{3\frac{1}{2}+\frac{4}{5}}{9-\frac{2}{5}}\times\dfrac{3\frac{1}{2}-\frac{4}{5}}{9+\frac{2}{5}}\times\dfrac{2-\frac{1}{19}}{7-\frac{1}{16}}$. 42. $\dfrac{3\frac{2}{3}+8\frac{1}{9}}{12\frac{1}{12}-9\frac{7}{8}}\div\dfrac{3\frac{1}{7}\text{ of }9\frac{7}{9}}{3\frac{13}{14}\text{ of }4\frac{2}{5}}$.

43. $\dfrac{1\frac{2}{9}\times(2\frac{1}{2}+3\frac{5}{6})}{2\frac{1}{9}\times(7-\frac{2}{3}-\frac{5}{6})}\div\dfrac{\frac{1}{2}-\frac{23}{26}\text{ of }\frac{1}{2}}{\frac{3}{4}-\frac{12}{13}\text{ of }\frac{3}{4}}$. 44. $\dfrac{3\frac{1}{5}+4\frac{1}{3}-(5\frac{1}{4}\times\frac{6}{7})}{3\frac{1}{2}-(4\frac{1}{3}\times\frac{7}{26})+(4\frac{1}{5}\times\frac{3}{7})}$.

45. $\dfrac{4\frac{2}{3}}{6\frac{6}{7}} \times 2\frac{2}{7} \times (5 - 3\frac{2}{7}) \div (1\frac{1}{2} + \frac{1}{10})$.   46. $1 - \frac{1}{6}$ of $\dfrac{1}{1 - \frac{13}{15}}$ of $\dfrac{\frac{4}{7} \text{ of } \frac{2}{9}}{\frac{4}{7} - \frac{2}{9}} \div \dfrac{\frac{1}{3} - \frac{1}{8}}{\frac{1}{3} + \frac{1}{8}}$.

47. $\frac{2}{139}\{(14 - \frac{2}{5}) \div 6\frac{4}{5} - \frac{3}{5} - \frac{1}{10}$ of $(\frac{2}{5} - \frac{3}{10})\}$.

48. $\{(\frac{1}{2})^3 + (\frac{1}{3})^3 + (\frac{1}{6})^3 - 3 \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{6}\} \div \{(\frac{1}{2})^2 + (\frac{1}{3})^2 + (\frac{1}{6})^2 - \frac{1}{6} - \frac{1}{12} - \frac{1}{18}\}$.

Fractions of Concrete Quantities.

77. EXAMPLE 1. **Find the value of $\frac{5}{56}$ of £1.**

$$\tfrac{5}{56} \text{ of } £1 = \tfrac{5}{56} \text{ of } 20s. = \tfrac{5 \times 5 \times 4}{14 \times 4} = \tfrac{25}{14} = 1\tfrac{11}{14}s.$$

$$\tfrac{11}{14} \text{ of } 1s. = \tfrac{11 \times 12}{14}d. = \tfrac{11 \times 6 \times 2}{7 \times 2} = \tfrac{66}{7} = 9\tfrac{3}{7}d.$$

$$\therefore \tfrac{5}{56} \text{ of } £1 = 1s.\ 9\tfrac{3}{7}d.$$

Or thus:

$$\begin{array}{r|l} & £1 \\ & \;\;5 \\ \hline 8 & \;\;5 \\ \hline 7 & 12s.\ 6d. \\ \hline & 1s.\ 9\frac{3}{7}d. \end{array}$$

EXAMPLE 2. **Find $\frac{4}{7}$ of 3 yds. 1 ft. 6 in.**

$$\tfrac{4}{7} \text{ of } 3\tfrac{1}{2} \text{ yds.} = \tfrac{4}{7} \text{ of } \tfrac{7}{2} = 2 \text{ yds.}$$

EXAMPLE 3. **Find the value of $2\frac{1}{11}$ of £7. 6s. 8d.**

$$2\tfrac{1}{11} \text{ of } £7.\ 6s.\ 8d. = \tfrac{23}{11} \text{ of } £7\tfrac{1}{3} = \tfrac{23}{11} \times \tfrac{22}{3} = \tfrac{46}{3}$$
$$= 15\tfrac{1}{3} = £15.\ 6s.\ 8d.$$

EXAMPLE 4. **Find the value of £16. 8s. $1\frac{1}{2}$d. $\times\, 8\frac{6}{35}$.**

$$\begin{array}{r|rcrcr} & £ & & s. & & d. \\ & 16 & . & 8 & . & 1\frac{1}{2} \\ & & & & & 6 \\ \hline 7 & 98 & . & 8 & . & 9 \\ \hline 5 & 14 & . & 1 & . & 3 \\ \hline & 2 & . & 16 & . & 3 \end{array}$$

£16. 8s. $1\frac{1}{2}$d, $\times 8$ = £131. 5s. 0d.
£16. 8s. $1\frac{1}{2}$d. $\times \frac{6}{35}$ = £2. 16s. 3d.
$\therefore$ £16. 8s. $1\frac{1}{2}$d. $\times 8\frac{6}{35}$ = £134. 1s. 3d.

EXAMPLE 5. **Multiply £26. 2s. $7\frac{3}{4}$d. by $\frac{5}{33}$.**

$$\begin{array}{r|rcrcr} & £ & & s. & & d. \\ & 26 & . & 2 & . & 7\frac{3}{4} \\ & & & & & 5 \\ \hline 11 & 130 & . & 13 & . & 2\frac{3}{4} \\ \hline 3 & 11 & . & 17 & . & 6\frac{35}{44} \\ \hline & 3 & . & 19 & . & 2\frac{35}{132} \end{array}$$

The result here would generally be expected to be given "to the nearest farthing" or perhaps "to the nearest penny," instead of its being encumbered with such a fraction of a penny as $\frac{35}{132}$.

$$\tfrac{35}{132} \text{ of } 1d. = \tfrac{35}{132} \text{ of } 4 \text{ farthings} = 1\tfrac{2}{33} \text{ of a farthing.}$$

Thus the answer *to the nearest farthing* is £3. 19*s.* $2\frac{1}{4}$*d.*

" " *to the nearest penny* is £3. 19*s.* 2*d.*

It will be found later that a question of this sort is most easily treated by using decimals of a penny.

EXAMPLE 6. **Divide £25. 6s. 9d. by $13\frac{4}{7}$.**

$13\frac{4}{7}$ reduced to one fraction, and inverted, becomes $\frac{7}{95}$.

$$\begin{array}{r|l}
 & \text{£.}\quad s.\quad d. \\
 & 25 \;.\; 6 \;.\; 9 \times \tfrac{7}{95} \\
 & \qquad\qquad 7 \\
5 & 177 \;.\; 7 \;.\; 3 \\
19 & 35 \;.\; 9 \;.\; 5\tfrac{2}{5}\;(\text{£}1.\ 17s.\ 4\tfrac{7}{95}d. \\
 & 19 \\
 & \overline{16} \\
 & 20 \\
 & \overline{329} \\
 & 19 \\
 & \overline{139} \\
 & 133 \\
 & \overline{6} \\
 & 12 \\
 & \overline{77\tfrac{2}{5}} \\
 & 76 \\
 & \overline{1\tfrac{2}{5}}
\end{array}
\qquad \frac{1\frac{2}{5}}{19} = \frac{7}{95}.$$

Or

$$\begin{array}{r|l}
 & \text{£.}\quad s.\quad d. \\
 & 25 \;.\; 6 \;.\; 9 \\
 & \qquad\qquad 7 \\
95 & 177 \;.\; 7 \;.\; 3\;(\text{£}1.\ 17s.\ 4\tfrac{7}{95}d. \\
 & 95 \\
 & \overline{82} \\
 & 20 \\
 & \overline{1647} \\
 & 95 \\
 & \overline{697} \\
 & 665 \\
 & \overline{32} \\
 & 12 \\
 & \overline{387} \\
 & 380 \\
 & \overline{7}
\end{array}$$

EXAMPLE 7. **Find the value of $\frac{5}{24}$ of £1 + $\frac{3}{8}$ of 1s. + $\frac{7}{18}$ of 2s. 6d.**

The simplest way to do this example is to find the value of each of the three terms and add these values together.

$$\tfrac{5}{24} \text{ of £}1 = 50d. = 4s.\ 2d.$$

$$\tfrac{3}{8} \text{ of } 1s. = \tfrac{3}{8} \text{ of } 12d. = 4\tfrac{1}{2}d.$$

$$\tfrac{7}{18} \text{ of } 2s.\ 6d. = 11\tfrac{2}{3}d.$$

$$\text{The sum} = 5s.\ 6\tfrac{1}{6}d.$$

$$= 5s.\ 6d. \text{ to the nearest penny.}$$

It would be possible to express it all in shillings, viz.

$$\tfrac{5}{24} \text{ of } 20 + \tfrac{3}{8} + \tfrac{7}{18} \text{ of } \tfrac{5}{2}; \quad i.e.\ (4\tfrac{1}{6} + \tfrac{3}{8} + \tfrac{35}{36}) \text{ of a shilling,}$$

and so obtain the sum of the three terms as a fraction of a shilling. It would be cumbrous in this way, and more liable to error.

EXAMPLES VIII. m.

Find the value of

1. $\frac{1}{5}$ of £1. **2.** $\frac{1}{8}$ of £1. **3.** $\frac{1}{3}$ of £1.

4. $\frac{1}{6}$ of £1. **5.** $\frac{2}{3}$ of £1. **6.** $\frac{5}{6}$ of £1.

Commit the results of these to memory for use in some of the following.

[*Show up at least a rough check of each result.*]

Find the value of

7. $\frac{5}{8}$ of £3. 12*s*. **8.** $\frac{1}{10}$ of £3. 6*s*. 8*d*. **9.** $2\frac{6}{7}$ of £3. 10*s*.

10. £4. 13*s*. 4*d*. $\times 6\frac{1}{7}$. **11.** $\frac{3}{19}$ of 3 tons 16 cwt.

12. £2. 15*s*. 8*d*. $\times 24\frac{5}{6}$. **13.** £1. 13*s*. 9*d*. $\times 16\frac{2}{3}$.

14. £16. 8*s*. $1\frac{1}{2}$*d*. $\times 2\frac{2}{35}$. **15.** £3. 16*s*. 8*d*. $\div 7\frac{2}{3}$.

16. 28 cwt. 14 lb. $\div 16\frac{2}{3}$. **17.** 3 weeks 6 days $\times (11 - \frac{1}{9})$.

18. 70 yds. 2 ft. 3 in. $\times 7\frac{1}{7}$. **19.** $\frac{3}{8}$ of $1\frac{2}{5}$ of 10 hrs. 40 min.

20. 1 ton 5 cwt. 35 lb. $\div 10\frac{1}{8}$. **21.** $\frac{1}{7}$ of $2\frac{6}{11}$ of 11 tons 2 cwt. 3 qrs.

22. $\frac{1}{15}$ of 9*s*. 6*d*. + $\frac{1}{5}$ of £140. 10*s*. 6*d*. + $\frac{4}{5}$ of 15*s*. 9*d*.

23. A rectangular area 38 yds. by 14 yds. $-\frac{1}{11}$ of an acre (in sq. yds.).

24. $\frac{1}{12}$ of £50 + $\frac{1}{3}$ of £1. 14*s*. + $\frac{1}{4}$ of £2. 2*s*. 7*d*.

25. £9. 9*s*. $-\frac{7}{8}$ of £1. 7*s*. + $\frac{5}{8}$ of $\frac{2}{3}$ of £1 $-\frac{2}{7}$ of $1\frac{5}{9}$ of 6*s*. 8*d*.

26. $\frac{1}{40}$ of a week $-\frac{3}{20}$ of an hour.

27. $\frac{1}{16}$ of 1 cwt. $-\frac{3}{49}$ of 3 qrs. 14 lb.

28. $\frac{1}{20}$ of a furlong + $\frac{3}{4}$ of a chain.

29. $\frac{1}{125}$ of 7 m. 5 dm. + $\frac{7}{15}$ of 4 dm. 5 cm.

30. $\frac{3}{20}$ of 1 Kg. $-\frac{3}{4}$ of 1 Hg.

31. $\frac{2}{75}$ of 9 Dm. 3 m. 1 dm. 5 cm. $-\frac{1}{625}$ of a Kilometre.

32. $\frac{3}{11}$ of 7 g. 5 dg. 2 cg. 4 mg. $-\frac{2}{50}$ of 6 g. 1 dg.

33. $\frac{2}{7}$ of 3 fr. 15 c. + $\frac{1}{8}$ of 70 fr. **34.** $\frac{3}{8}$ of £1 + $\frac{15}{94}$ of £3. 18*s*. 4*d*.

35. $\frac{31}{56}$ of 226 chains 24 links.

36. $\frac{15}{88}$ of $2\frac{1}{2}$ acres + $\frac{1}{2}$ of 1 ac. 895 sq. yds.

37. $\frac{7}{60}$ of £15. 1*s*. 3*d*. $\sim \frac{39}{49}$ of £3. 16*s*. $6\frac{3}{4}$*d*.

Reduction of One Quantity to the Fraction of Another. Ratio. ἀναλογία

78. The quantities must be of the *same sort*. (It will be seen that $\frac{7}{10}$ does not express 7 ounces as a fraction of 10 yards.)

Ratio. If one quantity a is three times another quantity b, then a is said to have to b the ratio 3 to 1.

This may be written thus : $a : b = 3 : 1$. ....(1)

Since $a=3b$, it is clear that

$$\frac{a}{b}=\frac{3b}{b}=\frac{3}{1}. \quad\text{.............................}(2)$$

The statement (2) expresses the same fact as (1), and the fractional form $\frac{a}{b}$ will be found more convenient than $a:b$.

Similarly if a were $\frac{3}{4}$ of b the ratio of a to b would be 3 to 4, and we should write

$$\frac{a}{b}=\frac{3}{4}.$$

79. There are many ways of wording a question on this subject; one way being the clearest to one learner, another to another learner. It is well therefore to give a specimen of a question put in different shapes.

EXAMPLE 1. **Reduce 13s. 7d. to the fraction of £1.**
Express 13s. 7d. as a fraction of £1.
Express 13s. 7d. in terms of £1.
Find the ratio of 13s. 7d. to £1.
What fraction of £1 is 13s. 7d.?
By what fraction must £1 be multiplied to become 13s. 7d.?

The last but one is perhaps the clearest to a beginner.

To work it out we must reduce both quantities to the same denomination.

$$13s.\ 7d.=163d.$$

$$\text{But a penny}=\tfrac{1}{240}\text{ of a £.}$$

$$\therefore\ 13s.\ 7d.=\tfrac{163}{240}\text{ of a £.}$$

$$\text{The answer } \tfrac{163}{240} \text{ is } \frac{\text{the number of pence in } 13s.\ 7d.}{\text{the number of pence in £1}}.$$

Similarly, if we have to find what fraction £1. 15*s.* 7*d.* is of £6. 7*s.* 1*d.*,

$$\text{the required fraction}=\frac{\text{the number of pence in £1. } 15s.\ 7d.}{\text{the number of pence in £6. } 7s.\ 1d.}$$

$$=\frac{427}{1525}=\frac{7\times61}{25\times61}=\frac{7}{25}.$$

A common mistake of a thoughtless sort is to get the result inverted. A little observation is enough to prevent that.

£1. 15*s.* 7*d.* is less than £6. 7*s.* 1*d.*

∴ it must be a *proper* fraction of £6. 7*s.* 1*d.*

∴ the answer cannot be $\frac{25}{7}$.

If the example were, "Reduce 17*s.* 6*d.* to the fraction of 11*s.* 3*d.*," the answer would be $\dfrac{17\frac{1}{2}}{11\frac{1}{4}}$, *i.e.* $\frac{14}{9}$.

Here we see that 17*s.* 6*d.* is not properly speaking a part of 11*s.* 3*d.*;

∴ 17*s.* 6*d.* is not a *proper* fraction of 11*s.* 3*d.*;

∴ the result cannot be $\frac{9}{14}$.

EXAMPLE 2. **What fraction of 3 miles 110 yds. is 1 mile 6 furlongs?**

$$\text{The required fraction}=\frac{1\frac{3}{4}}{3\frac{110}{1760}}=\frac{1\frac{3}{4}}{3\frac{1}{16}}=\frac{\frac{7}{4}\times 16}{\frac{49}{16}\times 16}=\frac{7\times 4}{49}$$
$$=\frac{4}{7}.$$

Check the answer by observing whether it ought to be a proper or an improper fraction.

EXAMPLE 3. **Express $\frac{2}{5}$ of £3. 16s. 8d. as a fraction of $\frac{3}{7}$ of £6. 2s. 6d.**

£3. 16*s*. 8*d*. = £$3\frac{5}{6}$ and £6. 2*s*. 6*d*. = £$6\frac{1}{8}$.

Thus we can express numerator and denominator in £ instead of reducing them to pence.

$$\text{The required fraction}=\frac{\frac{2}{5}\text{ of }3\frac{5}{6}}{\frac{3}{7}\text{ of }6\frac{1}{8}}=\frac{\frac{2}{5}\times\frac{23}{6}}{\frac{3}{7}\times\frac{49}{8}}=\frac{\frac{23}{15}}{\frac{21}{8}}$$
$$=\frac{\frac{23}{\cancel{15}}\times\cancel{15}\times 8}{\frac{21}{\cancel{8}}\times 15\times\cancel{8}}=\frac{184}{315}.$$

EXAMPLES VIII. n.

[*Examples* 1–39 *may be taken orally.*]

1. What fraction of £1 is a crown? 2. What fraction of £1 is 3*s*. 4*d*.?
3. „ „ 2*s*. 6*d*.? 4. „ „ 2*s*.?
5. „ „ 1*s*. 8*d*.? 6. „ „ 3*s*.?
7. „ „ 6*s*. 8*d*.? 8. „ „ 7*s*. 6*d*.?
9. „ „ 8*s*.? 10. „ „ 12*s*.?
11. „ „ 15*s*.? 12. „ „ 16*s*.?
13. What fraction of 1*s*. is 4*d*.? 14. What fraction of 1*s*. is 7*d*.?
15. „ „ 10*d*.? 16. „ „ $1\frac{1}{2}$*d*.?
17. „ „ $10\frac{1}{2}$*d*.? 18. „ „ $3\frac{3}{4}$*d*.?
19. What fraction of 1 yd. is 9 inches?
20. „ „ 27 „
21. Express 8 inches as a fraction of 1 foot.
22. „ 9 „ „ „ „
23. „ $1\frac{1}{2}$ „ „ „ „
24. „ 3 furlongs „ „ 1 mile.
25. „ 1 chain „ „ 1 furlong.
26. „ $5\frac{1}{2}$ yards „ „ 1 chain.
27. „ 14 lb. „ „ 1 cwt.
28. „ $\frac{1}{2}$ cwt. „ „ 1 ton.
29. „ 28 lb. „ „ 1 cwt.
30. „ 15° „ „ 1 right angle.

31. What fraction of £1 is 13*s*. 4*d*.?
32. „ „ 12*s*. 6*d*.?
33. „ „ 8*s*. 9*d*.?
34. „ „ 6*s*. 3*d*.?
35. „ „ 11*s*. 3*d*.?
36. „ „ 16*s*. 8*d*.?
37. „ „ 18*s*. 9*d*.?
38. „ „ 5*s*. 10*d*.?
39. „ „ 3*s*. $7\frac{1}{2}$*d*.?

[*Revise each example before proceeding to the next.*]

40. Express 6*s*. 9*d*. as a fraction of £4. 13*s*. 9*d*.
41. „ £2. 0*s*. 5*d*. „ „ £10. 10*s*. 2*d*.
42. „ 3 yds. 2 ft. „ „ 4 yds. 1 ft. 6 in.
43. „ £7. 16*s*. 8*d*. „ „ £6. 17*s*. 6*d*.
44. „ 16*s*. $10\frac{1}{2}$*d*. „ „ £5.
45. „ $82\frac{1}{2}$ sq. yds. „ „ 1 acre.
46. „ 3*s*. 2*d*. „ „ £19.
47. „ 2*s*. 10*d*. „ „ half a guinea.
48. „ 2 cwt. 17 lb. „ „ 1 ton.
49. „ 101° 15′ „ „ 180°
50. „ 15 chs. 15 links „ 113 chains 12 links.
51. What fraction is £2. 12*s*. 6*d*. of £3. 17*s*. 6*d*.?
52. „ „ 3*s*. $4\frac{1}{2}$*d*. of £1. 3*s*. $5\frac{1}{4}$*d*.?
53. „ „ 17 chains of 1 mile.
54. „ „ £30. 2*s*. 6*d*. of £37. 3*s*. 1*d*.
55. „ „ 14 hrs. 24 min. of 1 day.
56. Reduce 1 mile 7 fur. to the fraction of 3 miles 6 fur.
57. „ £1. 15*s*. $9\frac{1}{2}$*d*. „ „ £1. 10*s*.
58. „ £7. 13*s*. $1\frac{1}{2}$*d*. „ „ £2. 6*s*. $10\frac{1}{2}$*d*.
59. „ 25 yds. 2 ft. „ „ half a mile.
60. „ £7. 4*s*. 1*d*. „ „ 10 guineas.
61. „ 7 oz. 4 dwt. „ „ 3 lb. Troy.
62. „ 7 cwt. 3 qr. 14 lb. „ „ 2 tons.
63. „ 3 mls. 7 fur. 110 yds. „ 4 mls. 2 fur. 80 yds.
64. „ £125. 2*s*. 6*d*. „ „ £113. 15*s*.
65. „ $\frac{1}{2}$ of 3 cwt. 1 qr. 14 lb. „ $\frac{4}{7}$ of 5 cwt. 2 qr. 14 lb.
66. „ $\frac{7}{8}$ of $2\frac{1}{2}$ guineas „ „ $\frac{4}{5}$ of £1. 15*s*.
67. Find the ratio of $17\frac{1}{2}$ hours to 3 weeks 4 days.
68. „ „ £1. 17*s*. 4*d*. to £1. 3*s*. 4*d*.
69. „ „ 1 ro. 5 sq. po. to half an acre.

70. Find the ratio of $\frac{1}{5}$ of £4. 13*s*. 9*d*. to $\frac{2}{7}$ of £2. 3*s*. 9*d*.
71. " " $\frac{7}{8}$ of £3. 17*s*. 3*d*. to $\frac{5}{4}$ of £14. 3*s*. 3*d*.
72. " " $\frac{1}{2}$ of 3 qrs. 7 lb. to 2 cwt.
73. " " $\frac{4}{7}$ yd. to $\frac{3}{3850}$ of a furlong.
74. " " 3*s*. 6*d*. to $\frac{3}{11}$ of a guinea.
75. " " 4 min. 3 sec. to $\frac{1}{16}$ of a day.
76. " " $\frac{5}{11}$ of 6 miles to $\frac{3}{8}$ of 1000 yds.
77. What fraction of £5. 13*s*. $10\frac{3}{4}$*d*. is $1\frac{5}{6}$ of 4*s*. $5\frac{1}{4}$*d*.?
78. " " 2 tons is $\frac{13}{3}$ of 5 cwt. 2 qrs. 14 lb.?
79. A rectangular area is 30 yds. 9 in. by 20 yds. Express it as a fraction of an acre.
80. What part of a sq. mile does a railway company have to buy for a line 10 miles 10 chains in length, if the width required is 1 chain?
81. What fraction of 35 shillings is equal to the sum of $2\frac{3}{4}$ of 7*s*. 6*d*. and $\frac{2}{3}$ of $\frac{5}{4}$ of 3*s*.?
82. What weight is the same fraction of 1 ton as £2. 9*s*. 10*d*. is of £3. 16*s*. 8*d*.?

Problems in Fractions.

80. The following principle is of frequent use:

If $a=\frac{5}{6}$ of b, then $b=\frac{6}{5}$ of a.

Proof. Since $a=\frac{5}{6}$ of b,

it follows that $\frac{6}{5}$ of $a=\frac{6}{5}$ of $\frac{5}{6}$ of $b=b$.

Or thus: $a=\dfrac{5b}{6}$. Multiplying each of these equal quantities by 6, we have $6a=5b$.

Dividing each of these equal quantities by 5, we have $\dfrac{6a}{5}=b$.

Similarly, if $x=\frac{7}{24}$ of y, then $\frac{24}{7}$ of $x=\frac{24}{7}$ of $\frac{7}{24}$ of y,

$$i.e.\ y=\tfrac{24}{7}x.$$

EXAMPLE 1. **A man having a sum of money in his purse gives away $\frac{1}{3}$ of it to one person, $\frac{1}{6}$ to a second, $\frac{1}{8}$ to a third, $\frac{1}{12}$ to a fourth person, and then has 14 shillings left. What sum had he at first?**

He gives away $\frac{1}{3}+\frac{1}{6}+\frac{1}{8}+\frac{1}{12}$, *i.e.* $\frac{8+4+3+2}{24}$, *i.e.* $\frac{17}{24}$.

$\therefore$ he has remaining $\frac{7}{24}$ of the original sum.

Let x shillings be the sum of money.

$$\text{Then } \frac{7x}{24}=14.$$

Multiplying both sides by $\frac{24}{7}$, we get

$$x=14\times\tfrac{24}{7}=48.$$

$\therefore$ the sum of money $=48$ shillings $=$ £2. 8*s*.

Example 2. **One-third of the trees in a plantation are oaks, one-seventh are firs, and there are 66 trees besides. Of how many trees does the plantation consist?**

$$\tfrac{1}{3}+\tfrac{1}{7}=\tfrac{10}{21}.$$

$\therefore$ the rest are $\frac{11}{21}$ of the whole.

Let x be the number of trees.

Then $\frac{11}{21}x=66.$

Multiply both sides by $\frac{21}{11}$.

Then $x=\frac{66\times 21}{11}=126.$

Check.	Of these the oaks number	42
	the firs ,,	18
	the rest ,,	66
	Total	126

Example 3. **Find a number one-third of which exceeds its tenth by 63.**

$$\tfrac{1}{3}-\tfrac{1}{10}=\tfrac{7}{30}.$$

Thus $\frac{7}{30}$ of the number $=63$.

Let x be the number. Then $\frac{7}{30}x=63.$

$$\therefore\ x=63\times\tfrac{30}{7}=270.$$

Example 4. **Find a number which becomes 91 when increased by its twelfth part.**

$$1\tfrac{1}{12}=\tfrac{13}{12}.$$

Let x be the number. Then $\frac{13}{12}x=91.$

Multiplying both sides by $\frac{12}{13}$ we get

$$x=91\times\tfrac{12}{13}=84.$$

Example 5. **How much ore must I raise in order that after losing $\frac{5}{26}$ in carriage and $\frac{3}{7}$ of the residue in smelting, I may get 120 tons of metal?**

After a loss of $\frac{5}{26}$ the residue is $\frac{21}{26}$.

But after the smelting there remains only $\frac{4}{7}$ of this, viz. $\frac{4}{7}$ of $\frac{21}{26}$, *i.e.* $\frac{6}{13}$.

Let x be the number of tons raised.

Then $\frac{6}{13}x=120.$

$$\therefore\ x=120\times\tfrac{13}{6}=260.$$

Example 6. **By selling an estate for £3480 I lost $\frac{13}{100}$ of what it cost me. What did it cost me?**

As I lost $\frac{13}{100}$ of the cost, I must have received only $\frac{87}{100}$ of the cost.

$\therefore$ if £x = the cost,

$$\frac{87x}{100}=3480.$$

$$\therefore\ 87x=3480\times 100.$$

$$\therefore\ x=\frac{3480\times 100}{87}$$

$$=4000.$$

$\therefore$ the cost was £4000.

EXAMPLES VIII. o.

PROBLEMS.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

1. A man lost $\frac{1}{4}$ of his money, and then had 27 shillings left. What had he at first?

2. If $\frac{7}{10}$ of a piece of land be worth £245, what is the value of the whole?

3. Find a number such that its half exceeds its seventh by 25.

4. A cistern was three-fourths full of water, and after 55 gallons had been run off it was one-fifth full. How many gallons could it hold?

5. What number is that from which if its fifth part be taken the remainder is 56?

6. If $\frac{7}{8}$ of an estate be worth £9464, what is the value of the whole?

7. A tank was four-sevenths full of water. An addition of 51 gallons made it seven-eighths full. What was its capacity?

8. What number increased by its half and its tenth part = 128?

9. $\frac{11}{13}$ of a length of road is macadamised, $\frac{1}{15}$ is asphalted and the remaining 2992 yards are paved. Find the total length of the road in miles.

10. A tradesman adds $\frac{3}{10}$ of the cost in marking the price of an article. The price marked is £3. 18*s.* What was the cost?

11. In an estate $\frac{1}{20}$ is a lake, $\frac{1}{2}$ is pasture, $\frac{2}{5}$ is arable land and there are 23 acres of woods besides. What is the acreage of the whole?

12. $\frac{5}{9}$ of a number of people received 3*s.* each, the rest 5*s.* each. The sum distributed to the latter was £46. How many people were there? How much money was distributed altogether?

13. In an innings at cricket one man made half the runs, another $\frac{1}{6}$ and the rest of the eleven $\frac{1}{4}$: the extras came to 15. What was the score?

14. A man, after losing $\frac{5}{16}$ of his capital, has £6710 left. What was his original capital?

15. An executor, after paying away $\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{18}$ of a legacy, has still £104. 17*s.* 4*d.* in hand. How much was the legacy?

16. In selling a thing I gained $\frac{1}{8}$ of what it cost me. The selling price was £1. 9*s.* 3*d.* What did it cost me?

17. A man possessing $\frac{3}{4}$ of an estate sold $\frac{5}{7}$ of his share for £540. What was the value of the whole estate at this rate?

18. Of a congregation $\frac{11}{23}$ are women, $\frac{2}{3}$ of the remainder are men, and the rest, 20 in number, are children. Of how many does the congregation consist?

19. Find a number such that its third and fourth parts together exceed its seventh by 259.

20. After spending $\frac{17}{40}$ of my money and also $\frac{8}{19}$ of the remainder, I have £126. 10*s*. What had I at first?

21. The length of a line A is $\frac{9}{10}$ of that of another, B; also A and B together have a length of 5 cm. 7 mm. What are their respective lengths?

22. A does $\frac{3}{5}$ of a piece of work in 13 days, and, with the aid of B, finishes it in 6 days more. In what time could each do it separately?

23. An express train, which travels one-third as fast again as an ordinary train, performs a journey of 252 miles in $1\frac{1}{2}$ hours less than the ordinary train. Find the average speed of each.

IX. REVISION PAPERS.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

IX. a.

1. Simplify $14\frac{17}{21} - 2\frac{17}{35} - 1\frac{3}{7}$.

2. Find a quantity which, when added to $4\frac{3}{8}$ and then divided by $2\frac{3}{16}$, gives $2\frac{1}{2}$.

3. Find the value of $\frac{3}{4}$ of $10\frac{1}{2}d. + \frac{2}{5}$ of $\frac{7}{16}$ of $5s. + \frac{3}{8}$ of £1. 5*s*.

4. Divide 5 cwt. 2 qr. 14 lb. by $3\frac{1}{3}$.

5. Simplify $\dfrac{2\frac{1}{4} \times 2\frac{1}{4} \times 2\frac{1}{4} - 8}{2\frac{1}{4} \times 2\frac{1}{4} - 4}$.

6. Reduce $\frac{2}{7}$ of £4. 7*s*. 6*d*. to the fraction of $\frac{3}{8}$ of $\frac{9}{13}$ of £4. 17*s*. 6*d*.

7. A man's yearly income is £741. 11*s*. 2*d*. and his daily expenditure a guinea and a half. What fraction of his yearly income does he save?

IX. b.

1. Resolve 13332 and 2525 into prime factors, and find their H.C.F.

2. Arrange in ascending order of magnitude $\frac{3}{5}$, $\frac{8}{15}$, $\frac{7}{13}$.

3. Divide $4\frac{4}{5}$ of $5\frac{2}{5}$ by $\frac{1}{10} + \frac{17}{100}$.

4. Find the ratio of $\frac{2}{5}$ of 5 cwt. 1 qr. to $\frac{3}{4}$ of 3 tons 10 cwt.

5. If a mile be equal to 1600 metres, how many sq. metres are there in an acre?

6. One pipe can fill a cistern in 20 minutes, another can empty it in 18 minutes. If both are opened simultaneously when it is full, in what time will it be empty?

7. One-third of a flagstaff is painted red, $\frac{1}{4}$ is white and the remaining 35 feet are blue. What is its whole length?

IX. c.

1. Simplify $7362 \div \{30-(3^2+3)\} \sim (2^4+3^3)\times(3^2+1)$.

2. Find the L.C.M. of 231, 105, 165, leaving the result in factors.

3. Reduce $\frac{5481}{7105}$ to its lowest terms.

4. Simplify $\dfrac{\frac{7}{18}+\frac{2}{9}\div 3\frac{1}{2}}{6\frac{1}{14}-3\frac{1}{4}\text{ of }1\frac{6}{7}}$.

5. Multiply £4. 16*s.* $1\frac{3}{4}$*d.* by $110\frac{3}{5}$.

6. There are 11 allotments each 70 yds. square. Find what fraction the whole is of 14 acres.

7. Two men, who can separately dig over a garden in 5 and 7 days respectively, work for $1\frac{3}{4}$ days at it. What fraction of it remains to be dug?

IX. d.

1. Find the value of $8\frac{7}{11}$ of $\frac{3}{5}+9\frac{1}{2}$ of $\frac{13}{76}+3\frac{3}{4}$ of $\frac{8}{15}+6\frac{2}{3}$ of $\frac{15}{22}$.

2. Subtract $\frac{1}{5}$ of $3\frac{7}{17}$ of $4\frac{3}{29}$ from $11\frac{9}{20}$.

3. Find the value of $\frac{5}{18}$ of $\frac{16}{35}\times 2\frac{9}{20}$ of $2\frac{1}{4}\times 10\frac{15}{32}$.

4. Divide $12\frac{12}{13}$ by $1\frac{25}{39}$.

5. A and B working together can do a piece of work in 6 days. B alone would require 15 days. In what time could A do it?

6. What fraction of 14 acres is a field of 7 ac. 3 ro. 6 sq. poles?

7. Find $\frac{4}{55}$ of £6. 17*s.* 6*d.*, and take it from $\frac{1}{7}$ of £3. 11*s.* 2*d.*

IX. e.

1. Resolve 2436 into prime factors.

2. Express £3. 17*s.* 6*d.* as a fraction of £1; and find the value of 2436 oz. of gold at £3. 17*s.* 6*d.* an ounce.

3. Simplify $(2\frac{5}{9}$ of $1\frac{17}{46}-3)\times\frac{21}{44}\div(4\frac{2}{11}-2\frac{3}{4})$.

4. By selling goods for £130 a man gained $\frac{1}{12}$ of the cost. What was the cost?

5. A cistern supplied by 3 pipes can be filled by the first in 10 minutes, by the second in 12 and by the third in 15 minutes. In what time would it be filled by all three running together?

6. Express the sum of $7\frac{1}{2}$ guineas, $3\frac{1}{2}$ crowns and $5\frac{1}{4}$ florins as a fraction of £53.

7. Find the cost of carpet at 4*s.* 6*d.* a sq. yd. for a room 24 feet long and $16\frac{1}{2}$ feet wide.

IX. f.

1. Find the H.C.F. of 8775 and 12025.

2. The water in a tank 8 ft. long and 7 ft. wide is 4 ft. deep; and a cubic ft. of water weighs $62\frac{1}{2}$ lb. How many tons of water are there in the tank?

3. Reduce $\frac{427}{5490}$ and $\frac{6409}{7395}$ to their lowest terms, and find their sum.

4. Find the value of $(2\frac{7}{15}+3\frac{3}{8}+4\frac{11}{48})$ of £1.

5. Simplify $(\frac{1}{2}+\frac{2}{3}+\frac{3}{4})\times(2\frac{2}{3}-\frac{9}{20})\times(3\frac{2}{7}-1\frac{24}{161})\div 2\frac{13}{60}$.

6. The square of 188 is 35344. From this find by addition the square of 189.

7. Reduce 3 cwt. 3 qrs. 21 lb. to the fraction of 9 cwt. 21 lb. by expressing them both in quarters.

IX. g.

1. Express 723560 decimetres in kilometres and metres.

2. Find the L.C.M. of 12, 35, 65, 91.

3. Express in miles, furlongs and yds. $\frac{101}{220}$ of 3 miles.

4. $2\frac{4}{17}=\frac{11}{17}+\frac{21}{34}+\frac{32}{51}-\frac{1}{12}+\frac{*}{136}$. Fill up the missing numerator.

5. Simplify $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{15}{28}}{(\frac{5}{6}-\frac{3}{14})\times 1\frac{2}{13}}$.

6. Find the value of a cube, whose edge is $11\frac{1}{2}$ in., at 8*d.* a cubic inch.

7. If $\frac{5}{13}$ of an estate is worth £4500, what is the whole estate worth?

IX. h.

1. A sum of £82. 15*s.* was made up of equal numbers of sovereigns, crowns, half-crowns and pennies. How many coins of each sort were there?

2. Resolve 6552 and 2310 into prime factors and find their L.C.M., leaving it in factors.

3. Simplify $\dfrac{1-\frac{14}{25}-\frac{5}{12}}{1-\frac{1}{15}-\frac{9}{10}}$.

4. A man who owned $\frac{1}{4}$ of an estate sold $\frac{9}{10}$ of his share for £1350. What was the value of the whole estate?

5. What fraction multiplied by $3\frac{1}{2}$ of $\frac{4}{5}$ produces $2\frac{2}{5}$ of $\frac{4}{13}$?

6. From $\frac{1}{8}$ of $\frac{2}{3}$ of £57 take $\frac{5}{77}$ of $2\frac{3}{4}$ of £3. 6*s.* 6*d.*

7. Find the number of cubic feet in a cistern which would just hold 1440 bricks measuring 9 in. by 4 in. by 3 in. Find also the depth of the cistern if its length and breadth are 6 and 5 ft.

IX. k.

1. By what must 22 m. 9 dm. 5 cm. be divided to give 1 m. 5 dm. 3 cm.?

2. What is the cost of staining the floor of a room 22 ft. long and $16\frac{1}{2}$ ft. wide at $\frac{1}{2}d.$ per sq. ft.?

3. Find the L.C.M. of 42, 54, 63, 315.

4. Simplify $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}+2\frac{1}{7}}\times 4\frac{30}{53}$.

5. Find the ratio of 81 times £5. 4*s.* 2*d.* to 100 times £7. 16*s.* 9*d.*

6. By selling an article at 10 shillings I lose $\frac{1}{5}$ of the cost. What did it cost me?

7. Express in chains $\frac{2}{7}$ of 9 miles $-\frac{8}{21}$ of 3 furlongs $+1\frac{5}{7}$ of a chain.

IX. l.

1. A grocer buys 20 chests of tea, holding 85 lb. each, for £127. 10*s.*, and sells at 1*s.* 10*d.* per lb. What does he gain per lb.?

2. How many allotments each containing 3 ro. 7 sq. po. can be made out of a field of 99 ac. 0 ro. 35 sq. po.?

3. Find the H.C.F. of 4807 and 3781.

4. Simplify $\frac{3}{5}$ of $\dfrac{8\frac{1}{2}-5\frac{1}{3}}{15\frac{3}{4}-3\frac{2}{5}}$.

5. A rectangular field of 21 acres has a length of 15 chains. Find the breadth. Find also the cost of fencing it at 6*d.* a foot.

6. How many bricks measuring 8 in. by 4 in. by 3 in. are required for a wall 6 ft. high 1 ft. thick and 120 ft. long?

7. A cistern has 2 pipes which can fill it in 10 and 14 minutes respectively. Both are turned on, but, owing to the waste-pipe being open, the cistern takes 35 minutes to fill. How long would the waste-pipe alone take to empty the cistern?

IX. m.

1. A sum of £5. 14*s.* 10*d.* consists of equal numbers of crowns, half-crowns, shillings, threepences and pennies. How many are there of each sort of coin?

2. How many tons of water can be contained in a reservoir 64 ft. long, 21 ft. wide, 8 ft. deep, if a cubic foot of water weighs $62\frac{1}{2}$ lb.?

3. Multiply £6. 7*s.* $10\frac{1}{2}d.$ by $3\frac{3}{11}$.

4. Arrange in ascending order of magnitude $\frac{3}{4}$, $\frac{7}{9}$, $\frac{23}{30}$, $\frac{31}{40}$.

5. Simplify $\frac{4752}{10395}$.

6. Find the least number which, when divided by 8, 9, 12 or 15, leaves remainder 1.

7. A man finds that his necessary expenses absorb $\frac{4}{7}$ of his income, unnecessary expenses $\frac{6}{35}$, and he has £72 left over. Find his income.

IX. n.

1. Find the three numbers between 2500 and 3000 which are divisible by 21, 24 and 28.

2. Simplify $3\frac{7}{8}+1\frac{13}{14}+9\frac{31}{42}$.

3. Simplify $10\frac{5}{187}\times 2\frac{11}{20}\div 74\frac{5}{11}-\frac{1}{4}$.

4. A plot of building land containing 3 ac. 3 ro. 25 sq. po. is sold for £1312. 10*s*. Find the price per acre.

5. Find the number of feet in $3\frac{3}{19}$ of 8 miles 2 fur. 21 yds. 1 ft.

6. What fraction of 35*s*. is equal to the sum of $2\frac{3}{4}$ of 7*s*. 6*d*. and $\frac{3}{2}$ of $\frac{5}{4}$ of 3*s*.?

7. A man left $\frac{1}{2}$ his property to his wife, $\frac{1}{3}$ of the remainder to the eldest son and the rest equally among the five remaining children, who thus got £600 each. How much did the wife get?

IX. o.

1. A tradesman bought 10 tons 3 cwt. of sugar at £18 a ton, and sold it at $2\frac{1}{4}$*d*. a lb. What was his profit?

2. Divide £160. 4*s*. $8\frac{1}{4}$*d*. by $\frac{1}{5}$ of £1. 10*s*. $6\frac{1}{4}$*d*.

3. Find the L.C.M. of 78, 143, 91, 637, leaving the result in factors.

4. Simplify $\dfrac{1\frac{2}{3}+4\frac{3}{5}}{2\frac{2}{3}-2\frac{1}{5}}\div\left(1\frac{5}{8}-\frac{11}{14}\right)$.

5. Find the value of

$\frac{5}{8}$ of 1*s*. + $\frac{4}{15}$ of £1 + $\frac{5}{28}$ of 1 guinea + $\frac{3}{5}$ of £1. 6*s*. 8*d*.

6. The total area of three estates is 1768 acres. If the two smaller be respectively $\frac{3}{5}$ and $\frac{2}{3}$ of the largest, find the size of each.

7. Find the cost of painting at 4*d*. per sq. ft. the sides and bottom of a rectangular tank $2\frac{1}{2}$ yds. long, 4 ft. wide and $4\frac{1}{2}$ ft. deep.

IX. p.

1. A train 160 ft. long is travelling at 25 miles an hour. Find how long it takes in passing completely over a bridge 240 yds. long.

2. Add together $2\frac{2}{9}$, $3\frac{11}{12}$, $4\frac{3}{8}$, $2\frac{5}{6}$; and divide the result by the difference between $3\frac{7}{18}$ and $2\frac{5}{16}$.

3. Each of the children in a school received 3 apples and 2 pears. The apples were bought at 3*d*. a score, the pears at $2\frac{1}{2}$*d*. a dozen. What fraction of a penny did each child's share cost? If the total cost was 9*s*. 9*d*., how many children were there?

4. Find the number of sq. yds. in $\left(\frac{3}{4}-\frac{7}{11}\right)$ of an acre.

5. A room 30 ft. by 16 ft. has a margin 2 ft. 6 in. wide all round outside the carpet. What fraction of the whole floor is the margin?

6. A man loses $\frac{1}{4}$ of his money and afterwards loses $\frac{5}{12}$ of the remainder, and then has £56 left. What had he at first?

7. If 28 Kg. of luggage are allowed free to each passenger, how much will 5 passengers have to pay for over-weight at 75 centimes the Kg., if their luggage altogether weighs 209 Kg.?

IX. q.

1. A man walks at the rate of 2 steps, of 36 inches each, per second. How long will he take to walk 12 miles? Express the result in hours and minutes.

2. Find the greatest number such that 13850 and 17030 divided by it leave 17 each?

3. Express in factors the L.C.M. of 252, 322, 522, 667.

4. Find in its lowest terms the difference between $\frac{1638}{2205}$ and $\frac{910}{1911}$.

5. Find to the nearest halfpenny the value of

$$\tfrac{1}{3} \text{ of } 9s.\ 7d. + \tfrac{1}{4} \text{ of } £1.\ 6s.\ 9d. + \tfrac{1}{5} \text{ of } 14s.\ 6d.$$

6. If $\frac{7}{11}$ of an estate is worth £4655, what is the remainder worth?

7. A, B and C together can do a piece of work in 8 days, A alone in 30 days. After they have all worked together for 2 days A falls ill. How long do B and C take to finish it?

IX. r.

1. Determine, without performing the divisions, the remainders that result from dividing 48909661 by 8, 9 and 11; and give a reason for your statement in the case of the 8.

2. Find the H.C.F. of 4982 and 9823.

3. Find the cost of a carpet at 3*s.* 3*d.* per sq. yd. for a room 24 ft. long and $17\frac{1}{2}$ ft. wide, if a margin 2 ft. wide be left uncovered.

4. Simplify $\frac{1}{51}+\frac{4}{85}+\frac{3}{38}+\frac{5}{57}$. [Add them in pairs.]

5. A man owns $\frac{15}{28}$ of a field. If his share is $\frac{3}{8}$ of an acre, what is the size of the field in sq. yds.?

6. There are 4 divisions in a boy's school. Division I. contains $\frac{1}{7}$ of the whole number, Division II. contains $\frac{1}{4}$, Division III. $\frac{1}{5}$, and Division IV. 114 boys. What is the total number?

7. If 5 men or 8 women can do a piece of work in 7 days, what part can a man do, and what part can a woman do, in a day? How long would 1 man and 4 women take to do $\frac{1}{4}$ of the work?

IX. s.

1. Find the least number above 5000 which is a multiple of 63.

2. A house has 15 windows each containing 16 panes measuring 21 inches by 16 inches. Find the cost of glazing it at $10\frac{1}{2}d.$ per sq. ft.

3. Express in factors the L.C.M. of 2310, 6552, 2145.

4. Find the greatest and least of $\frac{6}{11}$, $\frac{19}{35}$, $\frac{31}{55}$, $\frac{43}{77}$.

5. Simplify $\dfrac{(\frac{1}{2}+\frac{1}{4})\div(\frac{5}{6}\text{ of }\frac{3}{8})}{2\frac{2}{3}\div(3\frac{1}{3}-2\frac{1}{2})}$.

6. An estate is occupied by 3 tenants, the 1st holding $\frac{2}{5}$ of it, the 2nd $\frac{3}{8}$ of it and the 3rd the remainder, viz. 25 ac. 2 ro. 24 sq. po. What is the area of the whole estate?

7. A can do a piece of work in 11 days, B in 20, C in 55. How long does the work take if A is assisted by B and C on alternate days?

IX. t.

1. The interior surface of a room 24 ft. by 17 ft. by 15 ft. is subjected to a pressure of 15 lb. per sq. inch. Find the total pressure in tons.

2. Prove that the value of a fraction is unaltered by multiplying the numerator and denominator by the same number; and that its value is altered by adding the same quantity to numerator and denominator.

3. Find the prime factors of 1881 and 4332, and hence their H.C.F.

4. Find the value of $\frac{5}{6}$ of half a guinea + $\frac{7}{15}$ of half a crown + $\frac{7}{8}$ of a florin − $\frac{3}{4}$ of 6*s.* 8*d.*

5. A building estate is to be let in plots so as to return a ground-rent of £65 an acre. What will be the ground-rent of a rectangular plot 49 ft. 6 in. wide and 286 ft. deep. [Find what fraction this plot is of an acre, and hence find the rent.]

6. A legacy is divided between 3 brothers, so that the eldest has $\frac{2}{5}$ of it, the second $\frac{3}{8}$ and the youngest the remainder, which is £35 less than the share of the eldest. Find the value of the legacy.

7. A cistern capable of holding 80 gallons can be filled by one pipe in $1\frac{1}{4}$ minutes and emptied by another in $1\frac{3}{4}$ minutes. If both pipes are open at the same time, how much water will be wasted before the cistern is filled?

IX. u.

1. Express each of the numbers 91, 119, 221 as the product of two whole numbers. Find the least number which on division by each of the three given numbers leaves the same remainder 6.

2. Find by how much $\frac{2}{3}+\frac{8}{21}-\frac{1}{2}$ exceeds $\frac{3}{4}+\frac{5}{14}-\frac{16}{21}$; find also the ratio of the difference to $\frac{4}{21}$.

3. Express in its lowest terms the ratio of £16. 1*s.* $10\frac{1}{2}$*d.* to £59. 0*s.* $2\frac{1}{2}$*d.*

4. Simplify $\frac{7}{15}\times 21\frac{9}{21}$ of 2*s.* $11\frac{1}{2}$*d.* − $(\frac{3}{5}-\frac{2}{7})$ of £1. 12*s.* 1*d.*

5. A man buys eggs at 1*s.* 3*d.* a dozen and sells them at 11*s.* 8*d.* per hundred. How many eggs must he sell to make a profit of £1?

6. After paying away $\frac{7}{240}$ of my income I find that there remains £939. 15*s.* 4*d.* What is my whole income?

7. A 40-gallon cask is under a tap which pours in water at the rate of $1\frac{9}{10}$ quarts per 10 seconds; it has a tap below, by which $2\frac{1}{10}$ quarts can be drawn off in 10 seconds. The cask was full, and it happened that both taps were left turned on. Supposing that the above-named rates continued steady, find how long it was before the cask was empty, and how much water ran to waste up to that instant.

X. UNITARY METHOD.

81. WHAT is meant by the **Unitary Method** is soon learnt by studying a few examples.

EXAMPLE 1. **Given that 14 yds. of calico cost 63 pence, find the price of 24 yds.**

14 yds. of calico cost 63 pence.

$\therefore$ 1 yd. ,, ,, $\frac{63}{14}$,,

$\therefore$ 24 yds. ,, ,, $\frac{63 \times 24}{14}$,,

i.e. the price required $= \frac{63 \times 24}{14} = \frac{9}{2} \times 24 = 9 \times 12$ pence

$= 9$ shillings.

EXAMPLE 2. **If I bought 7 ac. 3 roods of land for £325. 10s., how much did I pay per acre?**

7 ac. 3 roods ($=31$ roods) cost £$325\frac{1}{2}$ ($=$£$\frac{651}{2}$).

$\therefore$ 1 rood ,, £$\frac{651}{2 \times 31}$.

$\therefore$ 4 roods ($=1$ acre) ,, £$\frac{651 \times 4}{2 \times 31}$.

i.e. the cost per acre $= \frac{651 \times 2}{31} = 21 \times 2 =$ £42.

EXAMPLE 3. **If 27 men build a wall in 11 days, how many men would build the same wall in 3 days?**

N.B.—It will evidently take *more* men to do the same work in a *shorter* time; and *vice versa*, it will take *fewer* men to do the same work in a *longer* time.

27 men can build the wall in 11 days.

$\therefore$ 27×11 ,, ,, ,, 1 day.

$\therefore$ $\frac{27 \times 11}{3}$,, ,, ,, 3 days.

$\therefore$ the required number of men $= \frac{27 \times 11}{3} = 9 \times 11 = 99$.

EXAMPLE 4. **If a man pays income-tax at the rate of 10 pence in the £, what does he pay on an income of £750?**

On £1 he pays 10 pence.

$\therefore$,, £750 ,, 10×750 pence.

$$\begin{array}{r|l} 12 & \underline{7500} \text{ pence.} \\ 2,0 & \underline{62,5s.} \\ & \text{£31. 5s.} \end{array}$$

i.e. £31. 5*s.* is the amount he pays.

82. Bankruptcy. A man's possessions are called his **assets**. The amount he owes to other people is called his **liability**.

Thus if a bankrupt only has £570, but owes £920, £570 represents his assets, and £920 represents his liability.

If a bankrupt can only pay 11*s*. for every £ he owes, he is said to pay a **dividend of 11s. in the £.**

In problems on **Rates and Taxes**,* a man's **gross income** is his total income before any deductions for such things as rates and taxes have been made. His **net income** is his income after such deductions have been made.

i.e. Gross income = Net income + Rates and Taxes.

Thus if a man receives a salary of £621, and pays £15 in rates and taxes, his gross income is £621, and his net income £606 (= £621 − £15).

Interest and Capital. If I lend a man £500, and he pays me £20 per annum for the use of it, the £20 is called the **interest** on the £500, and the £500 is called **capital.**

EXAMPLE 1. **After paying the income-tax at the rate of 8d. in the £, a man has £580 left. What is his gross income?**

After paying the tax on 240 pence (£1) the man has 232 pence left.

∴ „ „ £240 „ „ £232 left.

∴ „ „ $£\frac{240}{232}$ „ „ £1 „

∴ „ „ $£\frac{240 \times 580}{232}$ „ „ £580 „

i.e. his total income $= \frac{240 \times 580}{232}$

$= \frac{30}{29} \times 580 = 30 \times 20 = \underline{£600}.$

EXAMPLE 2. **If a bankrupt pays 10s. 6d. in the £, how much does he pay on a debt of £172. 10s.?**

He pays 10*s*. 6*d*. instead of £1.

∴ he pays 10*s*. 6*d*. $\times 172\frac{1}{2}$ instead of £172. 10*s*.;

∴ the required amount = 10*s*. 6*d*. $\times 172\frac{1}{2}$

= 5*s*. 3*d*. × 345

= (5 × 345)*s*. + (3 × 345)*d*.

= $(1725 + 86\frac{1}{4})$*s*.

= $1811\frac{1}{4}$*s*.

= £90. 11*s*. 3*d*.

*Rates and taxes themselves have been subjected to many changes, but the principles on which problems dealing with them are based, remain unaltered.

EXAMPLES X. a. (*Oral.*)

1. If 15 yds. of calico cost 60 pence,
1 yd. „ costs how much?

2. If a dozen herrings cost 18 pence,
one herring costs how much?

3. If 10*d.* is the income-tax on £1,
what „ „ „ £9?

4. If 2*s.* 6*d.* is the District Rate on £1,
£1 „ „ how much?

5. If 6 men build a wall in 12 days, how long will one man take to build the same wall?

6. If one man can dig a piece of ground in 15 days, how long will 3 men take to dig it?

7. If a bankrupt pays 2*s.* 6*d.* in the £ how much does he pay on a debt of £8?

8. If £24 is the interest on a loan of £600, what is the interest on a loan of £100?

9. If 100 oranges cost 4*s.* 2*d.*, what is the price of (1) one orange, (2) 50 oranges, (3) 10 oranges?

10. If 3 men do a piece of work in 14 days, how many men will it take to do it in (1) 7 days, (2) 2 days?

11. If $\frac{1}{3}$ of a share is worth £3. 6*s.* 8*d.*, what is the value of (1) a whole share, (2) 9 shares?

12. If 7 kettles cost 7*s.* 7*d.*, what will be the cost of (1) one kettle, (2) 6 kettles of the same size?

13. If a train travels at the rate of 60 miles an hour, how far does it travel (1) in 3 hours, (2) in $1\frac{1}{2}$ hours?

14. If a bankrupt pays 13*s.* 4*d.* in the £, what does he pay on a debt of £3?

15. If a garrison has food which will last 1500 men for 14 days, how long would the food last for 3000 men?

16. If I buy eggs at 14 pence a score, how much do I give for 100 eggs?

17. What is the cost of 12 articles at $7\frac{1}{2}$*d.* each?

18. What is the cost of 20 things at 3*s.* each?

19. If 12 sauce-pans cost 9*s.* 6*d.*, how much did each cost?

20. If 23 eggs cost 18 pence, how much will 69 eggs cost?

21. If 6 men do a piece of work in 5 days, how long will 3 men take to do it?

22. If the wages of 8 men for a week amount to £21, what will be the wages of 2 men for (1) a week, (2) a fortnight?

23. If the income-tax is at 1*s.* in the £, what is the tax on an income of £200?

24. If 20 eggs cost 1*s.* 3*d.*, what will 80 eggs cost?

EXAMPLES X. b.

Write out the following, and fill in the blank spaces.

1. 7 lb. of coffee cost 154 pence.
∴ 1 lb. „ costs
∴ 9 lb. „ cost

2. 21 eggs can be bought for 18 pence.
∴ 1 egg „ „
∴ 56 eggs „ „

3. One shilling is the income-tax on £1.
∴ „ „ £250.

4. 6 men do a piece of work in 26 days.
∴ 1 man does the same
∴ 13 men do

5. A man pays no income-tax on £150, but on every £ above that amount he pays 8 pence. What does he pay when his income is £600?
He pays income-tax on (600 − 150 =) £450
On every taxable £ he pays 8 pence.
∴ on £450 „
i.e. „ „ shillings
= £

6. The wages of 9 men for a week amount to 144 shillings.
∴ „ „ 1 man „ „
∴ „ „ 7 men „ „
i.e. „ „ „ „ £. *s.*

7. $\frac{3}{8}$ of a share in a company is worth £57.
∴ $\frac{1}{8}$ „ „ the „ „
∴ one share in „ „ „

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Use factors if possible.*
(4) *Revise your work before proceeding to the next example.*

8. If 7 articles cost 7*s.* 10½*d.*, what is the price of 11 articles?

9. How much will 9 men earn in a week if 7 men earn £5. 8*s.* 6*d.* in the same time?

10. If I can travel 157 miles for 13*s.* 1*d.*, how far can I travel for 16*s.* 8*d.*?

11. How many oranges can I buy for 3*s.* 8*d.*, if 7 oranges cost 4*d.*?

12. If I can buy 6 lb. of apples for 1*s.* 4*d.*, how many lb. can I get for £1?

13. If 13 yards of linen cost 17*s.* 4*d.*, what will be the price of 20 yards?

14. How long will 7 men take to build a wall which 16 men can build in 14 days?

15. When the income-tax is at 7*d.* in the £, a man pays £9. 9*s.* 7*d.* What is his income?

16. If £13. 4*s.* 8*d.* is the interest on £350, what is the interest on £525?

17. If 254 centimetres = 100 inches, express 889 cm. in ft. and inches.

18. What is the cost of 9 lb. of tea if 4 lb. cost 7*s.* 4*d.*?

19. If 2 tons 10 cwt. of lead cost £78. 15*s.*, what is the price of 4 cwt.?

20. How long will a man take to walk $17\frac{1}{2}$ miles, if he walks at the rate of $3\frac{1}{2}$ miles an hour?

21. Taking 35 yards as equal to 32 metres, express 7 miles in metres.

22. If 13 shares in a company are worth £1235, how many shares can be bought for £1520?

23. If a field of 7 ac. 3 roods can be rented for £11. 12*s.* 6*d.*, what will be the rent of 9 ac. 2 roods?

24. If I buy eggs at 1*s.* 9*d.* a score, what do I pay for $7\frac{1}{2}$ dozen?

25. How much will it cost to keep 27 horses for a week, if the cost of 17 horses for the same time is £7. 13*s.*?

26. A man pays income-tax at the rate of 9*d.* in the £; what is the total tax when his income is £649. 10*s.*?

27. If 5 men plough a field in 8 days, how many men will it take to plough the same field in 10 days?

28. A man spends £74. 15*s.* in 65 days; what is his expenditure per annum?

29. If a train travels 56 miles in 64 minutes, what is its rate of travelling in miles per hour?

30. If 7 men plough a field in $5\frac{1}{2}$ days, how long would 11 men take to plough it?

31. A bankrupt pays 10*s.* 3*d.* in the £. How much would a man receive to whom the bankrupt owed £72. 10*s.*?

32. A bankrupt owes £2303, but can only pay £1353. 0*s.* 3*d.* How much does he pay in the £?

33. If £36. 5*s.* 6*d.* is the interest on £725. 10*s.*, what is the interest on £100?

34. A man pays interest at the rate of 4 per cent., *i.e.* he pays £4 for every £100. What interest does he pay on £637. 10*s.*?

35. When the income-tax is at 11*d.* in the £, a man pays £59. 11*s.* 8*d.* What is his income?

36. A bankrupt's assets amount to £650, and he pays 13*s.* 4*d.* in the £. How much does he owe?

37. A man gives £625 for some shares in a company and receives £25 each year as interest; how much does he receive for £100?

[This question is usually put thus: **How much does he receive per cent.?**]

38. A garrison of 1500 men has food which will provide for them for 39 days; how long would it last for 650 men?

39. A bicycle wheel in travelling 66 feet makes 9 revolutions; how many revolutions does it make in a mile?

40. A man pays income-tax on his rent, and hands over £54. 3*s.* to his landlord every year. What is the gross rental of the house, the income-tax being at the rate of 1*s.* in the £?

41. When the income-tax is at 10*d.* in the £, a man pays £3. 6*s.* 8*d.* more than when the tax is at 9*d.* in the £. What is his income?

42. A bankrupt's assets are £750, and his liabilities amount to £768. If the expense of winding up his affairs is £150, how much does he pay in the £? The expense of winding up has to be paid in full.

43. If I buy eggs at 15 pence a score, how much is this per dozen?

44. If I buy 75 sheep for £187. 10*s.*, how much more should I have to pay for 124 sheep?

45. 5 men can hoe a field in 9 days. After they have done half the field one extra man is employed. How many days will the extra man be employed?

46. If 6 men can plough a field in $3\frac{1}{2}$ days, how long will 7 boys take to plough the same field if 2 boys do the work of one man?

47. A man's net income, *i.e.* his income when he has paid his taxes, is £744. 5*s.* If he pays 11*d.* in the £, what is his gross income, *i.e.* his income before paying taxes?

48. A man's gross income is £635. 6*s.* 8*d.*, and his net income £603. 11*s.* 4*d.* What tax does he pay in the £?

49. If a man buys 500 eggs at 1*s.* 6*d.* a score, at how much a score must he sell them in order to make a profit of 12*s.* 6*d.*?

50. If 8 boys can do a piece of work in $9\frac{1}{2}$ days, and 1 man does the work of 2 boys, how long will 19 men take to do the work?

51. A man buys apples at 6 a penny and sells them at 4 a penny; how much does he gain by selling 168 apples?

52. 7 men can build a wall in 9 days. After half the wall is built, 2 extra men are employed. How long will the extra men be employed?

53. A man bought 400 oranges for £1, and found that 40 were bad. If he sold the rest at the rate of 12 for 9*d.*, what profit did he make?

XI. DECIMALS.

Powers of Ten.

83. The student has already learnt that $100 (= 10 \times 10)$ is often written as 10^2. 10^2 or 100 is called the *second power* of ten, the small 2 (written above) being called the *index*.

Similarly, $1000 = 10 \times 10 \times 10 = 10^3$, and 10^3 is called the third power of 10, 3 being the index.

In the same way, a million $= 1{,}000{,}000 = 10^6$ and is said to be the sixth power of 10.

Also $\frac{1}{100} = \frac{1}{10^2}$, $\frac{1}{1000} = \frac{1}{10^3}$ and so on, and these are said to be *inverse powers* of ten.

$\frac{1}{10^2}$ may be written as 10^{-2}, $\frac{1}{10^3}$ as 10^{-3}, and so on.

Now let us consider any number in the Arabic notation, say 7583.

Here the value denoted by the figure 7 is $7000 = 7 \times 10^3$.

" " " " 5 " $500 = 5 \times 10^2$.

" " " " 8 " $80 = 5 \times 10$.

We thus see that the value of any digit in a number depends upon its position.

84. The Decimal notation enables us to extend this principle to fractions.

In this system

·3 denotes $\frac{3}{10}$; read as "decimal 3" or "point 3."

·37 " $\frac{3}{10} + \frac{7}{100} = \frac{3}{10} + \frac{7}{10^2} = \frac{37}{100}$; read as "decimal three seven."

·374 " $\frac{3}{10} + \frac{7}{10^2} + \frac{4}{1000} = \frac{374}{1000}$.

·507 " $\frac{5}{10} + \frac{0}{10^2} + \frac{7}{10^3} = \frac{507}{1000}$.

·06 " $\frac{0}{10} + \frac{6}{10^2} = \frac{6}{100}$.

·0035 " $\frac{0}{10} + \frac{0}{10^2} + \frac{3}{10^3} + \frac{5}{10^4} = \frac{35}{10000}$.

$$73{\cdot}458 = 73 + \frac{4}{10} + \frac{5}{10^2} + \frac{8}{10^3} = 73\tfrac{458}{1000}.$$

$$60{\cdot}02 = 60 + \frac{0}{10} + \frac{2}{100} = 60\tfrac{2}{100}.$$

When there is no whole number, a cipher is often placed in front of the decimal point.

Thus ·35 may be written 0·35.

$$\text{Note that } \cdot 780 = \frac{7}{10} + \frac{8}{10^2} + \frac{0}{10^3} = \frac{78}{100}.$$

$$\text{Also } \cdot 7800 = \frac{78}{100}.$$

i.e. ciphers at the right hand of a decimal have no value. We can, however, sometimes use such ciphers with advantage.

N.B.—It is wrong to read ·37 as "decimal thirty-seven."

The numbers after (*i.e.* on the right-hand side of) the decimal point are called decimal places.

Thus in 3·457 there are 3 decimal places.
" 3·0468 " 4 "

The order of the powers of ten may be tabulated thus:

Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	
3	7	5	4	·2	6	8	= 3754·268.

85. To sum up the preceding:

A decimal fraction is equal to a vulgar fraction, whose numerator is the number obtained by omitting the decimal point and whose denominator is a power of 10. The index of the power is the same as the number of digits following the decimal point.

N.B.—The denominator is 1, followed by as many ciphers as there are decimal places.

When the denominator of a vulgar fraction is a power of 10, the fraction can at once be written in the form of a decimal.

$$\frac{7}{10} = \cdot 7. \quad \frac{7}{100} = \cdot 07.$$

$$3\tfrac{658}{1000} = 3 + \frac{600+50+8}{1000} = 3 + \frac{6}{10} + \frac{5}{10^2} + \frac{8}{10^3} = 3 \cdot 658.$$

$$7\tfrac{302}{1000} = 7 + \frac{300+2}{1000} = 7\cdot302.$$

$$5\tfrac{43}{1000} = 5 + \frac{0+40+3}{1000} = 5 + \frac{4}{10^2} + \frac{3}{10^3} = 5\cdot043.$$

$$6\tfrac{7}{10000} = 6 + \frac{0}{10} + \frac{0}{10^2} + \frac{0}{10^3} + \frac{7}{10^4} = 6\cdot0007.$$

86. Decimals are of the greatest use in dealing with the Metric System.

$$3 \text{ m. } 5 \text{ dm.} = 3\tfrac{5}{10} \text{ m.} = 3\cdot5 \text{ m.}$$

$$3 \text{ m. } 7 \text{ dm. } 8 \text{ cm.} = \left(3 + \frac{7}{10} + \frac{8}{10^2}\right) \text{m.} = 3\cdot78 \text{ m.}$$

Vice versa

$$658 \text{ cm.} = \frac{658}{100} \text{ m.} = 6\cdot58 \text{ m.}$$

$$37 \text{ cm.} = \frac{37}{100} \text{ m.} = 0\cdot37 \text{ m.}$$

$$35 \text{ mm.} = \frac{35}{1000} \text{ m.} = 0\cdot035 \text{ m.}$$

$$\begin{aligned} 6037 \text{ mm.} &= (6000+37) \text{ mm.} = \left(6 + \frac{37}{1000}\right) \text{m.} \\ &= 6\cdot037 \text{ m.} \\ &= \left(6 + \frac{0}{10} + \frac{3}{100} + \frac{7}{1000}\right) \text{m.} \\ &= 6 \text{ m. } 3 \text{ cm. } 7 \text{ mm.} \end{aligned}$$

$$\begin{aligned} 6\cdot257 \text{ m.} &= \left(6 + \frac{2}{10} + \frac{5}{10^2} + \frac{7}{10^3}\right) \text{m.} \\ &= 6 \text{ m. } 2 \text{ dm. } 5 \text{ cm. } 7 \text{ mm.} \end{aligned}$$

$$\begin{aligned} 3\cdot205 \text{ m.} &= \left(3 + \frac{2}{10} + \frac{0}{10^2} + \frac{5}{10^3}\right) \text{m.} \\ &= 3 \text{ m. } 2 \text{ dm. } 5 \text{ mm.} \end{aligned}$$

$$\begin{aligned} 3\cdot17 \text{ francs} &= \left(3 + \frac{17}{100}\right) \text{francs} \\ &= 3 \text{ fr. } 17 \text{ centimes.} \end{aligned}$$

$$7\cdot09 \text{ francs} = \left(7 + \frac{9}{100}\right) \text{francs} = 7 \text{ francs } 9 \text{ centimes.}$$

EXAMPLES XI. a. (*Oral.*)

The following are all powers of ten. Give the index in each case.

1. 1000. **2.** 10000. **3.** 10000000.

Express in words the value of each digit in the following :

4. 3·4. **5.** 46·825. **6.** ·016.
7. 625·002. **8.** 6·0035. **9.** 725·378.

10. Which is the greater? ·7 or ·65, and why?
11. „ „ ·04 or ·4, „
12. „ „ ·025 or ·006 „
13. „ „ ·6093 or ·72.

Read off the following as decimals :

14. Three and four-tenths. **15.** Forty and seven-tenths.
16. Six and 4 hundredths.
17. One hundred, 6 tenths and 3 hundredths.
18. 3 tenths and 4 thousandths. **19.** 5 ten-thousandths.

20. $\frac{86}{100}$. **21.** $\frac{806}{1000}$. **22.** $\frac{6}{10000}$. **23.** $\frac{63}{10^2}$. **24.** $\frac{871}{10^3}$.
25. $\frac{57}{10^3}$. **26.** $\frac{9}{10^3}$. **27.** $\frac{104}{10^4}$. **28.** $\frac{67}{10}$. **29.** $\frac{607}{10}$.
30. $\frac{8}{10^5}$. **31.** $\frac{65}{10}$. **32.** $\frac{691}{10}$. **33.** $\frac{5345}{100}$. **34.** $\frac{6394}{10}$.
35. $\frac{5026}{1000}$. **36.** $\frac{69347}{10}$. **37.** $\frac{8294}{1000000}$. **38.** $\frac{63025}{1000000}$.
39. $\frac{93004}{100}$. **40.** $\frac{625}{10^4}$. **41.** $\frac{621}{10^6}$. **42.** $\frac{623578}{10^6}$. **43.** $\frac{730502}{10^5}$.

Express each of the following as a vulgar fraction :

44. ·7. **45.** 0·2. **46.** 1·3. **47.** 1·06. **48.** 2·35. **49.** 3·406.
50. ·002. **51.** 1·07. **52.** 10·0406. **53.** 506·008.

Express each of the following as a decimal of a metre :

54. 5 cm. **55.** 7 dm. **56.** 8 mm. **57.** 5 cm. 3 mm.
58. 4 dm. 7 mm. **59.** 3 m. 8 cm. **60.** 3 m. 7 dm. 8 cm. 9 mm.
61. 8 cm. 4 mm.

Express each of the following as a vulgar fraction in its lowest terms :

62. ·5. **63.** ·4. **64.** ·2 **65.** 1·2.
66. 3·04. **67.** 0·25. **68.** 0·75. **69.** 1·6.

Express each of the following in francs and decimals of a franc.

70. 325 centimes. **71.** 209 centimes. **72.** 37 centimes.
73. 9 centimes.

($N.B.$—One-tenth of £1 = 2 shillings.)

74. What is the value of £2·5 in pounds and shillings?

75. „ „ £0·2 in shillings?

76. „ „ £0·5 „

77. „ „ £0·7 „

78. Express 2 shillings as a decimal of £1.

79. „ 8 „ „

80. „ 10 „ „

81. „ 14 „ „

82. Express £3. 2*s.* in pounds and decimals of a pound.

83. „ £5. 4*s.* „ „ „

84. „ £3. 10*s.* „ „ „

85. Express £0·4 in shillings.

86. Express £0·7 in shillings.

87. „ £1·6 „

Addition of Decimals.

87. Arrange the numbers to be added together underneath one another, so that the units are in the same column, the tenths are in the same column and so on. The decimal points thus appear in a column. Then use the method of addition employed for whole numbers.

EXAMPLE 1. **To add 30·754 and 1·892.**

$$\begin{array}{r} 30{\cdot}754 \\ 1{\cdot}892 \\ \hline 32{\cdot}646 \\ \hline \end{array}$$

Explanation. 2 thousandths + 4 thousandths = 6 thousandths.
9 hundredths + 5 hundredths = 14 hundredths
= 1 tenth + 4 hundredths.

Adding the one-tenth into the next column, we have
1 tenth + 8 tenths + 7 tenths = 16 tenths = 1·6 and so on.

EXAMPLE 2. **Add together 30·794, 1·6 and 314·02.**

$$\begin{array}{r} 30{\cdot}794 \\ 1{\cdot}600 \\ 314{\cdot}020 \\ \hline 346{\cdot}414 \\ \hline \end{array}$$

The beginner had better place ciphers after the decimal places where necessary to complete the columns. After a very little practice such ciphers may be omitted.

EXAMPLE 3. **Add together ·0065, ·0072 and ·072.**

$$\begin{array}{r} \cdot 0065 \\ \cdot 0072 \\ \underline{\cdot 0720} \\ \underline{\cdot 0857} \end{array}$$

Subtraction of Decimals.

88. Place the number to be subtracted underneath the other, so that the units are in the same column, the tenths in the same column and so on as in addition. Then use the method of subtraction employed in dealing with whole numbers.

EXAMPLE 1. **Subtract 7·6 from 13·285.**

$$\begin{array}{r} 13\cdot 285 \\ \underline{7\cdot 600} \\ \underline{5\cdot 685} \end{array}$$

(After a little practice the ciphers placed to complete the columns after the decimal places may be omitted.)

EXAMPLE 2. **Subtract 31·9065 from 43·89.**

$$\begin{array}{r} 43\cdot 8900 \\ \underline{31\cdot 9065} \\ \underline{11\cdot 9835} \end{array}$$

EXAMPLES XI. b. (*Oral.*)

Read off the answers to the following additions :

1. $\begin{array}{r} 1\cdot 36 \\ \underline{2\cdot 30} \end{array}$ **2.** $\begin{array}{r} 3\cdot 8562 \\ \underline{\cdot 6957} \end{array}$ **3.** $\begin{array}{r} \cdot 0068 \\ \underline{\cdot 083} \end{array}$ **4.** $\begin{array}{r} 6\cdot 7245 \\ \cdot 023 \\ \underline{1\cdot 0057} \end{array}$

5. $\begin{array}{r} 341\cdot 6531 \\ 21\cdot 035 \\ 15 \\ \underline{1\cdot 92} \end{array}$ **6.** $\begin{array}{r} 1\cdot 0002 \\ 251\cdot 98 \\ \underline{3\cdot 605} \end{array}$ **7.** $\begin{array}{r} \cdot 457 \\ \cdot 83 \\ \underline{\cdot 1256} \end{array}$ **8.** $\begin{array}{r} \cdot 001 \\ \cdot 0002 \\ \underline{\cdot 01} \end{array}$

9. ·3 + ·6. **10.** ·6 + ·2. **11.** ·3 + ·7. **12.** ·2 + ·3 + ·4.
13. ·5 + ·5. **14.** ·75 + ·25. **15.** ·02 + ·05 + ·06.
16. 1·2 + 1·8. **17.** 4·3 + 1·7. **18.** 4·1 + ·9.
19. 1·2 + 1·3 + 1·5. **20.** 3 + ·4 + ·6. **21.** ·001 + ·01.
22. ·0001 + ·001. **23.** ·025 + ·075. **24.** ·2 + ·02.
25. ·025 + ·7. **26.** ·02 + ·3 + ·004. **27.** ·7 + ·625.
28. 1·2 + 1·03 + ·045. **29.** ·01 + ·0001. **30.** 1·01 + 9·09.
31. ·0095 + ·001. **32.** ·625 + ·375 + 1·62. **33.** ·024 + ·076 + ·629.

Complete the following subtractions :

34. 2·46	35. 3·065	36. 1·0	37. 1·75	38. ·001
1·82	1·89	·5	1·25	·0001

39. ·7 – ·2. 40. 1·5 – ·3. 41. 3·2 – 1·4. 42. ·05 – ·03.
43. 8·2 – 7·5. 44. 8 – ·5. 45. 10 – ·7. 46. 9 – 0·4.
47. 12·723 – 9. 48. 7 – 5·23. 49. 7·5 – 5·7. 50. 2 – 0·03.
51. 6·09 – 1·2.
52. 3 m. – 5 dm. Give your answer in decimals of a metre.
53. 5 m. – 2·5 m. " " "
54. 7 m. – 3 dm. " " "
55. 9 m. – 3·5 dm. " " "
56. 1 m. – 7 cm. " " "
57. 3 m. – 7 dm. Give your answer in decimetres.
58. 3·7 m. – 9 dm. " " "
59. 3·75 m. – 5 dm. " " "
60. 2·34 m. – 7 cm. " " "

EXAMPLES XI. c.

Add together :

1. 3·54, 5·67, 0·3, 1·257.
2. ·0645, 81·236, 8·1236, 1·23.
3. 9·833471, 0·000625, 725, 3·62.
4. 125·006, 25·04, 6·9, 1·054.
5. ·1, ·01, ·001, ·0001.
6. 9·9, 0·09, 0·009, 6·001.
7. 6·0025, 5·53, 0·271, 2·1965.
8. ·35691, ·0248, ·625, ·0034.
9. 3·7205, 31·0382, 0·0381, 241.
10. 239·5, ·0006, 72·01, 3·456.

Find the sum of

11. 7 m. 2 cm. 5 mm., 6 dm. 1 cm. 8 mm., 2 cm. 3 mm. and 1 m. 4 cm. 4 mm. Give your answer in metres.
12. 9 Km. 7 Hm. 2 Dm., 1 Km. 1 Hm. 9 Dm., 2 Hm. 7 Dm. and 1 Km. 2 Dm. Give your answer in kilometres.

Find the value of

13. £6·35 + £3·2 + £1·45.
14. £6·25 + £2·15 in pounds and shillings.
15. 3·75*s.* + 0·5*s.* + 1·75*s.*
16. £6·025 + £5·31 + £7·165.
17. 4·05 pence + 3·2 pence + 1·75 pence.
18. 12·5*s.* + 4·3 pence + 15·7 pence.

Subtract

19. 6·25 from 8·7.
20. 3·025 from 10·8034.
21. 18·0003 " 21·92675.
22. 11·006 " 19·75.
23. 324·6895 " 1037·008.
24. 11·75 pence " 13·25 pence.
25. 8·375 shillings from 15·62 shillings.
26. 10 m. 7 dm. 3 mm. from 17 m. 9 dm. 1 mm.

Find the difference between

27. 3·06 and 7·825. 28. 0·1675 and 0·038.

29. 3 m. 7 dm. and 7 m. 2 dm. Give your answer in metres.

30. 3 cm. and 9 dm. ,, ,, centimetres.

31. What must be subtracted from 14·006 in order to leave 9·62 ?

32. What must be added to ·0036 in order to make ·5 ?

33. Find the difference between 17 m. 3 dm. 4 cm. and 156·7 dm. Give your answer in decimetres, also in centimetres.

Simplify the following :

34. $1·5+2·7-1·6$. 35. $3+0·0025-0·9$.

36. $6·5-(2+3·5)$. 37. $6·8-3·02+4·035$.

38. $7·83-(5-2·5)$. 39. $12·75s.-(3·2s.+1·05s.)$.

40. 4·6375 pence − (9·827 pence − 8·6895 pence).

Express each of the following in kilometres, and then find their sum :

41. 7 Km. 95 m., 25 Hm. 7 Dm., 19 Km. 7 Hm.

42. Find the difference between 3·7892 m. and 498 cm., giving your answer in metres.

43. Find the difference between 375806 cm. and 3·5832 Km. Give your answer in kilometres.

44. A room is 17·4 ft. long and 13·5 ft. wide ; what length of skirting board will be required to go all round the room ?

45. A piece of ground has sides of 43·3 m., 45·8 m., 37·6 m. and 29·3 m. ; how far does a man walk in going round it ? Give your answer in metres and also as a decimal of a kilometre.

Multiplication and Division of Decimals by 10 and powers of 10.

89. Take any decimal, say 658·34792, and compare it with 6583·4792.

The digit	6	in the first	denotes	600 ;	in the second	it denotes	6000.
,,	5	,,	,,	50 ;	,,	,,	500.
,,	8	,,	,,	8 units ;	,,	,,	80.
,,	3	,,	,,	3 tenths ;	,,	,,	3 units.
,,	4	,,	,,	4 hundredths ;	,,	,,	4 tenths,
				and so on.			

Thus we see that $658·34792 \times 10 = 6583·4792$.

The same evidently holds whatever decimal we use, and therefore we have the following rule :

To multiply a decimal by 10, move the decimal point one place to the right.

It follows that if we *multiply a decimal by* 100 ($=10^2$), we must merely move the decimal point **two** places to the right; if we multiply by 1000 ($=10^3$) we move the decimal point three places to the right, and so on.

We might express this in the following way:

$$658{\cdot}34792 = 658 + \frac{3}{10} + \frac{4}{10^2} + \frac{7}{10^3} + \frac{9}{10^4} + \frac{2}{10^5}.$$

Therefore $658{\cdot}34792 \times 10 = 6580 + 3 + \frac{4}{10} + \frac{7}{10^2} + \frac{9}{10^3} + \frac{2}{10^4}$

$$= 6583{\cdot}4792.$$

90. It is evident from the above that if we wish to ***divide a decimal by 10, we must move the decimal point one place to the left.***

If we divide a decimal by 100 ($=10^2$) we must move the decimal point 2 places to the left, and so on.

EXAMPLES. $1{\cdot}05706 \times 10 = 10{\cdot}5706$. $\quad {\cdot}3046 \times 10 = 3{\cdot}046$.

${\cdot}0052 \times 10 = 0{\cdot}052$. $\quad 72{\cdot}006 \times 100 = 7200{\cdot}6$.

${\cdot}00675 \times 1000 = 6{\cdot}75$.

$72{\cdot}853 \div 10 = 7{\cdot}2853$. $\quad 72{\cdot}853 \div 100 = {\cdot}72853$.

${\cdot}0685 \div 10 = {\cdot}00685$. $\quad {\cdot}0685 \div 100 = {\cdot}000685$.

$72{\cdot}329 \div 1000 = {\cdot}072329$.

Standard Form. A decimal is said to be in *standard form* when the decimal point appears immediately after the first digit reckoned from the left.

$3{\cdot}612$, $4{\cdot}025$ are examples of such.

$63{\cdot}54 = 6{\cdot}354 \times 10$, and is now in standard form

$329{\cdot}51 = 3{\cdot}2951 \times 10^2$, ,, ,, ,,

${\cdot}37 = 3{\cdot}7 \times \frac{1}{10}$, ,, ,, ,,

${\cdot}023 = 2{\cdot}3 \times \frac{1}{10^2}$, ,, ,, ,,

${\cdot}00257 = 2{\cdot}57 \times \frac{1}{10^3}$, ,, ,, ,,

We thus see that by using powers or inverse powers of ten, any decimal may be expressed in standard form.

EXAMPLES XI. d. (*Oral.*)

Multiply

1. 7·32 by 10. **2.** 0·542 by 10. **3.** 0·035 by 10.
4. 7·032 by 100. **5.** 0·56 by 100. **6.** 5·3 by 100.
7. ·005 by 100. **8.** ·005 by 1000. **9.** 6·305 by 1000.
10. 83·9 by 1000. **11.** 72·35 by 1000. **12.** 62·5361 by 1000.
13. 37·5892 by 10,000. **14.** ·035678 by 10,000. **15.** ·00006 by 10,000.
16. ·000354 by 1,000,000. **17.** 6·000354 by 1,000,000.
18. 18·005 by 1,000,000.

Divide

19. 7·5 by 10. **20.** ·35 by 10. **21.** 78·6 by 10.
22. ·0065 by 10. **23.** 1·0023 by 10. **24.** 35·6 by 100.
25. ·5 by 100. **26.** 50·7 by 100. **27.** 354·29 by 1000.
28. 6389·6 by 10. **29.** 527 by 10. **30.** 324 by 100.
31. 9·625 by 10. **32.** 90·3 by 100. **33.** 72·61 by 1000.
34. 8259·604 by 1000. **35.** ·02 by 100. **36.** 6·02 by 100.
37. 825·6 by 10,000. **38.** 82·89 by 10,000. **39.** 8·14 by 10,000.
40. 62571 by 10,000. **41.** 6914·3 by 10,000. **42.** ·5 by 10,000.

Express the following in standard form:

43. 56·2. **44.** 601·35. **45.** 11·374. **46.** 0·75.
47. 0·328. **48.** ·0379. **49.** ·01289. **50.** ·0035.
51. 628357·14. **52.** 93805·02. **53.** ·000006. **54.** 8375.

55. Multiply 3 m. 5 dm. 7 cm. by 10.
56. ,, ,, ,, 100.
57. Divide ,, ,, 10.
58. Multiply 1 Km. 7 Hm. 9 m. by 10.
59. ,, ,, ,, 100.
60. Divide ,, ,, 10.

Take the length 3 m. 4 dm. 7 mm.

61. What is its value in metres?
62. What is the result in metres when multiplied by 10?
63. ,, ,, ,, ,, 100?
64. What is the value of the same length in decimetres?
65. What is the result in decimetres when multiplied by 10?
66. ,, ,, ,, ,, ,, 100?

Take the length 5 m. 6 mm.

67. What is its value in metres?
68. What is the result in metres when multiplied by 10?
69. ,, ,, ,, ,, 100?

70. What is the value of the same length in decimetres?
71. What is the result in decimetres when multiplied by 10?
72. „ „ „ „ „ 100?
73. What is the value of the same length in millimetres?
74. „ result in millimetres when multiplied by 10?

Multiplication of Decimals.

91. (*a*) $3{\cdot}426 \times 4 = \frac{3426}{1000} \times 4 = \frac{13704}{1000} = 13{\cdot}704.$

(*b*) $13{\cdot}027 \times 7 = \frac{13027}{1000} \times 7 = \frac{91189}{1000} = 91{\cdot}189.$

We see that we may multiply as if the decimals were whole numbers, provided that in the result we place the decimal point in the correct position.

In example (*a*) we notice that the answer should be rather greater than $3 \times 4 (=12)$. This supplies a rough check to the work.

Similarly, in example (*b*) the answer should be somewhat greater than $13 \times 7 (=91)$.

(*c*) $\cdot 00347 \times 5 = \frac{347}{10^5} \times 5 = \frac{1735}{10^5} = \cdot 01735.$

(*d*) $125{\cdot}0245 \times 6 = \frac{1250245}{10^4} \times 6 = \frac{7501470}{10^4} = 750{\cdot}1470 = 750{\cdot}147.$

Again we see that we may multiply as in whole numbers, placing the decimal point in its correct position.

When we multiply by a single integral digit, the number of decimal places in the result is the same as the number of decimal places in the multiplicand.

EXAMPLES XI. e. (*Oral.*)

Read off the answers to the following multiplications:

1. $2{\cdot}6 \times 2$.	**2.** $3{\cdot}5 \times 3$.	**3.** $5{\cdot}7 \times 4$.	**4.** $\cdot 4 \times 4$.
5. $\cdot 06 \times 3$.	**6.** $\cdot 5 \times 2$.	**7.** $\cdot 4 \times 5$.	**8.** $1{\cdot}5 \times 4$.
9. $\cdot 003 \times 6$.	**10.** $6{\cdot}01 \times 5$.	**11.** $12{\cdot}32 \times 2$.	**12.** $\cdot 0065 \times 6$.
13. $\cdot 75 \times 4$.	**14.** $\cdot 125 \times 8$.	**15.** $\cdot 00125 \times 2$.	**16.** $71{\cdot}02 \times 7$.
17. $31{\cdot}5 \times 4$.	**18.** $1{\cdot}25 \times 8$.	**19.** $6{\cdot}25 \times 4$.	**20.** $67{\cdot}5 \times 8$.
21. $\cdot 25 \times 2 \times 4$.	**22.** $5 \times 1{\cdot}2 \times 2$.	**23.** $6 \times \cdot 6 \times 3$.	**24.** $4 \times \cdot 8 \times 3$.

Multiply

25. 4 m. 3 dm. 7 cm. by 3.

26. 8 Km. 7 Hm. 5 Dm. by 4.

27. 5 m. 4 cm. by 4.

28. 8 Km. 5 Dm. by 7.

29. 4 francs 25 centimes by 4.

30. 3 francs 25 centimes by 6.

31. 9 francs 7 centimes by 7.

32. 1 franc 5 centimes by 100.

33. 2 francs 20 centimes by 5.

92. In all the examples of the preceding article, the multiplier is in standard form. It is a convenient and safe plan, for reasons which will be better understood when approximations and contracted methods are used, to ***always arrange the multiplier in standard form,*** unless the multiplication is of a simple character.

Thus $84{\cdot}357 \times {\cdot}93 = 84{\cdot}357 \times \frac{9{\cdot}3}{10} = 8{\cdot}4357 \times 9{\cdot}3$ (9·3 is now in standard form)

$$= \frac{84357}{10^4} \times \frac{93}{10} = \frac{7845201}{10^5} = 78{\cdot}45201.$$

As a rough check to the above, we notice that the result should be somewhat greater than $9 \times 8\,(=72)$.

We again notice that the actual multiplication may be performed as if the numbers were whole numbers.

We may therefore arrange the work as below.

```
 8·4357
 9·3
-------
75·9213
 2·53071
--------
78·45201
```

→ *N.B.*—The number of decimal places *in the first line of the working* is the same as the number of decimal places in the multiplicand.

EXAMPLE 1. **Multiply 35·023 by 406·5.**

$$35{\cdot}023 \times 406{\cdot}5 = 35{\cdot}023 \times 4{\cdot}065 \times 10^2 = 3502{\cdot}3 \times 4{\cdot}065.$$

(The answer will be rather greater than 3502×4, *i.e.* 14008.)

```
 3502·3
    4·065
----------
14009·2
  210·138
   17·5115
----------
14236·8495
```

→ *N.B.*—See the remark made about the first line of the working in the above example.

Each line of multiplication should be checked by division.

EXAMPLE 2. **Multiply ·00267 by ·0625.**

$$\cdot00267 \times \cdot0625 = \cdot00267 \times \frac{6\cdot25}{10^2} = \cdot0000267 \times 6\cdot25.$$

$$\begin{array}{l} \cdot0000267 \\ \underline{6\cdot25} \\ \cdot0001602 \\ \cdot00000534 \\ \underline{\cdot000001335} \\ \cdot000166875 \end{array}$$

→ *N.B.*—7 decimal places.

As a check, we see that the result is a little less than

$\cdot00003 \times 6$, *i.e.* $\cdot00018$.

Another check. The number of decimal places in the result is always equal to the sum of the number of decimal places in the multiplier and the multiplicand.

93. As a general rule the multiplier should be reduced to standard form, but in some simple cases this may be dispensed with, and the fact mentioned above may be used to determine the position of the decimal point.

EXAMPLE 1. **Multiply ·02 by ·04.**

There will be 4 decimal places in the result, and $2 \times 4 = 8$.
Therefore the required product is ·0008.

EXAMPLE 2. **Multiply ·25 by ·008.**

There will be 5 decimal places in the result, and $25 \times 8 = 200$.
Therefore the required product $= \cdot00200 = \cdot002$.

N.B.—The two ciphers at the end of ·00200 count as decimal places.

EXAMPLE 3. **A room is 17·3 metres long and 13·5 metres wide. What is its area (1) in sq. m., (2) in sq. dekam., (3) in sq. hectom.?**

Area = length × breadth.

$17\cdot3 \times 13\cdot5 = 233\cdot55$ (the multiplication is left to be done by the student).

∴ the area of the room = 233·55 sq. m.
= 2·3355 sq. dekam., for 100 sq. m. = 1 sq. dekam.
= ·023355 sq. hectom., for 100 sq. dekam. = 1 sq. hectom.

Useful facts. $4 \times \cdot25 = 1$, and hence $4 \times 2\cdot5 = 10$ and $\cdot4 \times \cdot25 = \cdot1$.
$8 \times \cdot125 = 1$, „ $8 \times 1\cdot25 = 10$ and $\cdot8 \times \cdot125 = \cdot1$.
$8 \times \cdot25 = 2$.

EXAMPLES XI. f.

[*Give a rough check of the result where possible.*]

Find the product in each of the following:

1. $6\cdot125 \times 3\cdot12$.	**2.** $41\cdot63 \times 31\cdot5$.	**3.** $\cdot32 \times 26$.
4. $12\cdot36 \times \cdot042$.	**5.** $\cdot0037 \times 212\cdot5$.	**6.** $11\cdot625 \times \cdot0036$.
7. $40\cdot025 \times 102\cdot8$.	**8.** $3\cdot125 \times 4236\cdot2$.	**9.** $\cdot01 \times \cdot000625$.

Find the product in each of the following :

10. $2{\cdot}0631 \times {\cdot}00034$.
11. $6{\cdot}3875 \times 632$.
12. $1{\cdot}0036 \times {\cdot}0305$.
13. ${\cdot}000375 \times 1408$.
14. ${\cdot}00625 \times 524{\cdot}6$.
15. ${\cdot}0074 \times {\cdot}0025$.
16. $3964 \times {\cdot}00025$.
17. $1062{\cdot}5 \times {\cdot}0031$.
18. $170{\cdot}3418 \times {\cdot}0625$.
19. $3{\cdot}67859 \times {\cdot}0326$.
20. $183{\cdot}57 \times 1000{\cdot}1$.
21. $1{\cdot}2 \times {\cdot}5 \times 2{\cdot}4$.
22. $1{\cdot}3 \times 4 \times {\cdot}8$.
23. ${\cdot}02 \times {\cdot}8 \times {\cdot}6$.
24. $3{\cdot}5 \times {\cdot}4 \times {\cdot}001$.
25. ${\cdot}13 \times 1{\cdot}3 \times {\cdot}0001$.
26. $1{\cdot}125 \times 8 \times {\cdot}5$.
27. ${\cdot}000125 \times {\cdot}08$.
28. ${\cdot}0061 \times {\cdot}00345$.

29. Using as few figures as possible, find the difference between $20 \times 6{\cdot}2$ and $20 \times 6{\cdot}3$.

30. Find the sum of $13 \times 14{\cdot}7$ and $13 \times 5{\cdot}3$, using as few figures as possible.

31. Find the sum of $17 \times 3{\cdot}27$ and $17 \times 1{\cdot}73$.

32. What is the value of £6·125 in shillings?

33. " " 3·625*s.* in pence?

34. " " £3·125 in pence?

35. " " £1·25 in pence?

36. " " £2·0625 in shillings?

37. " " 1·026 tons in cwt.?

38. " " 1·342 lbs. Troy in dwt.?

39. " " 3·035 miles in yards?

40. A room is 13·5 feet long and 12·5 feet wide. What is its area in square feet?

41. What is the area in sq. ft. of a plank 13·4 ft. long and 0·75 ft. wide?

42. A piece of cardboard is ·24 m. long by 6·5 cm. wide. What is its area (1) in sq. cm., (2) in sq. m.?

43. The circumference of a circle may be taken to be its radius × 6·2832. What is the circumference (in inches) of a circle of 15 feet radius?

EXAMPLES XI. g. (*Oral.*)

Read off, or write down, the following products :

1. ${\cdot}2 \times {\cdot}3$.
2. ${\cdot}02 \times {\cdot}3$.
3. ${\cdot}01 \times {\cdot}02$.
4. $1{\cdot}04 \times {\cdot}2$.
5. $6{\cdot}25 \times {\cdot}4$.
6. $1{\cdot}25 \times {\cdot}8$.
7. $62{\cdot}5 \times {\cdot}04$.
8. $6{\cdot}025 \times 80$.
9. ${\cdot}0125 \times 800$.
10. ${\cdot}2 \times {\cdot}2 \times {\cdot}2$.
11. ${\cdot}2 \times {\cdot}02 \times {\cdot}002$
12. ${\cdot}3 \times {\cdot}3 \times {\cdot}3$.
13. ${\cdot}03 \times {\cdot}03 \times {\cdot}03$.
14. ${\cdot}3 \times {\cdot}03 \times {\cdot}003$.
15. $400 \times {\cdot}00125$.
16. $8000 \times 6{\cdot}125$.
17. $64 \times {\cdot}125$.
18. ${\cdot}125 \times 56$.
19. ${\cdot}13 \times {\cdot}13$.
20. $1{\cdot}3 \times 1{\cdot}3$.
21. $16 \times {\cdot}0125$.
22. $1{\cdot}6 \times {\cdot}125$.
23. $25 \times {\cdot}012$.
24. $2{\cdot}5 \times {\cdot}012$.

Division of Decimals.

94. The arguments and methods used in multiplication of decimals prove that in division of decimals we may perform the division as if the decimals were whole numbers, placing the decimal point in its correct position.

It will be found that the position of the decimal point is easily determined ***if the divisor is arranged in standard form.***

Before proceeding to general cases, some practice in dividing by single integers (which of course are in standard form) is advisable.

EXAMPLE 1. **Divide 125·048 by 8.**

$\frac{125}{8}=15+\text{a fraction}$. Therefore we see that the result required will be 15 + a decimal.

Hence
$$\begin{array}{r|l} 8 & 125{\cdot}048 \\ \hline & 15{\cdot}631 \end{array}$$

Read the following carefully :

$$125{\cdot}048 \div 8 = 15{\cdot}631 \text{ as above.}$$

Similarly, $12{\cdot}5048 \div 8 = 1{\cdot}5631.$ ....**(a)**

Now $1{\cdot}25048=\frac{1}{10}\times 12{\cdot}5048.$

Therefore $\frac{1{\cdot}25048}{8}=\frac{1}{10}\times\frac{12{\cdot}5048}{8}=\frac{1}{10}\times 1{\cdot}5631$ (See *a*)

$$={\cdot}15631.$$

Or thus,
$$\begin{array}{r|l} 8 & 1{\cdot}25048 \\ \hline & {\cdot}15631 \end{array}$$

Again, ${\cdot}125048=\frac{1}{10}(1{\cdot}25048).$

Therefore $\frac{{\cdot}125048}{8}=\frac{1}{10}\left(\frac{1{\cdot}25048}{8}\right)=\frac{1}{10}({\cdot}15631)={\cdot}015631.$

Or thus,
$$\begin{array}{r|l} 8 & {\cdot}125048 \\ \hline & {\cdot}015631 \end{array}$$

EXAMPLE 2. **Divide ·000125048 by 8.**

$$\begin{array}{r|l} 8 & {\cdot}000125048 \\ \hline & {\cdot}000015631 \end{array}$$

EXAMPLE 3. **Divide 125 by 8, expressing the result as a decimal.**

$$\begin{array}{r|l} 8 & 125{\cdot}000 \\ \hline & 15{\cdot}625 \end{array}$$

N.B.—We may always add as many ciphers as are necessary after the decimal point in an integral dividend.

95. ***We may multiply, or divide, both dividend and divisor by 10, or by any the same power of 10, without altering the result.***

In other words, we may move the decimal point right or left the same number of places in both dividend and divisor without altering the result.

EXAMPLES. $$\frac{3{\cdot}0456}{{\cdot}005}=\frac{3{\cdot}0456\times 1000}{{\cdot}005\times 1000}=\frac{3045{\cdot}6}{5}.$$

$$\frac{{\cdot}009826}{{\cdot}125}=\frac{9{\cdot}826}{125}.$$

$$\frac{6934{\cdot}02}{5000}=\frac{6{\cdot}93402}{5}.$$

Some Simple Cases of Division.

96. $${\cdot}8\div{\cdot}2=\frac{{\cdot}8}{{\cdot}2}=\frac{8}{2}=4.$$

$$\frac{{\cdot}08}{{\cdot}2}=\frac{8}{20}=\frac{4}{10}={\cdot}4.$$

$$\frac{{\cdot}000169}{{\cdot}000013}=\frac{169}{13}=13.$$

$$\frac{{\cdot}625}{1{\cdot}25}=\frac{625}{1250}=\frac{1}{2}={\cdot}5$$

$$\text{or}\quad =\frac{62{\cdot}5}{125}=\frac{125\times{\cdot}5}{125}={\cdot}5.$$

$${\cdot}8\div{\cdot}05=\frac{{\cdot}8}{{\cdot}05}=\frac{80}{5}=16.$$

$$\frac{6{\cdot}125}{{\cdot}5}=\frac{61{\cdot}25}{5}=12{\cdot}25.$$

$$\frac{6{\cdot}25}{{\cdot}125}=\frac{6250}{125}=50.$$

$$\frac{{\cdot}00096}{{\cdot}0128}=\frac{9{\cdot}6}{128}=\frac{\cancel{8}\times 1{\cdot}2}{\cancel{8}\times 16}=\frac{\cancel{4}\times{\cdot}3}{\cancel{4}\times 4}$$
$$={\cdot}075.$$

EXAMPLES XI. h. (*These may be taken orally.*)

Divide

1. 12·48 by 3.	**2.** 5·602 by 2.	**3.** 7·804 by 4.
4. 13·025 by 5.	**5.** 126·028 by 7.	**6.** 1263·125 by 5.
7. 1·26 by 2.	**8.** 1·35 by 3.	**9.** 2·684 by 4.
10. 1·565 by 5.	**11.** ·56 by 2.	**12.** ·378 by 3.
13. ·568 by 4.	**14.** ·168 by 4.	**15.** 1·7892 by 7.
16. ·546 by 6.	**17.** ·744 by 8.	**18.** ·966 by 7.
19. ·024 by 2.	**20.** ·056 by 4.	**21.** ·3642 by 6.
22. ·0025 by 2.	**23.** ·0039 by 3.	**24.** ·00135 by 5.
25. 12·56 by ·2.	**26.** 1·356 by ·3.	**27.** 16·8 by ·4.
28. ·625 by ·5.	**29.** 6·25 by ·5.	**30.** 62·5 by ·5.
31. 10 by 8.	**32.** 1 by 8.	**33.** ·1 by 8.
34. ·1 by ·8.	**35.** ·1 by ·08.	**36.** ·01 by ·8.
37. ·625 by ·125.	**38.** 6·25 by ·125.	**39.** 625 by ·125.
40. 6·034 by ·2.	**41.** 1·68 by 1·2.	**42.** 1·68 by ·12.

97. EXAMPLE 1. **Divide 9·729 by 2·3.**

The divisor 2·3 is in standard form.

$\frac{9}{2} = 4 +$ a fraction. Thus the answer appears to be $4 +$ a decimal.

Hence the work will stand thus:

```
      4·23  (quotient)
2·3 ) 9·729
      9·2
      ---
       52
       46
       --
        69
        69
        --
```

Explanation. 23 goes 4 times into 97. Therefore we place 4 above the **9.** Then follows the decimal point, as shown above, and the rest of the working is the same as in ordinary division of whole numbers.

It is always advisable, in long division, to write the quotient **above the dividend**, and to arrange it so that the decimal point in the quotient comes immediately above the decimal point in the dividend.

Read the following carefully, and note how in each case the position of the decimal point is arrived at.

```
       ·0002...              13· etc.              1074·
3·25 ) ·0007289     7·05 ) 97·348       8·71 ) 9354·602...
           650             70 5                871
           ---             ----                ---
            789            26 84                644 6
            etc.           21 15                609 7
                           -----                -----
                             etc.                34 90
                                                   etc.
```

EXAMPLE 2. **Divide ·09729 by 2·3.**

The divisor is already in standard form.

Rough estimate. Evidently $\frac{\cdot 09729}{2} = \cdot 04$, etc., showing us that there is no whole number in the result and that there will be one cipher immediately after the decimal point.

```
      ·0423  (quotient)
2·3 ) ·09729
        92
        --
         52
         46
         --
          69
          69
          --
```

EXAMPLE 3. **Divide 887·25 by 27·3.**

We must first arrange the divisor in standard form.

We do this by dividing both divisor and dividend by ten.

Thus $\dfrac{887{\cdot}25}{27{\cdot}3}=\dfrac{88{\cdot}725}{2{\cdot}73}$.

Rough estimate. Also $\dfrac{88{\cdot}725}{2}=44+\text{a}$ decimal, showing us that there will be a whole number in the result, consisting of two digits; but we must notice that as we are dividing by 2·73 (more than 2) the result will not be as great as 44.

$$\begin{array}{r@{\,}l}
 & \underline{\;32{\cdot}5\;\;\;\;\;\;} \\
2{\cdot}73\,) & 88{\cdot}725 \\
 & \underline{81{\cdot}9\;\;\;} \\
 & \;\;6\;82\; \\
 & \;\;\underline{5\;46\;} \\
 & \;\;1\;365 \\
 & \;\;\underline{1\;365}
\end{array}$$

Useful check. ***The sum of the number of decimal places in the quotient and divisor is equal to the number of decimal places in the dividend (including any ciphers added to the dividend).***

EXAMPLE 4. **Divide ·040702 by 173·2.**

$$\frac{{\cdot}040702}{173{\cdot}2}=\frac{{\cdot}00040702}{1{\cdot}732}.$$

$$\begin{array}{r@{\,}l}
 & \underline{{\cdot}000235\;\;\;} \\
1{\cdot}732\,) & {\cdot}000407020 \\
 & \;\;\;\;\;\underline{3464}\;\;\;\; \\
 & \;\;\;\;\;\;\;6062\;\;\; \\
 & \;\;\;\;\;\;\;\underline{5196}\;\;\; \\
 & \;\;\;\;\;\;\;\;\;8660 \\
 & \;\;\;\;\;\;\;\;\;\underline{8660}
\end{array}$$

Sometimes a *long* division may be avoided.

EXAMPLE 5. **Divide 1·5375 by ·00125.**

$$[8\times{\cdot}125=1.]$$

$$\frac{1{\cdot}5375}{{\cdot}00125}=\frac{153{\cdot}75}{{\cdot}125}=\frac{153{\cdot}75\times 8}{1}=1230{\cdot}00.$$

We may sometimes use division by factors, in which case it is advisable to reduce the divisor to a whole number.

EXAMPLE 6. **Divide 3·60258 by ·24.**

$$\frac{3{\cdot}60258}{{\cdot}24}=\frac{360{\cdot}258}{24}=\frac{60{\cdot}043}{4}=15{\cdot}01075.$$

EXAMPLES XI. k.

The following examples, in all of which the actual division is very simple, will afford practice in determining the position of the decimal point:

Divide

1. ·008 by ·002.
2. ·64 by ·4.
3. ·5 by ·2.
4. ·028 by ·4.
5. ·015 by ·03.
6. ·125 by ·05.
7. 6·125 by ·5.
8. 12 by ·3.
9. 12·5 by ·5.
10. ·0128 by ·008.
11. ·01 by ·1.
12. ·003 by ·0003.
13. ·67125 by ·005.
14. ·1 by ·001.
15. 6 by ·06.
16. ·0441 by ·07.
17. 90 by ·003.
18. ·0001 by ·01.
19. 1 by ·002.
20. ·031 by 100.

EXAMPLES XI. l.

[*Check each result, especially as regards the position of the decimal point.*]

Divide

1. 7·02 by 2·6.
2. 2·106 by ·39.
3. 57·5 by 2·3.
4. 3702·3 by 12·3.
5. 3·339 by ·126.
6. 6·01344 by ·2304.
7. 164·4464 by 52·04.
8. 1952·748 by 602·7.
9. 7605960 by 21·04.
10. ·05814085 by ·06205.
11. ·00135 by 27.
12. ·513 by ·0012.
13. 31·5785 by 137.
14. 36·6663 by 406·5.
15. ·00192 by ·00016.
16. ·00195 by ·013.
17. 45·752 by 15·2.
18. 45·752 by ·152.
19. 45·752 by ·00152.
20. ·45752 by 152.
21. 8721 by 127·5.
22. 3·399844 by ·0316.
23. ·158125 by 126·5.
24. 164605·8 by ·0502.
25. ·952791 by 2307.
26. 3849300 by 98·7.

In Examples 27–36 a *long* division can be avoided with advantage.

Divide

27. 3·40725 by ·25.
28. 6·348 by 2·4.
29. 19·347 by ·125.
30. 1·0325 by 1·25.
31. 7·237008 by ·36.
32. 5·67 by ·72.
33. 9·233 by ·0056.
34. ·80347 by ·025.
35. ·001709664 by ·00132.
36. ·5689278 by 9·45.
37. A rod 3 m. 54 cm. long is divided into 6 equal parts. What is the length of each part?
38. A man walks 17 Km. 25 m. in 3 hours. What is his rate per hour? Give the result in kilometres and decimals of a kilometre, and also in kilometres, hectometres, etc.

39. The sum of 27 francs 5 centimes is divided equally amongst 5 children. How much does each child receive?

40. A man walked 48·6 miles in 15 hours. Express decimally his rate of walking in miles per hour.

41. A wheel has a circumference of 81·5 inches. How many revolutions does it make in travelling 16·3 miles?

42. If 5 feet = 1·523985 metres, how many centimetres are there in one inch?

43. If 219·675 sq. ft. is the area of the floor of a room and 17·4 ft. the length of the room, what is its width in feet?

44. A rectangular piece of paper has an area of 90·3 sq. in. and a width of 8·4 in.; what is its length?

45. A packet of paper containing 500 sheets is 2 inches thick. What is the thickness of each sheet as a decimal of an inch?

46. Taking a mile as equal to 1·6093 kilometres, find the length of a yard in metres.

47. How many pieces of thread each 19 cm. long, can be cut from a length of 16 m. 15 cm.?

48. What length of carpet 2·75 ft. wide will cover a floor whose area is 262·9 sq. ft.?

Divide

49. 13·78 by 11	as far as	3	decimal places.
50. 147·25 by 13·2	„	2	„
51. ·00073 by ·006	„	4	„
52. ·006 by 97·5	„	6	„
53. 9·4107 by 306	„	4	„
54. 1257·31 by 34·1	„	2	„
55. 1 by 1728 (by factors)	„	6	„
56. 1 by $\lfloor\underline{6}$, *i.e.* $1\times2\times3\times4\times5\times6$	„	7	„
57. 1 by $\lfloor\underline{9}$	„	7	„
58. 1 by 3^7	„	6	„
59. 1 by 5^7	„	7	„

Multiply

60. ·0345 by $3\frac{1}{5}$. **61.** 3·048 by $2\frac{1}{8}$. **62.** ·00341 by $7\frac{1}{11}$.

63. 18·046 by $11\frac{3}{7}$. **64.** 7·005 by $16\frac{2}{3}$. **65.** ·3276 by $11\frac{1}{9}$.

Divide

66. ·0064 by $3\frac{1}{5}$. **67.** ·00128 by $2\frac{2}{3}$. **68.** ·624 by $9\frac{1}{11}$.

69. 9·0246 by $8\frac{2}{3}$.

Simplify the following expressions, giving your results in decimal form.

[Before doing the multiplication and division, see if the expression cannot be simplified by the use of factors and cancelling.]

70. $\dfrac{1{\cdot}25 \times 4{\cdot}2}{{\cdot}05}$. **71.** $\dfrac{3{\cdot}06 \times 3{\cdot}6}{10{\cdot}8}$. **72.** $\dfrac{0{\cdot}25 \times 17{\cdot}28}{14{\cdot}4}$.

73. $\dfrac{0{\cdot}631 \times 6{\cdot}4}{{\cdot}16}$. **74.** $\dfrac{1{\cdot}7 \times 0{\cdot}0702}{13{\cdot}6}$. **75.** $\dfrac{{\cdot}00625}{{\cdot}8 \times {\cdot}125}$.

76. $\dfrac{{\cdot}063}{{\cdot}007} + \dfrac{6{\cdot}04}{{\cdot}04}$. **77.** $\dfrac{{\cdot}00625}{{\cdot}0025} - \dfrac{{\cdot}792}{1{\cdot}98}$.

78. $(1{\cdot}2 + {\cdot}5 - {\cdot}03) \times {\cdot}09$. **79.** $(1{\cdot}7 + 3{\cdot}02 - 1{\cdot}708) \times \dfrac{{\cdot}4}{{\cdot}16}$.

Conversion of Vulgar Fractions into Decimals.

98. EXAMPLE 1. **Express $\frac{5}{16}$ as a decimal.**

The question might be worded thus : *Divide* 5 *by* 16.

Hence, using factors,

$$\begin{array}{r|l} 4 & 5{\cdot}00 \\ \hline 4 & 1{\cdot}2500 \\ \hline & \underline{{\cdot}3125} \end{array}$$

$$\therefore \ \tfrac{5}{16} = {\cdot}3125.$$

EXAMPLE 2. **Express $3\frac{15}{128}$ as a decimal.**

The whole number in the decimal will evidently be 3, and we have to divide 15 by 128. This might be done by the rules for long division of decimals, but the use of factors is to be preferred.

$128 = 8 \times 8 \times 2.$

$$\begin{array}{r|l} 8 & 15{\cdot}00\ldots \\ \hline 8 & 1{\cdot}875 \\ \hline 2 & {\cdot}234375 \\ \hline & \underline{{\cdot}1171875} \end{array}$$

$$\text{Therefore } 3\tfrac{15}{128} = 3{\cdot}1171875.$$

99. Sometimes we shall find that a vulgar fraction cannot be expressed as a decimal which *terminates*.

Let us try to express $\frac{2}{3}$ as a decimal

$$\begin{array}{r|l} 3 & 2{\cdot}000000000 \\ \hline & {\cdot}66666666\ldots \end{array}$$

Thus we see that $\frac{2}{3} = {\cdot}666\ldots$, where the figure 6 goes on repeating itself for ever.

We can find the value of $\frac{2}{3}$ as a decimal to as many places as we please, but we cannot express it as an exact decimal in the ordinary way.

If we try to convert $\frac{37}{108}$ into a decimal, we shall find that it is equal to ·214259259259..., where the figures 2, 5, 9 go on repeating themselves for ever.

EXAMPLE. **Express $\frac{14}{135}$ as a decimal as far as 4 places**

$$\frac{14}{135} = \frac{\cdot 14}{1\cdot 35}.$$

$$\begin{array}{r@{\,}l}
 & \underline{\cdot 1037} \\
1\cdot 35\,) & \cdot 14000 \\
 & \underline{135} \\
 & 500 \\
 & \underline{405} \\
 & 950 \\
 & \underline{945}
\end{array}$$

Conversion of Decimals into Vulgar Fractions.

100. EXAMPLE 1. **Express 3·6125 as a vulgar fraction in its lowest terms.**

Good method. $3\cdot 6125 = 3\frac{6125}{10000} = 3\frac{\cancel{5}\times 1225}{\cancel{5}\times 2000} = 3\frac{\cancel{5}\times 245}{\cancel{5}\times 400} = 3\frac{\cancel{5}\times 49}{\cancel{5}\times 80}.$

Poor method. $3\cdot 6125 = 3\frac{\overset{\overset{\overset{49}{\cancel{245}}}{\cancel{1225}}}{\cancel{6125}}}{\underset{\underset{\underset{80}{\cancel{400}}}{\cancel{2000}}}{\cancel{10000}}} = 3\frac{49}{80}.$

The first method is the easier to check.

EXAMPLE 2. **Express $\frac{14\cdot 85}{3\cdot 465}$ as a vulgar fraction in its lowest terms.**

$$\frac{14\cdot 85}{3\cdot 465} = \frac{14850}{3465} = \frac{\cancel{5}\times 2970}{\cancel{5}\times 693} = \frac{9\times 330}{9\times 77}$$
$$= \frac{30}{7} = 4\tfrac{2}{7}.$$

Useful Facts in connection with Decimals.

101. $\cdot 25 = \frac{25}{100} = \frac{1}{4}.$ Thus $7\cdot 624 \times \cdot 25 = \frac{7\cdot 624}{4} = 1\cdot 906.$

$25 = \frac{100}{4}.$ Hence $89\cdot 162 \times 25 = \frac{89\cdot 162 \times 100}{4}$

$$= \frac{8916\cdot 2}{4} = 2229\cdot 05.$$

$$\frac{1}{\cdot 25} = \frac{4}{4 \times \cdot 25} = 4. \qquad \therefore \ \frac{2\cdot 87}{\cdot 25} = 2\cdot 87 \times 4 = 11\cdot 48.$$

$$\cdot 025 = \frac{25}{1000} = \frac{1}{40}. \qquad \therefore \ 976\cdot 5 \times \cdot 025 = \frac{976\cdot 5}{40}$$

$$= \frac{97\cdot 65}{4} = 24\cdot 4125.$$

$$\frac{1}{125} = \frac{8}{1000}. \qquad \therefore \ \frac{963\cdot 42}{125} = \frac{963\cdot 42 \times 8}{1000}$$

$$= \cdot 96342 \times 8 = 7\cdot 70736.$$

$$\cdot 125 = \frac{\cdot 125 \times 8}{8} = \frac{1}{8}. \qquad \therefore \ 60\cdot 16 \times \cdot 125 = \frac{60\cdot 16}{8} = 7\cdot 52.$$

EXAMPLE.
$$\frac{893\cdot 25}{6\cdot 25} = \frac{178\cdot 65 \times \cancel{5}}{1\cdot 25 \times \cancel{5}} = \frac{178\cdot 65 \times 8}{10}$$

$$= 17\cdot 865 \times 8 = 142\cdot 92.$$

EXAMPLES XI. m.

[*Revise each example before proceeding to the next.*]

Express each of the following as a decimal:

1. $\frac{1}{2}$.	**2.** $\frac{3}{4}$.	**3.** $\frac{5}{8}$.	**4.** $\frac{1}{16}$.	**5.** $\frac{13}{128}$.
6. $3\frac{3}{8}$.	**7.** $4\frac{5}{16}$.	**8.** $125\frac{1}{25}$.	**9.** $\frac{1}{125}$.	**10.** $\frac{1}{1250}$.
11. $\frac{3}{640}$.	**12.** $3\frac{21}{1280}$.	**13.** $3\frac{1}{8}$.	**14.** $17\frac{3}{64}$.	**15.** $\frac{13}{650}$.
16. $19\frac{11}{275}$.	**17.** $19\frac{1}{16}$.	**18.** $3\frac{1}{32}$.	**19.** $\frac{3}{128}$.	**20.** $\frac{9}{8}$.
21. $\frac{13}{5}$.	**22.** $\frac{193}{4}$.	**23.** $\frac{1728}{25}$.	**24.** $\frac{69375}{32}$.	

Express each of the following as a decimal, as far as 3 decimal places:

25. $\frac{2}{9}$.	**26.** $\frac{3}{17}$.	**27.** $\frac{61}{319}$.	**28.** $\frac{602}{31}$.	**29.** $4\frac{11}{13}$.
30. $5\frac{1}{42}$.	**31.** $6\frac{4}{7}$.	**32.** $9\frac{11}{21}$.	**33.** $15\frac{41}{108}$.	**34.** $17\frac{9}{44}$.

Express the following as vulgar fractions, in their lowest terms:

35. $6\cdot 25$.	**36.** $3\cdot 105$.	**37.** $14\cdot 012$.	**38.** $16\cdot 128$.	**39.** $17\cdot 32$.

40. $3\cdot 0625$.	**41.** $23\cdot 52$.	**42.** $\cdot 78125$.	**43.** $\frac{\cdot 5}{\cdot 2}$.
44. $\frac{3\cdot 5}{1\cdot 4}$.	**45.** $\frac{1\cdot 44}{21\cdot 6}$.	**46.** $\frac{315}{17\cdot 85}$.	**47.** $\frac{3\cdot 6 + 1\cdot 3}{2}$.
48. $\frac{1\cdot 1 - \cdot 05}{3}$.	**49.** $\frac{3\cdot 1 + 4\cdot 5}{0\cdot 4}$.	**50.** $\frac{4\cdot 2 - 1\cdot 5}{3\cdot 3}$.	**51.** $\frac{5\cdot 3 - 1\cdot 06}{30\cdot 7 - 25\cdot 4}$.

Simplify the following, using as few figures as possible, and giving the results in decimal form:

52. $8\cdot 24 \times \cdot 25$.	**53.** $\frac{93\cdot 7}{\cdot 25}$.	**54.** $\frac{562\cdot 5}{25}$.	**55.** $96\cdot 32 \times \cdot 025$.

Simplify the following, using as few figures as possible, and giving the results in decimal form :

56. $2{\cdot}24 \times 1{\cdot}25$. **57.** $\frac{\cdot 3125}{125}$. **58.** $(\cdot 125)^2$. **59.** $\frac{3{\cdot}49}{\cdot 125}$.

60. $7{\cdot}05 \times \cdot 125$. **61.** $\frac{\cdot 06}{25}$. **62.** $9{\cdot}347 \times 125$. **63.** $\cdot 00692 \times \cdot 0025$.

64. $\frac{937}{250}$. **65.** $\frac{83641}{1250}$. **66.** $\frac{\cdot 92875}{\cdot 625}$. **67.** $\cdot 0125 \times 7{\cdot}839$.

68. $\frac{56{\cdot}325}{\cdot 3125}$ (using factors). **69.** $\frac{75{\cdot}3}{\cdot 75}$ (using factors).

70. $\frac{7{\cdot}924}{\cdot 35}$. **71.** $\frac{680{\cdot}4}{22{\cdot}5}$.

XII. APPROXIMATE VALUES.

102. Let us consider the number 34·825.

34·825 is greater than 34, but less than 35.

Also it is nearer to 35 than to 34, for ·825 is greater than ·5 $(=\frac{1}{2})$.

∴ we say that 34·825 = 35 to the nearest whole number.

In the same way, 34·825 is greater than 34·8, but less than 34·9, and it is nearer to 34·8 than to 34·9.

∴ we say that 34·825 = 34·8 correct to one decimal place.

Again, 34·875 is nearer to 34·9 than to 34·8.

∴ we say that 34·875 = 34·9 correct to one decimal place.

103. In finding approximate values,

One half, or more than one half, counts as unity.

i.e. ·5	,,	,,	·5	,,	,,		
·05	,,	,,	·05 and less than ·1			counts as	·1.
·005	,,	,,	·005	,,	,,	·01 ,,	·01,

and so on.

Hence if $x = 9{\cdot}48527$,

$x = 9$ to the nearest whole number,
$= 9{\cdot}5$ correct to one decimal place,
$= 9{\cdot}49$,, two ,, places,
$= 9{\cdot}485$,, three ,, ,,
$= 9{\cdot}4853$,, four ,, ,,

Significant Digits.

104. The significant digits of a number are those which remain when all ciphers at the beginning and end have been removed.

Thus the significant digits of ·0007045, of 704500 and of 70·45 are 7045.

The term is often used in connection with approximate values.

For instance, the mean distance of the moon from the earth is 238,833 miles.

Hence the mean distance correct to three significant digits is 239,000 miles, for 238,833 is greater than 238,000, less than 239,000 and nearer the latter than the former.

1 metre = 39·37043 inches.

∴ 1 metre = 39·37 inches correct to 4 significant digits.

Note that this is the same as 39·37043 in. correct to 2 decimal places.

Errors.

105. When we say that 9·485 is the value of 9·48527 correct to 3 decimal places, it is evident that the error in so doing is ·00027 (= 9·48527 − 9·485). This error is less than ·0003 ($=\frac{3}{10000}$).

Hence we see that, compared with the original number, which is greater than 9, the error is very small.

Taking 9·49 as the correct value to 2 decimal places, we see that the error is ·00473 (= 9·49 − 9·48527).

In dealing with approximations in whole numbers, we follow a similar rule to that employed with decimals.

The value of 6937283 is 6937280 to the nearest ten.

,,	,,	,,	6937300	,,	hundred.
,,	,,	,,	6937000	,,	thousand,

and so on.

The circumference of the earth at the equator is 24,899 miles.

This is 24900 miles to the nearest 10 miles,

or 24900	,,	,,	100	,,
or 25000	,,	,,	1000	,,

The error in taking 25000 miles as the equatorial circumference of the earth is 101 miles ($=25000-24899$).

106. The question as to whether an error is small or great is a relative one. For instance, sometimes an error of £1 may be considerable, sometimes it may be very small.

An error of £1 in £10 (*i.e.* $\frac{1}{10}$th of the whole) is considerable.

" £1 " £100000 (*i.e.* $\frac{1}{100000}$th of the whole) is quite small.

The fraction representing $\dfrac{\text{actual error}}{\text{total amount or quantity}}$ is called the **relative** or **fractional error.**

In the above case of the circumference of the earth, the actual error is 101 miles, a very considerable distance, but the relative error $=\frac{101}{24899}$, or roughly $=\frac{100}{25000}=\frac{1}{250}=\cdot004$, a quite small quantity.

N.B.—Unless the error is a very large one, the fraction

$$\text{Relative error}\left(=\frac{\text{actual error}}{\text{total amount}}\right)=\frac{\text{actual error}}{\text{approximate value}} \text{ very nearly.}$$

E.g. $34969=35000$ to two significant digits.

Here the relative error $=\frac{31}{34969}=\frac{31}{35000}$ very nearly.

The mean, or average, distance of the sun from the earth $=92965000$ miles.

This is 93000000 miles correct to 2 significant digits.

In this approximation the actual error is 35000 miles, a very considerable distance, but we shall find that the *relative* error, *i.e.* the error compared with the total distance, is quite a small one.

The relative error $=\frac{35000}{92965000}$ or approximately $=\frac{35000}{93000000}$ $=\frac{35}{93000}=\frac{7}{18600}$ or roughly $\frac{1}{2700}$.

Thus we see that the *relative* error is quite small though the actual error is as much as 35000 miles.

Again, the value of $\cdot00479$ correct to 3 decimal places $=\cdot005$.

The actual error here $=\cdot00021$, a small amount.

The relative error $=\dfrac{\cdot00021}{\cdot00479}=\dfrac{21}{479}$ or roughly $\dfrac{1}{20}$, not a very small amount.

EXAMPLE. **Find the value of 318·4 ÷ 34·7 correct to two decimal places.**

$$\frac{318 \cdot 4}{34 \cdot 7} = \frac{31 \cdot 84}{3 \cdot 47}$$

```
          9·175...
3·47 ) 31·84000
       31 23
          610
          347
         2630
         2429
          2010
          1735
          ......
```

Thus $\frac{318 \cdot 4}{34 \cdot 7} = 9 \cdot 175\ldots$

$= 9 \cdot 18$ correct to two decimal places.

EXAMPLES XII. a.

What is the value of

1. ·0365 correct to 2 decimal places?
2. ·6325 „ 3 „
3. 7·67249 „ 3 „
4. 9375 to the nearest hundred?
5. 10345 „ „
6. 961·07281 correct to 3 decimal places?
7. „ „ 4 significant figures?
8. ·0625 „ 2 „
9. 93601825 „ 5 „
10. „ to the nearest million?
11. $3 \cdot 0468 + 4 \cdot 6251 + 9 \cdot 6024$ correct to 3 decimal places?
12. $16 \cdot 94 + 0 \cdot 0089 + 7 \cdot 18025$ „ 2 „
13. $6 \cdot 25 + 7 \cdot 08 + 0 \cdot 61 + 1 \cdot 02$ to the nearest whole number?
14. $7 \cdot 84625 + 0 \cdot 00738 - 1 \cdot 0263$ correct to 2 decimal places?
15. $19 \cdot 625 - 7 \cdot 0009$ „ 3 „
16. $3 \cdot 72895 \times \cdot 6$ „ 3 „
17. $3 \cdot 25^2$, *i.e.* $3 \cdot 25 \times 3 \cdot 25$ „ 2 „
18. $10 \cdot 0001 \times \cdot 009$ „ 3 „
19. $\cdot 8^3$ „ 2 „
20. $7896 \cdot 25 \times \cdot 25$ to the nearest whole number?
21. $\frac{65}{7}$ correct to 3 decimal places?
22. $\frac{\cdot 005}{18}$ correct to 5 decimal places?
23. $\frac{\cdot 782}{36}$ „ 3 „
24. $6 \cdot 848$ m. $\times 7$ to the nearest centimetre?

What is the value of

25. £117·8035 × 6 to the nearest £?

26. $£\frac{3124·437}{25}$ „ £?

27. $\frac{94 \text{ francs}}{17}$ „ centime?

One lb. Avoirdupois = 453·593 grams.

28. What is the value (in grams) of 10 lb. to the nearest gram?

29. „ „ „ 100 lb. „ „

30. „ „ „ one-tenth of a lb. to the nearest tenth of a gram?

One metre = 39·37043 inches.

31. What is the value of a metre to the nearest hundredth of an inch?

32. „ „ „ „ tenth of a foot?

33. „ „ 10 metres „ tenth of a foot?

34. „ „ 100 „ „ foot?

35. „ „ a kilometre „ inch?

36. „ „ „ „ foot?

37. Express one yard ($\frac{1}{1760}$ of a mile) as a decimal of a mile, correct to 5 decimal places.

Find the magnitude of the error in each of the following statements:

38. 31·9 × 0·11 = 3·5.

39. 13 × 6·26 = 81·4.

40. $\frac{1}{625}$ = ·002.

41. 317·7 × 4·92 = 1563·2.

[A rough estimate is often sufficient, *e.g.* $\frac{1}{25981} = \frac{1}{26000}$ very nearly.]

(i) Find as a vulgar fraction and (ii) express as a decimal to two significant digits the magnitude of the relative error when we take

42. 2500 as the approximate value of 2501.

43. 50 „ „ „ 50·006.

44. 40000 „ „ „ 39979.

45. ·03 „ „ „ ·03126.

46. ·008 „ „ „ ·007825.

47. 36000000 „ „ „ 36121000.

Recurring Decimals, sometimes called Repeating Decimals.

107. $\frac{1}{8}$ = ·125. But let us try to express $\frac{1}{3}$ as a decimal.

$$\begin{array}{r|l} 3 & 1·000000\ldots \\ \hline & ·333333\ldots \end{array}$$

Thus we see that the digit 3 goes on repeating itself (recurring) for ever. We call such a decimal a *recurring* or *repeating* decimal.

The value of $\frac{1}{3}$ correct to 2 decimal places $= \cdot 33$.

" " " 3 " " $= \cdot 333$, and so on.

We therefore see that though we cannot express the value of $\frac{1}{3}$ as a decimal exactly, we can obtain its value correct to as many decimal places as we please.

EXAMPLE 1. **Express $\frac{1}{27}$ as a decimal.**

Using factors,

$$\begin{array}{r|l} 9 & 1\cdot000000\ldots \\ \hline 3 & \cdot111111\ldots \\ \hline & \cdot037037\ldots \end{array}$$

Hence the value of $\frac{1}{27}$ correct to 2 decimal places $= \cdot 04$.

" " " 3 " " $= \cdot 037$.

" " " 4 " " $= \cdot 0370$.

EXAMPLE 2. **Express $\frac{7}{15}$ as a decimal correct to 3 decimal places.**

$$\begin{array}{r|l} 5 & 7\cdot00000 \\ \hline 3 & 1\cdot40000 \\ \hline & \cdot46666\ldots \end{array}$$

$\therefore \frac{7}{15} = \cdot 467$ correct to 3 decimal places.

108. We have seen that if a vulgar fraction has a denominator which is ten, or any power of ten, we can express it as a decimal which *terminates*, *i.e.* has no recurring figures.

Now the prime factors of 10 are 2 and 5, and hence the prime factors of any power of 10 are all 'twos' and 'fives.'

We shall find that if a vulgar fraction *in its lowest terms* has a denominator whose prime factors include anything but 'twos' and 'fives,' the equivalent decimal will have recurring figures.

$$\frac{3}{75} = \frac{3}{5^2 \times 3} \quad \text{(Here the 'threes' cancel)}$$

$$= \frac{1}{5^2} = \frac{2^2}{10^2} = \frac{4}{100} = \cdot 04.$$

Hence $\frac{3}{75}$ can be expressed as a terminating decimal.

But $\frac{1}{75}\left(=\frac{1}{5^2 \times 3}\right)$ will become a recurring decimal.

$$\begin{array}{r|l} 25 & 1\cdot000000 \\ \hline 3 & \cdot040000 \\ \hline \frac{1}{75} = & \cdot013333\ldots \end{array}$$

Recurring decimals are expressed shortly by placing dots over the first and last digits of the part which recurs.

Thus $3{\cdot}4037037037\ldots = 3{\cdot}4\dot{0}3\dot{7}$.

$6{\cdot}333\ldots = 6{\cdot}\dot{3}$.

The figures which repeat themselves are called the **repetend** or **recurring period.**

E.g. In $6{\cdot}\dot{3}$ 3 is the repetend.

„ $6{\cdot}0\dot{2}\dot{4}$ 24 „ „

„ $3{\cdot}72\dot{9}8\dot{1}$ 981 „ „

109. The following simple cases of recurring decimals are important:

$\frac{1}{3} = {\cdot}3333\ldots = {\cdot}\dot{3}$. ${\cdot}\dot{9} = {\cdot}9999\ldots = 1$.

This is proved as follows:

Let $x = {\cdot}99999\ldots$.

Multiplying these equal quantities by 10, we get

$$10x = 9{\cdot}9999\ldots.$$

$\therefore$ by subtraction $9x = 9$.

$\therefore\ x = 1$. *i.e.* ${\cdot}\dot{9} = 1$.

Again, to prove that ${\cdot}42\dot{9} = {\cdot}43$.

Let $x = {\cdot}4299999\ldots$.

Multiply these equal quantities by 10:

$$10x = 4{\cdot}299999\ldots$$

But $x = \ {\cdot}429999\ldots$

$\therefore\ 9x = 3{\cdot}87$.

Divide both sides by 9:

$x = {\cdot}43$. *i.e.* ${\cdot}42\dot{9} = {\cdot}43$.

In the same way, ${\cdot}381\dot{9} = {\cdot}382$.

${\cdot}64\dot{9} = {\cdot}65$, and so on.

EXAMPLES XII. b.

What is the value of

1. 3·617617617... correct to 2 decimal places?
2. „ „ 3 „
3. „ „ 4 „
4. $5{\cdot}\dot{3}$ 3 „ 2 „
5. $5{\cdot}\dot{7}\dot{2}$ „ 2 „
6. $6{\cdot}\dot{8}1\dot{7}$ „ 2 „
7. „ „ 4 „

8.	$1{\cdot}03\dot{6}$	correct to	3 decimal places?	
9.	"	"	4	"
10.	$4{\cdot}1\dot{3}\dot{8}$	"	4 significant digits?	
11.	"	"	4 decimal places?	
12.	$\cdot 0\dot{9}$	"	1 decimal place?	
13.	"	"	3 decimal places?	
14.	"	"	5	"
15.	$\cdot 62\dot{7}$	"	3 significant digits?	
16.	$\cdot 32\dot{9}$	"	2 decimal places?	
17.	"	"	4	"
18.	$\cdot\dot{7}4581\dot{6}$	"	4	"

Express the following as decimals (divide by factors where possible):

19. $\frac{1}{15}$ correct to 3 decimal places.
20. $\frac{1}{75}$ " 4 "
21. $\frac{19}{135}$ " 3 "
22. $\frac{135}{71}$ " 3 "
23. $\frac{1}{31}$ " 3 "
24. $\frac{17}{81}$ " 5 "
25. $\frac{1}{\underline{|6}}$, *i.e.* $\frac{1}{1\times2\times3\times4\times5\times6}$, correct to 5 decimal places.

[$\underline{|6}$ is called "factorial 6."]

26. $\frac{2}{\underline{|7}}$ correct to 5 decimal places.

Negative Quantities.

110. Any quantity with the sign + prefixed, or understood, is called a *positive* quantity, and any quantity with the sign − prefixed is called a *negative* quantity.

Arithmetically we cannot subtract 5 from 3, but nevertheless the expression 3 − 5 has an intelligible interpretation.

This is best seen by considering a few examples.

If a farmer buys 7 cows, and sells 4 cows, he has 3 *more* than he had at first. On the other hand, if he buys 4 cows, and sells 7, he has 3 *less* than at the start.

We express this by saying that

$$7 \text{ cows} - 4 \text{ cows} = +3 \text{ cows},$$
$$4 \text{ cows} - 7 \text{ cows} = -3 \text{ cows}.$$

Again, if a man gains £10 and loses £6, he has £10 − £6, *i.e.* £4, *more* than at first. If, on the other hand, he gains £6 and loses £10, he has £4 *less* than at first,

$$\textit{i.e.}\quad £10 - £6 = +£4,$$
$$£6 - £10 = -£4.$$

Moreover, if he loses £10 and then gains £6, he will then have £4 *less* than at first,

$$\textit{i.e.}\quad -£10 + £6 = -£4.$$

If a man runs 120 yds. along a road, and then runs 90 yds. towards his starting point he will be 30 yds. from his starting point. But if he runs 90 yds. and then 120 yds. in the opposite direction, he will still be 30 yds. from his starting point, but *on the opposite side of it.*

$$120 - 90 = 30, \qquad 90 - 120 = -30.$$

This might be expressed thus:

$$120 - 90 = 30 + \cancel{90} - \cancel{90} = 30,$$
$$90 - 120 = \cancel{90} - \cancel{90} - 30 = -30.$$

Thus we see that $+4$ and -4 are the exact opposite of one another.

If we consider a man's income, +£4 will represent an *increase*, whilst −£4 will represent an equal *decrease*. +4 yds. and −4 yds. represent 4 yds. *in opposite directions*, and so on.

If a man loses first £10, and then again loses £4, he is £14 poorer than at first.

That is, $\quad -£10 - £4 = -£14.$

Thus, $\quad -3 - 2 = -5$ and $-5 - 6 = -11.$

Graphical Illustrations.

111. Take a straight line XOX′ of unlimited length, and let all distances measured *to the right* be considered positive, whilst all distances measured in the opposite direction, from right to left, are taken as negative.

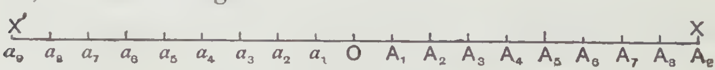


Fig. 16.

Take $\qquad OA_1 = A_1A_2 = A_2A_3 = \ldots = b$ along OX

and $\qquad Oa_1 = a_1a_2 = a_2a_3 = \ldots = b$ along OX′.

If we take O as the starting point in each case,

OA_6 denotes $+6b$, whilst Oa_6 denotes $-6b$, and so on.

Also A_3A_7 denotes $+4b$, whilst A_7A_3 denotes $-4b$.

Thus $6b$ is denoted by OA_6 (6 spaces *to the right*), and A_6A_4 denotes $-2b$ (2 spaces *to the left*);

$$\therefore\ 6b-2b=OA_4=4b.$$

Again, still starting from O, $-2b$ is denoted by Oa_2 (2 spaces *to the left*) and $+5b$ by a_2A_3 (5 spaces *to the right*).

$$\therefore\ -2b+5b=OA_3=3b.$$

Again, $-3b$ is denoted by Oa_3, and $-4b$ by a_3a_7, both distances being measured *to the left*,

$$\therefore\ -3b-4b=Oa_7=-7b.$$

Once more,

$-7b$ is denoted by Oa_7 (7 spaces in the negative direction),
$+4b$ „ a_7a_3 (4 „ positive „).

$$\therefore\ -7b+4b \text{ is denoted by } Oa_3,$$

$$i.e.\ \ -7b+4b=-3b.$$

EXAMPLES XII. c. (*Oral.*)

What is the value of

1. $7-4$?	**2.** $4-7$?	**3.** $11-5$?
4. $5-11$?	**5.** $4a-2a$?	**6.** $2a-4a$?
7. $-2a-4a$?	**8.** $-2a+4a$?	**9.** $7x-15x$?
10. $-7x+3x$?	**11.** $-7x-3x$?	**12.** $x-3x$?
13. $-x-5x$?	**14.** $6x-4x-3x$?	**15.** $-4x-3x+2x$?
16. $-7x+4x+x$?	**17.** $-x-6x-9x$?	**18.** $-7x+4x-3x+6x$?

19. A train travels 50 miles north and then 70 miles south. Describe its position at the end of the journey. How far has it travelled?

20. A bicyclist rides 15 miles east, then 9 miles west, 5 miles east again, and lastly 14 miles west. Describe his position at the end of the journey. How far did he travel?

Limits of Error.

[*The following should be omitted unless the student has learnt Algebra up to multiplication.*]

112. If 6·742 is the value of a quantity correct to 3 decimal places, the exact quantity must be less than 6·7425, and cannot be less than 6·7415, by the rules for approximation.

Now $\qquad 6{\cdot}742 - 6{\cdot}7425 = -{\cdot}0005$

and $\qquad 6{\cdot}742 - 6{\cdot}7415 = +{\cdot}0005.$

$\therefore$ we say that $\pm{\cdot}0005$ ($= \pm\frac{1}{2000}$) are the **limits of error.**

[By $\pm{\cdot}0005$ is meant plus or minus ${\cdot}0005$.]

In other words the error may be less than ${\cdot}0005$, but cannot be greater. Also the error may be positive or negative, *i.e.* the estimated value may be too great or too small.

The reasoning holds for any quantity correct to 3 decimal places.

In the same way, if a quantity is correct to 5 decimal places, the limits of error are $\pm{\cdot}000005$, *i.e.* $\pm\frac{1}{200000}$.

Suppose that $\pm x$ are the limits of error in taking a as the value of a quantity, and $\pm y$ are the limits of error in taking b as value of another quantity.

The greatest possible value of their sum is

$$(a+x)+(b+y)=(a+b)+(x+y).$$

The least possible value of their sum is

$$(a-x)+(b-y)=(a+b)-(x+y).$$

$\therefore$ the limits of error are $\pm(x+y)$.

In taking the difference of these quantities, a being the greater, the greatest possible difference is $(a+x)-(b-y)=(a-b)+(x+y)$,
" least " " $(a-x)-(b+y)=(a-b)-(x+y)$.

$\therefore$ as with their sum, the limits of error are $\pm(x+y)$.

Dealing with the same quantities, we know by Algebra that the product of $a+x$ and $b+y$ is $ab+ay+bx+xy$, and the product of $a-x$ and $b-y$ is $ab-ay-bx+xy$.

If the errors x and y are small, their product xy is *very small.*

Hence, *this small error being neglected*, the limits of error in the product are $\pm(ay+bx)$.

For instance, if a and b are calculated correct to one decimal place, the limits of error in each are $\pm{\cdot}05$.

$\therefore$ the limits of error in the product will be $\pm(a+b)\times{\cdot}05$ or $\pm\dfrac{a+b}{20}$.

In this case we *neglect* the small quantity $({\cdot}05)^2$ *i.e.* $\frac{1}{400}$.

In the same way, if a and b are correct to 3 decimal places, the limits of error will be $\pm(a+b)\times{\cdot}0005$.

Here we neglect $({\cdot}0005)^2$, *i.e.* $\frac{1}{4000000}$

EXAMPLE 1. **Two measurements of 3·78 in., 4·36 in. are correct to 2 decimal places. To how many decimal places is their sum correct? And what are the limits of error?**

Their sum $\begin{array}{r}3{\cdot}78\\+4{\cdot}36\end{array}=8{\cdot}14.$ Their greatest possible sum $\begin{array}{r}3{\cdot}785\\+4{\cdot}365\end{array}=8{\cdot}15$ in.

$\therefore$ we can only be sure that the measured sum is correct to one decimal place.

As in Art. 112, the limits of error are $\pm{\cdot}01$ in.

EXAMPLE 2. **To how many decimal places is the sum of 3·45, 1·63 and 2·09 correct, if they are each known to be correct to 2 decimal places? Also what are the limits of error?**

$$\begin{array}{rcl}3{\cdot}45 & \quad & 3{\cdot}455\\ 1{\cdot}63 & & 1{\cdot}635\\ 2{\cdot}09 & & 2{\cdot}095\\ \hline 7{\cdot}17 & & 7{\cdot}185 = \text{the greatest possible sum.}\end{array}$$

$\therefore$ we can only be sure that the estimated sum is correct to one decimal place.

The limits of error in each case are $\pm{\cdot}005$.

$\therefore$ the limits of error in their sum are $\pm({\cdot}005+{\cdot}005+{\cdot}005)$
$= \pm 3({\cdot}005) = \pm{\cdot}015.$

EXAMPLE 3. **Two quantities 3·78 and 4·23 are known to be correct to two decimal places. To how many decimal places is their product correct? And what are the limits of error?**

$$\begin{array}{rrr}3{\cdot}78 & 3{\cdot}785 & 3{\cdot}775\\ 4{\cdot}23 & 4{\cdot}235 & 4{\cdot}225\\ \hline 15{\cdot}12 & 15{\cdot}140 & 15{\cdot}100\\ {\cdot}756 & {\cdot}7570 & {\cdot}7550\\ {\cdot}1134 & {\cdot}11355 & {\cdot}07550\\ \hline 15{\cdot}9894 & {\cdot}018925 & {\cdot}018875\\ & \overline{16{\cdot}029475} & \overline{15{\cdot}949375}\end{array}$$

Hence we see that the product is not necessarily correct beyond the tens digit.

The greatest possible error is, by subtraction, ·040075 in the first case and ·040025 in the second case.

Thus we see that the limits of error may be taken to be $\pm{\cdot}04$ approximately.

EXAMPLES XII. d.

To what degree of accuracy, *i.e.* to how many decimal places, can the following be with certainty calculated? State the limits of error in each case. It may be assumed that the quantities are correct to the number of decimal places given.

1. $2{\cdot}41+3{\cdot}02$. **2.** $7{\cdot}36+8{\cdot}64$. What is the greatest possible relative error in fractional form?

3. $3{\cdot}89+1{\cdot}05+2{\cdot}57$. **4.** $19{\cdot}72-18{\cdot}91$. **5.** ${\cdot}08-{\cdot}02$. What is the greatest possible relative error in fractional form?

6. $3{\cdot}923+14{\cdot}829+11{\cdot}087$. **7.** $3{\cdot}921+9{\cdot}372+8{\cdot}013$.

8. $3{\cdot}0062+1{\cdot}7001+2{\cdot}8901$.

9. Two pieces of paper are respectively 4·53 and 3·71 inches long. Their lengths are taken to be 4·5 and 3·7 inches. What is the error thus made in (1) the sum of their lengths, (2) in the difference of their lengths? Find the relative error in each case.

10. To how many decimal places can we be certain that the product $\cdot 05 \times \cdot 87$ is correct, both quantities being correct to 2 decimal places? If each quantity is too small by ·002, what is the error in the product?

11. To how many decimal places is the product $\cdot 08 \times \cdot 09$ correct, both quantities being correct to 2 decimal places? Show that the greatest possible relative error is a little more than $\frac{1}{9}$.

12. A wall to be papered is 5·4 m. by 3·7 m. correct to one decimal place. Show that more than half a square metre of extra paper should be obtained to provide for possible errors in measurement.

13. Find the error (to 2 decimal places) if $\dfrac{\cdot 68}{\cdot 13}$ is used instead of $\dfrac{\cdot 683}{\cdot 125}$.

14. One metre = 39·37043 inches correct to 5 decimal places. Find the length of 1000 m. in feet correct to as many decimal places as are reliable from the datum.

15. One mile = 1·6093 kilometres correct to 4 decimal places. Find the length of one yard in metres correct to as many decimal places as are reliable.

Decimalisation.

113. The following examples will show how we may express one quantity as the decimal of another quantity.

If we are dealing with concrete quantities, the two quantities must be of the same kind. We cannot express 3*s.* 4*d.* as a decimal of 17 yards.

EXAMPLE 1. **Express 3s. 10½d. as the decimal of £1.**

$$10\tfrac{1}{2}d. = 10\cdot 5d. = \frac{10\cdot 5}{12}s. = \cdot 875s.;$$

$$\therefore\ 3s.\ 10\tfrac{1}{2}d. = 3\cdot 875s. = £\frac{3\cdot 875}{20} = £\frac{\cdot 3875}{2}$$
$$= £\cdot 19375.$$

The work may be shortened thus:

$$\begin{array}{r|l} 12 & 10\cdot 5 \text{ pence} \\ \hline 2,0 & 3\cdot 875s. \\ \hline & £\cdot 19375 \end{array}$$

N.B.—After reducing the 10·5 pence to the decimal of a shilling, we add on the 3*s.*, and then reduce the shillings to the decimal of £1.

Example 2. **Reduce £3. 15s. 9d. to the decimal of £4.**

$$\begin{array}{r|l} 12 & 9 \text{ pence} \\ 2,0 & 15{\cdot}75s. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1) \\ 4 & 3{\cdot}7875£ \quad \ldots\ldots\ldots\ldots\ldots\ldots (2) \\ & {\cdot}946875 \text{ of } £4. \end{array}$$

$\therefore$ £3. 15*s.* 9*d.* = ·946875 of £4.

In line (1), after reducing 9 pence to the decimal of a shilling, we add on the 15*s.*

In line (2), after reducing 15·75*s.* to the decimal of £1, we add on the £3.

114. The above short method cannot be used when the second quantity contains more than one denomination.

Example 1. **Reduce £1. 3s. 5d. to the decimal of £9. 7s. 4d.**

We shall first reduce the two quantities to the same denomination.

£1. 3*s.* 5*d.* = 23*s.* 5*d.* = 281*d.*

£9. 7*s.* 4*d.* = 187*s.* 4*d.* = 2248*d.*

$$\therefore \text{ the required decimal} = \frac{281}{2248} \text{ expressed as a decimal}$$

$$= \frac{{\cdot}281}{2{\cdot}248}.$$

$$\begin{array}{r}
{\cdot}125 \\
2{\cdot}248\,)\ {\cdot}28100 \\
2248 \\
\hline
5620 \\
4496 \\
\hline
11240 \\
11240 \\
\hline
\end{array}$$

$\therefore$ £1. 3*s.* 5*d.* = ·125 of £9. 7*s.* 4*d.*

Example 2. **Express £3. 13s. 9½d. as a decimal of £6 correct to 3 decimal places.**

As the result is to be correct in the third decimal place, we must work to 4 decimal places.

$$\begin{array}{r|l} 12 & 9{\cdot}5000 \\ 2,0 & 13{\cdot}7916\ldots \\ 6 & 3{\cdot}6895\ldots \\ & {\cdot}6149\ldots \end{array}$$

$\therefore$ £3. 13*s.* 9½*d.* = ·615 of £6 correct to 3 decimal places.

The following are examples of the reverse process:

EXAMPLE 3. **Find the value of £0·7125 in shillings and pence.**

$$\begin{array}{rr} \text{£.} & \cdot 7125 \\ & 20 \\ \hline s. & 14\cdot 2500 \\ & 12 \\ \hline d. & 3\cdot 00 \end{array}$$

N.B.—In the third line, we leave the 14*s.* and multiply only the decimal part by 12 in order to reduce it to pence.

£0·7125 is evidently the same as 0·7125 of £1.

Also ·35 of £2 = £(·35 × 2).

EXAMPLE 4. **Find the value of 0·493261 of £1 in shillings, etc., to the nearest farthing.**

$$\begin{array}{rr} \text{£.} & \cdot 493261 \\ & 20 \\ \hline s. & 9\cdot 865220 \\ & 12 \\ \hline d. & 10\cdot 38264 \\ & 4 \\ \hline f. & 1\cdot 53056 \end{array}$$

= 2 farthings to the nearest farthing.

∴ £0·493261 = 9*s.* $10\frac{1}{2}d.$ to the nearest farthing.

EXAMPLE 5. **Find the value of ·91725 of £5 to the nearest penny.**

$$\begin{aligned} \cdot 91725 \text{ of £5} &= \text{£}(\cdot 91725 \times 5) \\ &= \text{£}4\cdot 58625 \end{aligned}$$

$$\begin{array}{r} 20 \\ \hline 11\cdot 72500 \\ 12 \\ \hline 8\cdot 700 \end{array}$$

= £4. 11*s.* 9*d.* to the nearest penny.

EXAMPLE 6. **Find the value of ·325 of £3. 14s. $6\frac{1}{2}$d. to the nearest farthing.**

$$\begin{array}{l} \text{£.} \quad s. \quad d. \\ 3 \;.\; 14 \;.\; 6\frac{1}{2} \\ 20 \\ \hline 74 \\ 12 \\ \hline 894\cdot 5 \text{ pence} \\ \quad \cdot 325 \\ \hline 268\cdot 35 \\ \;\, 17\cdot 890 \\ \;\;\; 4\cdot 4725 \\ \hline 290\cdot 7125 \text{ pence} \\ \qquad\quad 4 \\ \hline 2\cdot 8500 \text{ farthings.} \end{array}$$

(We now have to multiply this by ·325.)

∴ ·325 of £3. 14*s.* $6\frac{1}{2}d.$ = $290\frac{3}{4}$ pence to the nearest farthing
= 24*s.* $2\frac{3}{4}d.$,, ,,
= £1. 4*s.* $2\frac{3}{4}d.$,, ,,

EXAMPLE 7. **Find the value of £0·8126 + 0·324s. to the nearest penny.**

$$\begin{array}{r} £0{\cdot}8126 \\ 20 \\ \hline 16{\cdot}2520s. \\ {\cdot}324 \\ \hline 16{\cdot}576s. \\ 12 \\ \hline 6{\cdot}912 \end{array} \quad \text{(Here we add } 0{\cdot}324s.\text{)}$$

∴ 16*s*. 7*d*. is the required value.

115. The following are useful and should be committed to memory.

$$\textbf{A florin} = 2\text{s.} = £\tfrac{1}{10} = £{\cdot}1.$$
$$1\text{s.} = £\tfrac{1}{20} = £{\cdot}05.$$
$$6\text{d.} = £\tfrac{1}{40} = £{\cdot}025.$$

EXAMPLES. £0·8 = 8 florins = 16*s*.
£0·3 = 3 florins = 6*s*.
£·65 = 6 florins + 1*s*. = 13*s*.
£·725 = 7 florins + 1 sixpence = 14*s*. 6*d*.

EXAMPLES XII. e. (*Oral.*)

Express the sums of money in the following 8 examples as decimals of £1:

1. 10*s*.	**2.** 5*s*.	**3.** 2*s*.	**4.** 1*s*.
5. 8*s*.	**6.** 17*s*.	**7.** 2*s*. 6*d*.	**8.** 15*s*.

Express

9.	10*s*.	as the decimal of	£2.	**10.**	£1. 10*s*.	as the decimal of	£3.
11.	£3	,,	£4.	**12.**	7 lb.	,,	1 qr.
13.	15 cwt.	,,	a ton.	**14.**	7 centimes	,,	a franc.
15.	50 centimes	,,	2 francs.				
16.	2*s*. 6*d*.	,,	10*s*.	**17.**	7*s*. 6*d*.	,,	£1.
18.	5 dwt.	,,	1 oz.	**19.**	7 dwt.	,,	1 oz.
20.	7 dwt.	,,	2 oz.	**21.**	1 guinea	,,	£1.
22.	1 guinea	,,	£2.				

Give the values of the following in shillings and pence:

23. £0·25.	**24.** £0·75.	**25.** £0·95.
26. £0·625.	**27.** £0·725.	**28.** £0·075.

29. £0·64 to the nearest shilling. **30.** £0·39 to the nearest shilling.
31. £0·71 ,, ,, ,, **32.** £0·426 ,, ,, sixpence.
33. 0·823 ,, ,, sixpence.

EXAMPLES XII. f.

[*Avoid side sums, and revise your work before proceeding to the next example.*]

Express

1. 7*s*. 9*d*. as the decimal of £1.

2. 5*s*. 3*d*. as the decimal of £1.

3. 13*s*. 6*d*. " "

4. 18*s*. $10\frac{1}{2}$*d*. " "

5. £1. 14*s*. 6*d*. " £2.

6. £1. 11*s*. $4\frac{1}{2}$*d*. " £2.

7. 9*s*. $3\frac{3}{4}$*d*. " £1.

8. 14*s*. $6\frac{3}{4}$*d*. " £1.

9. £2. 14*s*. $2\frac{1}{4}$*d*. " "

10. £4. 13*s*. $11\frac{1}{4}$*d*. " £5.

11. 15*s*. $9\frac{3}{4}$*d*. " "

12. £3. 13*s*. $4\frac{1}{2}$*d*. " £2.

13. £2. 7*s*. $2\frac{1}{4}$*d*. " £4.

14. 6*s*. $0\frac{3}{4}$*d*. " £1.

15. £7. 15*s*. $11\frac{1}{4}$*d*. " £10.

16. £12. 13*s*. $0\frac{3}{4}$*d*. " £20.

17. £13. 15*s*. " £25.

Reduce

18. 11*s*. to the decimal of 13*s*. 9*d*.

19. 3*s*. $1\frac{1}{4}$*d*. " £1. 4*s*. 10*d*.

20. £1 " £1. 6*s*. 8*d*.

21. 5*s*. $1\frac{1}{2}$*d*. " £4. 2*s*.

22. 1*s*. $1\frac{3}{4}$*d*. " £1. 16*s*. 8*d*.

23. a guinea " £13. 2*s*. 6*d*.

24. 280 lbs. " 1 ton.

25. 6 cwt. 1 qr. " 1 ton.

26. 1 ton 17 cwt. 2 qrs. " 3 tons.

27. 1 qr. 7 lb. " 1 ton.

28. 9 grains " 1 lb. Troy.

29. 1 dwt. 21 gr. " 1 lb. Troy.

30. 4 chains 25 links " 1 mile.

31. 5 furlongs 165 yards " 1 mile.

32. 15*s*. 9*d*. " 1 guinea.

In Examples 33 to 45 give the result correct to 3 decimal places.

Reduce

33. 16*s*. 11*d*. to the decimal of £1.

34. 3*s*. 4*d*. to the decimal of 10*s*.

35. 15*s*. $7\frac{3}{4}$*d*. " £1.

36. £2. 3*s*. $6\frac{1}{2}$*d*. " £1.

37. £3. 6*s*. 11*d*. " £5.

38. £1. 16*s*. 7*d*. " £2. 10*s*.

39. £1. 13*s*. 4*d*. " £3. 10*s*.

40. £3. 14*s*. 6*d*. " £5. 10*s*. 4*d*.

41. 10*s*. $6\frac{3}{4}$*d*. " £1.

42. £3. 15*s*. 2*d*. " £10. 6*s*.

43. 15*s.* 11*d.* to the decimal of £10.
44. 335 yards ,, 1 mile.
45. $3\frac{1}{2}$ yards ,, 1 mile.

Find the value of

46. ·675 of £1.
47. ·675 of 10*s.*
48. ·2625 of £1.
49. ·728125 of £1.
50. 1·834375 of £2.
51. ·0025 of £20.
52. 3·075 of £3.
53. ·0625 of £3.
54. 1·025 of £3. 10*s.*
55. ·08125 of £15.
56. ·25 of £6. 3*s.* 4*d.*
57. ·625 of £12. 3*s.* 4*d.*
58. ·625 of 1 ton.
59. 1·625 of 1 cwt.
60. 2·125 of 2 lb. Troy.
61. ·324 of £1 to the nearest penny.
62. ·619 of £2. 10*s.* to the nearest farthing.
63. ·382471 of £7 ,, ,,
64. ·333 of £6. 13*s.* 4*d.* ,, penny.
65. 2·632 guineas ,, ,,
66. ·002 of a mile ,, inch.
67. ·523 of a guinea ,, penny.
68. ·5 of a guinea + ·2 of half a crown.
69. ·075 of £2 + ·85 of £1.
70. 1·23 of £3 + 1·56 of £2 + ·19 of £1.
71. ·625 of £3. 10*s.* − ·625 of £1.
72. ·3125 of £3 + ·3125 of £2 − ·3125 of £1.
73. £0·6024 + 0·952*s.*
74. £0·7218 + 0·632*s.* − 0·816*d.*
75. ·001 of a ton + ·98 of a cwt.
76. ·325 of a guinea + 2·175*s.*
77. £0·3246 + 0·34*s.* to the nearest penny.

Decimalisation of Money at Sight.

116. One florin $= 2s. = £\frac{1}{10} = £\cdot 1.$

$$2 \text{ florins} = 4s. = £\tfrac{2}{10} = £\cdot 2, \text{ and so on.}$$

$$1s. = £\tfrac{1}{20} = £\cdot 05.$$

$$6 \text{ pence} = £\tfrac{1}{40} = £\cdot 025.$$

Thus $4s.\ 6d. = £\cdot 225.$

$$5s. = £\cdot 25.$$

$$5s.\ 6d. = £(\cdot 25 + \cdot 025) = £\cdot 275.$$

$$1s.\ 6d. = £(\cdot 05 + \cdot 025) = £\cdot 075.$$

Vice versa, $£\cdot 4 = 4 \text{ florins} = 8s.$

$$£\cdot 35 = 3 \text{ florins} + 1s. = 7s.$$

$$£\cdot 425 = 4 \text{ florins} + 6 \text{ pence} = 8s.\ 6d.$$

$$£\cdot 375 = 3 \text{ florins} + 1s. + 6d. = 7s.\ 6d.$$

EXAMPLES XII. g. *(Oral.)*

Read off, in shillings and pence, the value of

1. £0·7.	**2.** £0·5.	**3.** £0·9.	**4.** £0·05.	**5.** £0·15.
6. £0·75.	**7.** £0·45.	**8.** £0·95.	**9.** £0·35.	**10.** £·625.
11. £·225.	**12.** £·725	**13.** £·275.	**14.** £·425.	**15.** £·55.
16. £·175.	**17.** £·975.	**18.** £·825.		

Express as a decimal of £1:

19. 4*s.*	**20.** 7*s.*	**21.** 9*s.*	**22.** 2*s.* 6*d.*	**23.** 4*s.* 6*d.*
24. 8*s.* 6*d.*	**25.** 3*s.* 6*d.*	**26.** 5*s.* 6*d.*	**27.** 9*s.* 6*d.*	**28.** 12*s.* 6*d.*
29. 16*s.* 6*d.*	**30.** 13*s.* 6*d.*	**31.** 15*s.* 6*d.*	**32.** 17*s.* 6*d.*	**33.** 19*s.* 6*d.*

117. 1 penny $= £\frac{1}{240} = £·004166\ldots.$

1 farthing $= £\frac{1}{960} = £·001041666\ldots.$

Let us examine the table below:

Pence	£		£	
1 =	·004	166... =	·004	correct to 3 decimal places.
2 =	·008	333... =	·008	" " "
3 =	·012	5 =	·013	" " "
4 =	·016	666... =	·017	" " "
5 =	·020	833... =	·021	" " "
6 =	·025	=	·025	" " "
7 =	·029	166... =	·029	" " "
8 =	·033	333... =	·033	" " "
9 =	·037	5 =	·038	" " "
10 =	·041	666... =	·042	" " "
11 =	·045	833... =	·046	" " "

From this we see that the following rule holds for converting any number of pence, less than 12, into a decimal of £1 correct to 3 decimal places.

The number of farthings + one extra for every sixpence (counting 3 pence or more as 6 pence) = the number of thousandths of £1.

Thus 7 pence	= 28 farthings	= 28 + 1	thousandths of £1.
5 "	= 20 "	= 20 + 1	" "
9 "	= 36 "	= 36 + 2	" "
$3\frac{1}{2}$ "	= 14 "	= 14 + 1	" "
$11\frac{1}{2}$ "	= 46 "	= 46 + 2	" "

118. The following are correct to 3 decimal places.

After a little practice, the result can be written down without any working.

$4s.\ 7\frac{1}{2}d. = 2$ florins $+ 30$ farthings $= £(\cdot 2 + 31$ thousandths$) = £\cdot 231.$
$5s.\ 3\frac{1}{2}d. = 2$ fl. $+ 1s. + 14f.$ $= £(\cdot 25 + 15$ „ $) = £\cdot 265.$
$7s.\ 11\frac{3}{4}d. = 3$ fl. $+ 1s. + 47f.$ $= £(\cdot 35 + 49$ „ $) = £\cdot 399.$

The converse, viz.: *To convert a decimal to 3 places of £1 into shillings and pence.*

The tenths give the number of florins.

If there are 5 hundredths, they will give a shilling.

Count the remaining thousandths as farthings, subtracting one farthing for every sixpence thus obtained, counting 3d., or more than 3d., as 6d.

$£\cdot 423 = 4$ florins $+ (23 - 1)$ farthings $= 8s.\ 5\frac{1}{2}d.$
(23 farthings $= 5\frac{3}{4}d.$ $\therefore$ we subtract 1 farthing.)
$£\cdot 587 = 5$ florins $+ 1s. + (37 - 2)$ farthings $= 11s.\ 8\frac{3}{4}d.$
(37 farthings $= 9\frac{1}{4}d.$ $\therefore$ we subtract 2 farthings.)

EXAMPLES XII. h. (*Oral.*)

Read off as a decimal of £1 correct to 3 decimal places:

1. $4\frac{1}{2}d.$ **2.** $6\frac{3}{4}d.$ **3.** $9d.$ **4.** $9\frac{3}{4}d.$ **5.** $10\frac{1}{2}d.$
6. $5\frac{1}{2}d.$ **7.** $11\frac{3}{4}d.$ **8.** $2\frac{1}{4}d.$ **9.** $2s.\ 4d.$ **10.** $3s.\ 4d.$
11. $10s.\ 6d.$ **12.** $4s.\ 5d.$ **13.** $8s.\ 2d.$ **14.** $7s.\ 10d.$
15. $6s.\ 8d.$ **16.** $1s.\ 5d.$ **17.** $6s.\ 4\frac{1}{2}d.$ **18.** $3s.\ 4\frac{1}{2}d.$
19. $5s.\ 3\frac{1}{4}d.$ **20.** $4s.\ 10\frac{1}{2}d.$ **21.** $9s.\ 7\frac{1}{4}d.$ **22.** $12s.\ 3d.$
23. $8s.\ 4\frac{3}{4}d.$ **24.** $13s.\ 3\frac{1}{2}d.$ **25.** $14s.\ 8\frac{1}{4}d.$ **26.** $11s.\ 2\frac{3}{4}d.$
27. $9s.\ 11\frac{1}{2}d.$ **28.** $13s.\ 6\frac{3}{4}d.$ **29.** $17s.\ 5\frac{1}{4}d.$ **30.** $15s.\ 9\frac{1}{2}d.$
31. $1s.\ 6\frac{1}{2}d.$ **32.** $9s.\ 10\frac{1}{4}d.$ **33.** $18s.\ 10\frac{1}{4}d.$ **34.** $19s.\ 11\frac{3}{4}d.$

Read off, correct to the nearest farthing, the value of

35. £·027. **36.** £·039. **37.** £·015. **38.** £·065.
39. £·047. **40.** £·089. **41.** £·528. **42.** £·314.
43. £·213. **44.** £·314. **45.** £·726. **46.** £·945.
47. £·704. **48.** £·845. **49.** £·432. **50.** £·06.
51. £·082. **52.** £·09. **53.** £·02. **54.** £·07.
55. £·18. **56.** £·24. **57.** £·37. **58.** £·79.
59. £·156. **60.** £·264. **61.** £·308. **62.** £·068.
63. £·385. **64.** £·223. **65.** £·847. **66.** £·322.
67. £·117. **68.** £·698. **69.** £·992. **70.** £·999.

EXAMPLES XII. k.

PROBLEMS INVOLVING THE USE OF DECIMALS.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

1. From a strip of paper 15 inches long, I cut three pieces each 4·3 inches long. How much remains?

2. If 4·35 metres of wire weigh 6·5 grammes, what will be the weight of 2·61 metres?

3. On a map 1 inch represents 1 mile: how many yards will 0·35 in. represent?

4. If a garden is 33·05 metres long and 17·2 metres wide, what is its area in square metres?

5. Given that 1 metre = 1·09363 yards, express 12 cm. as a decimal of an inch.

6. Find the value of ·00135 of a ton in lb. and decimals of a lb.

7. Find the sum of ·025 of a week, ·35 of a day and ·3 of 3 hours.

8. If 1 kilogram = 2·2 lb. and 1 metre = 39·37 inches, express in pounds per sq. inch a pressure of 10·5 kilograms per sq. centimetre. Give your result to the nearest pound.

9. Find the difference in hours between ·2345 of a day and 2·32 of an hour.
What is the difference to the nearest hour?
What is the difference to the nearest half-hour?

10. Add together $\frac{7}{9}$ of a guinea, $\frac{3}{8}$ of 10*s.* and $\frac{7}{8}$ of 2*s.*; and reduce the result to the decimal of £1 correct to 3 decimal places.

11. Find the value of ·0015 of a mile in feet.

12. How many exact lengths of ·125 of a foot can be cut from a length of 37·36 feet. What length is left?

13. Simplify the expression $\dfrac{\frac{3}{7} \text{ of } 3{\cdot}26}{\frac{5}{14} \text{ of } 2{\cdot}445}$.

14. If sound travels at the rate of 1125 ft. per second, find, to the nearest second, how long it takes to travel 1·025 miles.

15. A book of 350 pages is 18·5 mm. thick; find the thickness (correct to 2 decimal places of a millimetre) of the paper of which it is made.

16. A piece of cloth measured with a yard measure which is 0·6 inch too long appears to be $10\frac{1}{2}$ yards long. What is its true length to the nearest foot?

17. Reduce 12 ft. $4\frac{1}{2}$ in. to the decimal of a mile.

18. If we changed our coinage so that £1 = 10 florins = 100 cents, what would be the value of £3. 7*s.* 3*d.* in £, florins and cents?

19. A bankrupt pays £·625 in the £, and his assets are £324. 10*s.* How much does he owe?

20. Express £17. 7*s.* $4\frac{1}{2}$*d.* in florins and decimal of a florin.

21. If 1 foot = 30·48 cm., find the measure of 1 square foot in square centimetres correct to 2 decimal places.

22. A room is 7 m. 6 dm. long and 5 m. 7 dm. wide; what is its area (1) in sq. metres, (2) in sq. decimetres?

23. Express 4 cwt. 3 qr. 21 lb. as a decimal of 1 cwt., and find (to the nearest penny) the cost of this quantity at £2. 13*s.* 6*d.* per cwt.

24. What is the error if we take 27 m. as the sum of 3·472 m., 4·235 m. and 19·213 m.? What fraction is the error of the sum?

25. A wire is 14·23 metres long. How many lengths of 35 cm. can be cut from it? What is the length of the remainder?

26. The front wheel of an old-fashioned bicycle had a diameter of 54 inches. How far (to the nearest foot) did a rider travel during each revolution of the wheel?

(Circumference of a circle = diameter × 3·1416.)

How many complete revolutions would the wheel make in one mile?

27. Dorando, in New York, ran 26 miles 385 yds. in 2 hrs. 44 min. $20\frac{2}{5}$ secs. What was his average time for a mile to the nearest second?

28. A circular running track is one quarter of a mile in length. Find its diameter to the nearest foot.

(Circumference of a circle = diameter × 3·1416.)

29. A gardener wishing to make a circular flower-bed containing 120 sq. ft. makes its radius 6 ft. long. Find the error to the nearest square foot.

[Area of a circle = $(\text{radius})^2 \times 3·1416$.]

30. Coal weighs 1·4 times its bulk of water, and water weighs 62·3 lb. per cubic foot. Find the weight of 9 cubic feet of coal to the nearest lb.

31. Reduce 3 tons 4 cwt. 3 qrs. to the decimal of a ton, and find the cost of this quantity of coal at £1. 2*s.* 6*d.* a ton to the nearest penny.

32. A motor car travels for 27 minutes at an average rate of 13 miles per hour, for 29 minutes at 21 miles per hour and for 43 minutes at 33 miles per hour. Find the total distance travelled in miles and decimal of a mile, and the average speed in miles per hour (to the nearest mile).

33. Policemen timed a motor car to travel 1 furlong in $17\frac{3}{5}$ sec. By how many miles per hour did the driver exceed a speed of 20 miles per hour? Give your result to the nearest tenth of a mile per hour.

34. The circumference of a circle = diameter $\times \pi$, where $\pi = 3{\cdot}14159$ correct to 5 decimal places. Find the error, correct to 4 decimal places, in the circumference of a circle of 5 ft. diameter, if $\frac{22}{7}$ is used for the value of π. Give the result as a decimal of a foot.

35. A boy is asked to multiply 3·0425 by 7·62. He takes down 3·0325 by mistake, and then does the multiplication correctly. Find his error.

36. Which is the more valuable? And by how much?

£·52345 or 2·7345 of 3*s.* 4*d.* (neglect all farthings).

Contracted Work in Approximations.

119. When approximate results only are required, the work may usually be shortened by the omission of unnecessary figures.

It is necessary, as a rule, to work (often only mentally) to 2 *more* decimal places than required in the result. If we require a result correct to 3 decimal places, we work to 5, and so on.

Addition and Subtraction.

EXAMPLE 1. **Find, correct to 3 decimal places, the sum of 6·34570321, 3·04823145 and 4·129814.**

Contracted Work.		Full Work.
6·345	70	6·34570321
3·048	23	3·04823145
4·129	81	4·129814
13·524		13·52374866

$\therefore$ the sum = 13·524 correct to 3 decimal places.

Explanation. We draw a line down the page immediately after the third decimal places. The third decimal place must be correct; therefore we write down 5 columns of decimal places.

We add up the 2 columns on the right of the line mentally.

In the fourth column, $7+2+8=17$. $\therefore$ in full work we should put down 7 and carry 1. As our result is to be correct in the 3rd decimal place, we carry 2, for 17 is nearer to 20 than to 10.

EXAMPLE 2. **Find the difference, correct to 3 places, between 9·0347812 and 7·846024.**

Contracted Work.		Full Work.
9·034	78	9·0347812
7·846	02	7·846024
1·189		1·1887572

$\therefore$ the difference = 1·189 correct to 3 decimal places.

EXAMPLE 3. **Find the value of $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}$ correct to 3 decimal places.**

$$\begin{array}{ll|l} \frac{1}{3} & = \cdot 333 & 33\ldots \\ \frac{1}{3^2}=\frac{\cdot 333\ldots}{3} & = \cdot 111 & 11\ldots \\ \frac{1}{3^3}=\frac{\cdot 11111\ldots}{3} & = \cdot 037 & 03\ldots \\ \frac{1}{3^4}=\frac{\cdot 037037}{3} & = \cdot 012 & 34\ldots \\ \hline \end{array}$$

∴ the value = ·494 correct to 3 decimal places.

EXAMPLES XII. 1.

Find the sum, correct to three decimal places, of

1. 3·40789, 41·72301, 3·9876253 and 17·006258.
2. 13·0068712, ·00719034, 62·158575 and 3·04.
3. 14·7310421, ·00005, 12·82346 and 7·93567.
4. ·00235, ·008791, ·0638275 and ·000125.
5. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{9}$ and $\frac{1}{11}$.
6. $3\frac{1}{9}$, $7\frac{1}{8}$, $2\frac{1}{12}$ and $1\frac{1}{7}$.
7. $\frac{1}{6}, \frac{1}{6^2}, \frac{1}{6^3}, \frac{1}{6^4}$.
8. $\frac{1}{7}, \frac{1}{7^2}, \frac{1}{7^3}, \frac{1}{7^4}$.
9. $\frac{1}{\lfloor 2}, \frac{1}{\lfloor 3}, \frac{1}{\lfloor 4}, \frac{1}{\lfloor 5}, \frac{1}{\lfloor 6}$.

(*N.B.*—$\lfloor 4 = 1\times 2\times 3\times 4$, and is called "factorial 4.")

10. $\frac{1}{3}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}$.

Find the difference, correct to two decimal places, between

11. 2·7291 and 3·078.
12. 81·93456 and 7·0358.
13. 11·7138 and 31·62571.
14. 4·3856 and 6·2.
15. 6 and 3·0999.
16. 1·9 and ·0999.

Multiplication.

120. EXAMPLE 1. **Find the product of 30·45627 and ·4825 correct to 3 decimal places.**

We first reduce the multiplier ·4825 to standard form.

$$30\cdot 45627 \times \cdot 4825 = 3\cdot 045627 \times 4\cdot 825.$$

[3 × 4·8 = 14·4 gives a rough estimate of the answer.]

CONTRACTED WORK. (With uncorrected figures.)			FULL WORK.
3·045	627		3·045627
4·825			4·825
12·182	48	...(*a*)	12·182508
2·436	48	...(*b*)	2·4365016
·060	90	...(*c*)	·06091254
·015	20	...(*d*)	·015228135
14·695			14·695150275

Requiring a result correct to 3 decimal places, we draw a line down the page immediately after the 3rd decimal place in the multiplicand.

Explanation. Line (a). We work only to 5 decimal places. Hence, we reject the 7 and multiply the rest by 4.

Line (b). $6 \times 8 = 48$. Put down 8 under the 8 in line (a) and carry 4. We see that we no longer need the 2 in the multiplicand, and therefore reject it here.

Line (c). We begin with $5 \times 2 = 10$. Put down 0 and carry 1, rejecting the 6 in the multiplicand.

Line (d). We begin with $4 \times 5 = 20$. Put down 0 and carry 2, rejecting the 5 in the multiplicand.

In the addition at the end $4+4+9+2+1=20$. $\therefore$ we carry 2. *If the addition came to* 26, *we should carry* 3.

A comparison between the full and the contracted work shows the saving of labour.

Comparing line (a) with the corresponding line in the full work, we see that the figures are the same only as far as the third decimal place. This is because the 4, 8 are *uncorrected figures.*

We might have proceeded thus.

3·045	$\cancel{6}\cancel{2}\cancel{7}$	
4·825		
12·182	51	... $7 \times 4 = 28$. $\therefore$ we carry 3 on to 2×4.
2·436	50	... $2 \times 8 = 16$, nearer to 20 than to 10. $\therefore$ we carry 2 to 6×8.
·060	91	... $6 \times 2 = 12$. $\therefore$ we carry 1 to 5×2.
·015	23	... $5 \times 5 = 25$. $\therefore$,, 3 to 4×5.
14·695		

The two columns to the right of the line drawn down the page are now *corrected figures.* This method of using corrected figures is evidently more accurate than the other, and is to be recommended.

It is advisable to cross out the figures on the right in the multiplicand as they are done with.

EXAMPLE 2. **Multiply 34·890625 by 57·0253178 correct to 2 decimal places.**

[$30 \times 60 = 1800$ gives a rough estimate of the answer.]

$$34{\cdot}890625 \times 57{\cdot}0253178 = 348{\cdot}90625 \times 5{\cdot}70253178.$$

348·$\cancel{9}\cancel{0}$	$\cancel{6}\cancel{2}\cancel{5}$	
5·70	253178	
1744·53	13	......(a)
244·23	43	......(b)
·69	78	......(c)
·17	45	......(d)
·01	04	......(e)
·00	03	......(f)
1989·65		

Line (a) begins with $2 \times 5 + 3$ (the 3 is carried from 5×5). We now reject the 2 and the 5 in the multiplicand.

Line (b) begins with $6 \times 7 + 1$ carried, and we now reject the 6 and the 0, for we need not multiply by 0.

Line (*c*) begins with 9×2, and we reject the 9 afterwards.
,, (*d*) ,, $8\times 5+5$ carried, and we now reject the 8.
,, (*e*) ,, $4\times 3+2$,, ,, ,, 4.
,, (*f*) ,, 3×1.

We need not multiply by the 7 and 8 at all, for the result would not affect the 2nd decimal place.

121. *Factors may often be used with advantage.*

EXAMPLE 1. **Find the product of 17·8345662 and ·025 correct to 4 decimal places.**

$$17{\cdot}8345662\times{\cdot}025={\cdot}0178345662\times 25.$$

$$\begin{array}{l|l} {\cdot}0178 & 345662 \\ & \quad\quad\;\; 5 \\ \hline {\cdot}0891 & 73 \\ & \;5 \\ \hline {\cdot}4459 & \end{array}$$

Here $7\times 5+2=37$. $\therefore$ we carry 4.

$\therefore$ the product, correct to 4 decimal places $={\cdot}4459$.

Or we might use the following method :

$$17{\cdot}8345662\times{\cdot}025={\cdot}0178345\ldots\times 25={\cdot}0178345\ldots\times\frac{100}{4}$$

$$=\frac{1{\cdot}78345\ldots}{4}$$

$$={\cdot}4459 \text{ correct to 4 decimal places.}$$

EXAMPLE 2. **Find the product of 670312 and 78639 to the nearest million.**

$$670312\times 78639=6{\cdot}70312\times 7863{\cdot}9\times 10^6.$$

We now find the value of $6{\cdot}70312\times 7863{\cdot}9$ to the nearest integer, as in the above examples. This is 52713.

$\therefore$ the required product $=52{,}713{,}000{,}000$.

EXAMPLES XII. m.

[*Use factors whenever convenient, and form a rough estimate of the answer before doing the work.*]

Find the value of

1.	$6{\cdot}0735\times 4{\cdot}8$	correct to 2 decimal places.
2.	${\cdot}63705\times{\cdot}24$	,, 3 ,, ,,
3.	$144\times{\cdot}06258$	,, 3 ,, ,,
4.	$81{\cdot}92384\times{\cdot}036$	,, 4 ,, ,,
5.	$8{\cdot}03\dot{7}\times 3{\cdot}5$	,, 3 ,, ,,
6.	$8{\cdot}93471\times 2{\cdot}61$	,, 2 ,, ,,
7.	$21{\cdot}63892\times{\cdot}125$	,, 3 ,, ,,
8.	$312{\cdot}70304\times{\cdot}00629$	,, 4 ,, ,,
9.	$762{\cdot}058\times{\cdot}0324$	,, 4 ,, ,,
10.	${\cdot}80395\times{\cdot}5296$	,, 2 ,, ,,
11.	$1000{\cdot}25\times{\cdot}0030205$	,, 3 ,, ,,
12.	$6{\cdot}093571\times 3{\cdot}2456$	,, 2 ,, ,,

Find the value of

13. 34612×1234 to the nearest thousand.

14. 6281456×347 " " million.

15. $346{\cdot}025 \times {\cdot}892$ " " integer.

16. $45{\cdot}602 \times 89{\cdot}3$ correct to 4 significant figures.

17. $8{\cdot}926 \times 3{\cdot}54$ " 3 " "

18. ${\cdot}0892 \times {\cdot}0354$ " 3 " "

19. $347{\cdot}031 \times {\cdot}060305$ correct to 2 decimal places.

20. $890{\cdot}204 \times {\cdot}43205$ " 3 " "

Division.

122. We divide by factors whenever convenient, in which case it is best to convert the divisor into a whole number.

Example 1. **Divide 7·34 by 3·6 correct to 3 decimal places.**

$$\frac{7{\cdot}34}{3{\cdot}6} = \frac{73{\cdot}4}{36}. \qquad \begin{array}{r|l} 6 & 73{\cdot}4 \\ \hline 6 & 12{\cdot}2333\ldots \\ \hline & 2{\cdot}0388\ldots \end{array}$$

$\therefore$ the required quotient $= 2{\cdot}039$.

In all cases it is necessary to work to one more decimal place than is required in the result.

A multiplier may sometimes be used with advantage.

Example 2. **Divide 732·05718 by 1·25 correct to 4 decimal places.**

$$\frac{732{\cdot}05718}{1{\cdot}25} = \frac{732{\cdot}05718 \times 8}{1{\cdot}25 \times 8} = \frac{5856{\cdot}45744}{10}$$

$= 585{\cdot}6457$ correct to 4 decimal places.

Example 3. **Divide 9432·56 by 302·78 correct to 3 decimal places.**

$$\frac{9432{\cdot}56}{302{\cdot}78} = \frac{94{\cdot}3256}{3{\cdot}0278}. \quad \text{[Rough estimate of answer} = 31 + \text{a decimal.]}$$

Contracted Work.

$$\begin{array}{r|l} & 31{\cdot}1531 \\ \hline 3{\cdot}0278\ )\ 94{\cdot}3256 & \\ 90\ 834 & \\ \hline 3\ 4916 & \\ 3\ 0278 & \\ \hline 4638 & \ldots(a) \\ 3028 & \ldots(b) \\ \hline 1610 & \\ 1514 & \ldots(c) \\ \hline 96 & \\ 91 & \\ \hline 5 & \\ 3 & \end{array}$$

Full Work.

$$\begin{array}{r} 31{\cdot}1531 \\ \hline 3{\cdot}0278\ )\ 94{\cdot}3256 \\ 90\ 834 \\ \hline 3\ 4916 \\ 3\ 0278 \\ \hline 46380 \\ 30278 \\ \hline 161020 \\ 151390 \\ \hline 96300 \\ 90834 \\ \hline 54660 \\ 30278 \\ \hline \end{array}$$

$\therefore$ the required quotient is **31·153**.

The work must be carefully done in **straight** columns.

Explanation. We work to 4 decimal places, and therefore draw a straight line down the page immediately after the 4th decimal place in the dividend.

Line (a). Here we reject the 0 which would follow the 8. Therefore we also reject the 8 in the divisor, remembering that the last figure in line (b) must be *corrected.*

Line (c). To obtain line (c) we reject the 7 in the divisor. Here again the 4 is a *corrected* figure, the next rejected figure being 9.

EXAMPLE 4. **Divide ·076823461 by ·0035462732 correct to 4 decimal places.**

$$\frac{\cdot076823461}{\cdot0035462732}=\frac{76\cdot823461}{3\cdot5462732}.\quad\left[\text{Rough estimate of answer}=\frac{770}{35}=22.\right]$$

```
               21·66315
3·5462732 ) 76·82346 | 1
            70 92546
            5 89800
            3 54627
            2 35173
            2 12776 | ...7 × 6 = 42.        ∴ we carry 4 to 2 × 6.
              22397
              21278 | ...2 × 6 + 4 = 16.    ∴      ,,     2 to 6 × 6.
               1119
               1064 | ...6 × 3 = 18.        ∴      ,,     2 to 4 × 3.
                 55
                 35
                 20
                 18 | ...5 × 5 = 25.        ∴      ,,     3 to 3 × 5.
```

$\therefore$ the required quotient is 21·6632.

Here we need not use the 2, the last figure in the divisor.

EXAMPLE 5. **Find the value of $\frac{65\cdot43\times172\cdot8947}{140\cdot5}$ correct to 3 decimal places.**

The expression $=\frac{\cdot6543\times172\cdot8947}{1\cdot405}$. (We reduce the divisor to standard form.)

When we do the division, we shall have to work to 4 decimal places.

Therefore, doing the multiplication first, we find the product correct to 4 decimal places, and then perform the division.

The product $=113\cdot1250$ correct to 4 decimal places.

$\therefore$ the required value $=\frac{113\cdot1250}{1\cdot405}=80\cdot516$ correct to 3 decimal places.

The working is left to the student.

123. Reciprocals.

$\frac{1}{a}$ is said to be the reciprocal of a,

$\frac{1}{13}$ " " " 13, and so on.

In some branches of Physical work the reciprocals of numbers enter largely into the calculations; so much so, that Tables of Reciprocals expressed as decimals are published.

For instance, in Mensuration it is extremely useful to know that the value of $\frac{1}{\pi}$ correct to five decimal places is ·31831.

EXAMPLE. **Find the value, correct to 6 decimal places, of $\frac{1}{7835}+\frac{1}{2903}$.**

By division or from tables, $\frac{1}{7835}=\cdot000127\,|\,63$

" $\frac{1}{2903}=\cdot000344\,|\,47$

$\therefore\ \frac{1}{7835}+\frac{1}{2903}=\cdot000472$ correct to 6 dec. places.

EXAMPLES XII. n.

Divide

1. 3·0452 by 35	correct to	6	decimal places.
2. 5·03216 by 18	"	4	"
3. ·41021 by ·24	"	3	"
4. 31·7971 by 3·6	"	5	"
5. 703·912 by 12·5	"	3	"
6. ·09831 by ·75	"	4	"
7. ·00372 by 7·2	"	6	"
8. ·00345 by ·000144	"	2	"
9. ·0031 by 10·5	"	4	"
10. 920·83 by 151·6	"	3	"
11. 91·380125 by 3·1416	"	2	"
12. ·7922568 by ·031	"	3	"
13. 153·62091 by 179·21	"	3	"
14. ·0003125 by ·17	"	5	"
15. ·341025 by ·6345	"	3	"
16. 135·0009 by 29	"	3	"

[*Where possible simplify the expression by cancellation before performing multiplication or division.*]

Find the value of

17. $\frac{\cdot 34 \times \cdot 71825}{1\cdot 7}$ correct to 3 decimal places.

18. $\frac{\cdot 125 \times 3\cdot 78792}{\cdot 25}$ " 3 "

19. $\frac{\cdot 375 \times 91\cdot 3719}{1\cdot 25}$ " 4 "

20. $\frac{10\cdot 4 \times 9\cdot 02304}{6\cdot 4}$ " 3 "

21. $\frac{185 \times 6\cdot 020499}{111}$ " 4 "

22. $\frac{1}{11}+\frac{1}{17}+\frac{1}{19}+\frac{1}{31}$ " 3 "

23. $\frac{1}{2703}+\frac{1}{3961}$ " 2 significant digits.

24. $\frac{1}{5801}-\frac{1}{8362}$ " " "

25. $\frac{1}{97}+\frac{1}{37}+\frac{1}{119}$ " " "

26. $\frac{3}{5732}+\frac{4}{8921}$ " " "

27. $\frac{5}{2032}+\frac{6}{8065}$ " " "

XIII. HARDER EXAMPLES ON TIME AND WORK.

124. EXAMPLE 1. **A can do a piece of work in 10, B in 12, C in 9 days. If A work for 2 days, then B and C for 4 days, in what time would A finish the work?**

A works for 2 days, and in that time does $\frac{2}{10}\left(=\frac{1}{5}\right)$ of the work.

B " 4 " " $\frac{4}{12}\left(=\frac{1}{3}\right)$ "

C " 4 " " $\frac{4}{9}$ " "

$\frac{1}{5}+\frac{1}{3}+\frac{4}{9}=\frac{9+15+20}{45}=\frac{44}{45}$.

$\therefore$ $\frac{1}{45}$ of the work remains to be done by A.

A does the whole in 10 days.

$\therefore$ he does $\frac{1}{45}$ in $\frac{2}{9}$ of a day.

EXAMPLE 2. **Of three pipes in a cistern, the first can fill it in 20 min., the second in 15 min., and the third can empty it in 30 min. All three pipes are opened until the cistern is half full, when the third is shut off. What is the total time taken to fill the cistern?**

In one minute the three pipes fill $\frac{1}{20}+\frac{1}{15}-\frac{1}{30}$, *i.e.* $\frac{3+4-2}{60}\left(=\frac{1}{12}\right)$.

$\therefore$ they would take 12 min. to fill the cistern.

$\therefore$ " " 6 " half the cistern.

The first two pipes in one min. fill $\frac{1}{20}+\frac{1}{15}$, *i.e.* $\frac{3+4}{60}\left(=\frac{7}{60}\right)$.

$\therefore$ they would fill the cistern in $\frac{60}{7}$ min.

$\therefore$,, half ,, $\frac{30}{7}\left(=4\frac{2}{7}\right)$ min.

$\therefore$ the total time required $=6+4\frac{2}{7}=10\frac{2}{7}$ min.

EXAMPLES XIII.

1. A can do a piece of work in 6 days, B in 8 days and C in 12 days. B and C work together for 2 days, and then C is replaced by A. How long do A and B take to finish the work?

2. A cistern can be emptied by one tap in 20 minutes and by another in 15 minutes. The first, after running for 6 minutes, is closed, and the second is then opened. How long is the cistern in being emptied?

3. A can do a piece of work in 10 days, B in $12\frac{1}{2}$, C in 15. If A and B work for 2 days, then A and C for 3 days, in what time would B and C finish the work? Check your result by adding together the fractions of work done by each pair.

4. A and B working together can do a piece of work in 7 days, and B alone can do it in 13 days. If B stops after 3 days, how long afterwards will A take to finish the work which he began with B?

5. A cistern can be filled by two taps in 4 minutes and 6 minutes respectively. Both taps are turned full on for 3 minutes, while the waste pipe is also left open. If the cistern is now just full, and the two taps are turned off, how long will it be before the cistern is again empty?

6. A can do a piece of work in 27 days. B can do it in 15 days. A works 12 days at it, then B works 5 days at it, and C finishes it in 4 days more. In what time could C have done it all?

7. A piece of work can be done by 3 men and 4 boys in 6 days, by 3 men and 1 boy in 8 days and by 4 women and 8 boys in 5 days. How long would a woman take to complete the work single-handed?

8. Two men and a boy can do a piece of work in 5 days, whilst a man and 2 boys can do it in 6 days. If a man is paid at the rate of 28*s.* a week, what should be the wages of a boy?

9. If A and B can do a piece of work in 9 days which A, B and C can do in 6 days, and A and C in 8 days, find the time which each alone would take.

10. A cistern can be emptied by one tap in 20 minutes and by another in 15 minutes. After the first has been opened 6 minutes, the second is also opened. How long do they take to empty the cistern?

11. A can do a piece of work in 6 days; B can do thrice as much in 16 days; C can do five times as much in 24 days. If A begins the work on Monday, B joins him on Tuesday, and C is also put on on Wednesday, on which day will they finish the work?

12. A and B can reap a field in 10 days, A and C in 12 days, B and C in 15 days. They all work together for 4 days, when A strikes. B and C work together for 6 days more, when B strikes. In how many days can C finish the work?

13. A pipe which can fill a cistern in 50 min. is set running. After 10 min. a second pipe which can fill the cistern in 20 min. is also set running. After another 10 min. a waste pipe which can empty the cistern in 10 min. is opened. In what time from the start is the cistern again empty?

14. Three pipes in a cistern are opened simultaneously. The first can fill it in 10 minutes, the second in 9 minutes, and the third can empty it in 20 minutes. When the cistern is two-thirds full, the third pipe is shut off. What is the total time taken to fill the cistern?

XIV. SQUARE ROOT. CUBE ROOT BY FACTORISATION.

125. THE student has learnt in Chapter V. that the **square root** of a number is that number which, multiplied by itself, gives the original number.

Thus $\sqrt{25}=5$, for $5\times5=25$.

Every quantity has two square roots, equal in value but opposite in sign. *E.g.* $\sqrt{4}$ is $+2$ or -2,

$$\text{for } (+2)^2=4 \text{ and } (-2)^2=4.$$

In Arithmetic we deal only with positive square roots.

We can often determine a square root by factorising.

$$625=5\times125=5^2\times25. \quad \therefore \sqrt{625}=25.$$

$$576=4\times144=4^2\times36. \quad \therefore \sqrt{576}=24.$$

$$8281=7\times1183=7^2\times169. \quad \therefore \sqrt{8281}=7\times13=91.$$

126. When a square root cannot be found by using factors, the following method must be used. It is based on the algebraic method, a proof of which will be found in most text-books on Algebra.

EXAMPLE 1. **Find the square root of 942841.**

First mark off the digits, two at a time, beginning on the right-hand side.

$$\begin{array}{rl}
 & 94\,28\,41\ (\,971 \\
 & 81\ldots\ldots\ldots\ldots\ldots\ldots(a) \\
2\times90+7=187\ldots(b) & \overline{1328} \\
187\times7\ = & 1309 \\
2\times970+1=1941 & \overline{\ \ 1941} \\
1941\times1\ = & \ \ 1941 \\
 & \overline{\qquad}
\end{array}$$

Explanation. Line (*a*). The nearest *square* below 94 is 9^2 $(=81)$.

We therefore put down 81 under 94 and subtract, leaving 13. We now bring down 28.

Line (*b*). We now multiply the result, as far as obtained, *i.e.* 9, by 10 and by 2. $2 \times 90 = 180$.

180 goes 7 times into 1328.

$\therefore$ 7 is the next figure in the result.

Add 7 to 180, and multiply the result by 7, obtaining 1309.

This we subtract from 1328, leaving 19.

We now bring down 41.

We now repeat the process of line (*b*), *i.e.* multiply 97 by 10 and by 2, obtaining 1940.

1940 goes once into 1941.

$\therefore$ we add 1 to 1940, and multiply the result by 1, obtaining 1941.

There is no remainder on subtracting, which shows that 971 is the exact square root of 942841.

EXAMPLE 2. **Find the square root of 9369721.**

$$
\begin{array}{rrl}
 & 9,36,97,21 & (\,3061 \\
 & \underline{9} & \\
2 \times 30 = 60 & 3697 & \text{60 will not divide 36.} \ \therefore \text{ we add 0 to} \\
2 \times 300 + 6 = 606 & & \text{the result, and bring down 97.} \\
606 \times 6 = & \underline{3636} & \\
 & 6121 & \\
2 \times 3060 + 1 = 6121 \quad = & \underline{6121} &
\end{array}
$$

Cube Root.

127. The *cube root* of a quantity is that quantity whose **third power** is equal to the original quantity.

$$2^3 = 2 \times 2 \times 2 = 8. \quad \therefore \ 2 \text{ is the cube root of } 8.$$

This is written thus : $2 = \sqrt[3]{8}$.

$$3^3 = 27. \quad \therefore \ \sqrt[3]{27} = 3.$$

We shall only deal here with quantities which are exact cubes and whose cube roots can be found by factorisation, or inspection.

EXAMPLE 1. **Find the cube root of 2744.**

$$2744 = 7 \times 392 = 7^2 \times 56 = 7^3 \times 8 = 7^3 \times 2^3.$$

$$\therefore \ \sqrt[3]{2744} = 7 \times 2 = 14.$$

EXAMPLE 2. **Find the cube root of 91125.**

$$91125 = 5 \times 18225 = 5^2 \times 3645 = 5^3 \times 729$$
$$= 5^3 \times 9^3.$$
$$\therefore \ \sqrt[3]{91125} = 45.$$

We can often determine a cube root by inspection.

If the number whose cube root is required ends in the figure

1,	last digit in cube root is 1;	no other units digit, *when cubed*, ends in 1.
2,	,, ,, 8;	,, ,, ,, ,, 2.
3,	,, ,, 7;	,, ,, ,, ,, 3.
4,	,, ,, 4;	,, ,, ,, ,, 4.
5,	,, ,, 5;	,, ,, ,, ,, 5.
6,	,, ,, 6;	,, ,, ,, ,, 6.
7,	,, ,, 3;	,, ,, ,, ,, 7.
8,	,, ,, 2;	,, ,, ,, ,, 8.
9,	,, ,, 9;	,, ,, ,, ,, 9.

Thus $\sqrt[3]{1331}$ ends with the figure 1.

Also 1331 is greater than 1000 (10^3) and less than $(20)^3$;

$$\therefore \sqrt[3]{1331} = 11.$$

We must, however, be careful to see **by checking** that the number has an exact cube root. *E.g.* we might otherwise take $\sqrt[3]{1431}$ to be 11.

$\sqrt[3]{12167}$ ends with the figure 3.

Also 12167 is greater than $(20)^3$, and less than $(30)^3$.

$\therefore$ if 12167 has a cube root its value is 23.

If we cube 23, we shall see that this is a correct cube root. Again $\sqrt[3]{103823}$ ends with the figure 7.

Also 103823 is greater than $(40)^3$, and less than $(50)^3$.

$\therefore$ if 103823 has a cube root its value is 47.

EXAMPLES XIV. a.

Find (by factorisation whenever convenient) the square root of

1. 729. **2.** 1296. **3.** 2025. **4.** 5184. **5.** 625.

6. 1225. **7.** 11025. **8.** 1764. **9.** 2916. **10.** 16129.

11. 116281. **12.** 388129. **13.** 40401. **14.** 9138529.

15. 12313081. **16.** 67749361. **17.** 25270729.

18. 1002001. **19.** 27352900. **20.** 9541921.

Find the cube root of

21. 125. **22.** 729. **23.** 512. **24.** 3375. **25.** 4096.

26. 42875. **27.** 1157625. **28.** 35937. **29.** 10648. **30.** 250047.

31. 166375. **32.** 157464. **33.** ·125. **34.** 4·096.

35. ·001. **36.** ·000064. **37.** ·042875. **38.** 157·464.

Determine the cube roots of the following numbers by inspection :

39. 4913. **40.** 6859. **41.** 24389. **42.** 29791.
43. 389017. **44.** 79507. **45.** 1·331.
46. ·729. **47.** ·000001728. **48.** ·000343.

Square Root of a Decimal.

128. The process of finding the square root of a decimal is the same as that for whole numbers, but we must remember to mark off the digits, two at a time, **starting from the decimal point**, and marking them off to the right as well as to the left.

EXAMPLE 1. **Find the square root of 5·5225**

$$\begin{array}{rl} & 5,\cdot52,25\ (\ 2\cdot35 \quad \text{(Rough estimate : rather greater than 2.)} \\ & \underline{4} \\ 2\times20+3=43 & 152 \\ 43\times3\ = & \underline{129} \\ 2\times230+5=465 & \ \ 2325 \\ 465\times5\ = & \ \ \underline{2325} \end{array}$$

The position of the decimal point can be determined by inspection, when the pairs of digits are marked off.

Thus, in $\sqrt{10,69,\cdot29}$, there will evidently be two digits in the integral part.

EXAMPLE 2. **Find the square root of 24 correct to 2 decimal places.**

To determine the correct second decimal place, we must work to 3 places.

$$\begin{array}{rl} & 24,\cdot00,00,00\ (\ 4\cdot898 \\ & \underline{16} \\ 2\times40+8=80+8 & \ \ 800 \\ 88\times8\ = & \ \ \underline{704} \\ 2\times480+9=960+9 & \ \ \ \ 9600 \\ 969\times9\ = & \ \ \ \ \underline{8721} \\ 2\times4890+8=9780+8 & \ \ \ \ \ \ 87900 \\ 9788\times8\ = & \ \ \ \ \ \ \underline{78304} \end{array}$$

$\therefore\ \sqrt{24}=4\cdot90$ correct to 2 decimal places.

Note the following carefully :

$\sqrt{\cdot00,00,36}=\cdot006$, by inspection.

$\sqrt{\cdot00,03,6}\ =\cdot018\ldots.$ (See the working below.)

$$\begin{array}{rl} & \cdot00,03,60,00\ (\ \cdot018\ldots \\ & \underline{1} \\ 2\times10+8=20+8 & 260 \\ 28\times8\ = & \underline{224} \\ & \ \ 3600 \end{array}$$

The Square Root of a Vulgar Fraction.

129.
$$\sqrt{2\tfrac{1}{4}}=\sqrt{\frac{9}{4}}=\frac{\sqrt{9}}{\sqrt{4}}=\frac{3}{2}.$$

Check.
$$\left(\frac{3}{2}\right)^2=\frac{9}{4}=2\tfrac{1}{4}.$$

$$\sqrt{39\tfrac{1}{16}}=\sqrt{\frac{625}{16}}=\frac{25}{4}=6\tfrac{1}{4}.$$

130. Some quantities have no exact square root.

The square of a fraction (in its lowest terms) is also in its lowest terms.

Thus, $\left(\frac{2}{3}\right)^2=\frac{4}{9}$ and $\left(\frac{5}{3}\right)^2=\frac{25}{9}$.

Hence, we see that the square of a fraction in its lowest terms cannot be a whole number.

Now $\sqrt{2}$ is evidently greater than 1, and less than 2. Therefore, if 2 had an exact square root, that root would be a fraction. Since, therefore, the square of a fraction cannot be a whole number, we see that 2 has no exact square root.

Let us use the ordinary process to try and find $\sqrt{2}$.

$$\begin{array}{rcl}
 & & 2\cdot00\,00\,00\ (\,1\cdot4142\ldots \\
 & & 1 \\
2\times10+4=20+4 & & 100 \\
24\times4 & = & 96 \\
2\times140+1=280+1 & & 400 \\
281\times1 & = & 281 \\
2\times1410+4=2820+4 & & 11900 \\
2824\times4 & = & 11296 \\
 & & 60400,\ \text{etc.}
\end{array}$$

Thus we see that we can find the square root of $\sqrt{2}$ to as many decimal places as we please, though we cannot find its exact value. When a root of an arithmetical quantity cannot be *exactly* determined, such a root is called a **surd** or **irrational quantity**.

$\sqrt{7}$, $\sqrt{3}$, $\sqrt{13}$ are all surds. $\sqrt{4}$ is a *rational* quantity.

131. When $n+1$ digits of a square root (of $2n+1$ digits) have been found, the remaining n digits may be found by division.

Or, when more than half the required number of digits have been found, the rest may be found by division.

EXAMPLE. **Find the square root of 3687525625.**

We see that there will be 5 digits in the result. Hence, when we have found 3 digits by the ordinary process, we shall find the remaining 2 digits by division.

$$
\begin{array}{rr}
 & 36,87,52,56,25\ (\ 60725 \\
 & \underline{36} \\
1200+7=1207 & 8752 \\
1207\times 7\ = & \underline{8449} \\
607\times 2=1214\ ) & 3035625\ (\ 25^{*} \\
 & \underline{2428} \\
 & 6076 \\
 & \underline{6070}
\end{array}
$$

* Here we use long division, taking as divisor the result (as far as obtained, *i.e.* 607) multiplied by 2.

EXAMPLES XIV. b.

[*Revise your work before proceeding to the next example.*]

Find the square root of

1. 18·1476. **2.** 4·2025. **3.** 2·566404. **4.** 189·0625.

5. 22710·49. **6.** 3881·29. **7.** 2527·0729. **8.** ·040401.

9. ·00002916. **10.** ·00001225. **11.** 1·002001.

12. ·00011025. **13.** ·0004.

14. ·004 correct to 3 decimal places. **15.** ·0025.

16. ·00025 " 3 " **17.** ·0169.

18. ·00169 " 3 " **19.** ·000144.

20. ·0001443 " 3 "

21. 23 " 3 "

22. 691 " 2 "

23. 69·1 " 2 "

24. 7941 " 2 "

25. 794·1 " 2 "

26. ·003361 " 3 "

27. ·0003361 " 3 "

28. $6\frac{1}{4}$. **29.** $10\frac{9}{16}$. **30.** $182\frac{1}{4}$. **31.** $12\frac{37}{49}$.

32. $252\frac{1}{64}$. **33.** $333\frac{108}{121}$. **34.** $621\frac{4}{625}$. **35.** $1374\frac{355}{729}$.

Area of a Triangle.

Definition. A **triangle** is a figure which is bounded by three straight lines.

132. *A right-angled triangle.*

Let ABC be a triangle, right-angled at B. Draw CD parallel to BA, and AD parallel to BC, so as to form the rectangle ABCD. Let the rectangle be now cut out with a pair of scissors, and then divided into two parts by cutting along the line AC. If the parts ABC, CDA are now placed upon one another, so that AD falls on CB, we shall find that they exactly coincide with one another and are therefore equal in area.

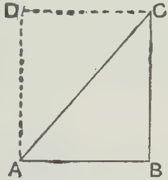


FIG. 17.

$\therefore$ if we call AB the base, and CB the altitude of the triangle ABC, its area $= \frac{1}{2}$ area of ABCD $= \frac{1}{2}\text{AB} \times \text{BC}$

$= \frac{1}{2}$ base $\times$ altitude.

Next take *any* triangle ABC, and let AD be drawn perpendicular to the base BC. There are two cases to consider, (1) when the point D lies in CB, and (2) when it lies in CB produced.

In both cases area of $\triangle\text{ACD} = \frac{1}{2}\text{CD} \times \text{AD}$,

and area of $\triangle\text{ABD} = \frac{1}{2}\text{BD} \times \text{AD}$.

In case (1)

$$\triangle\text{ABC} = \triangle\text{ACD} + \triangle\text{ADB} = \tfrac{1}{2}\text{CD} \times \text{AD} + \tfrac{1}{2}\text{BD} \times \text{AD}$$
$$= \tfrac{1}{2}(\text{CD} + \text{BD})\text{AD} = \tfrac{1}{2}\text{BC} \times \text{AD} = \tfrac{1}{2}\text{base} \times \text{altitude}.$$

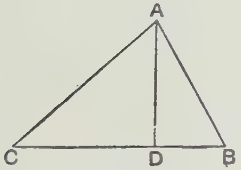


FIG. 18.

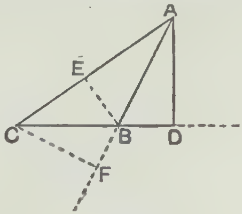


FIG. 19.

In case (2)
$$\triangle\text{ABC} = \triangle\text{ADC} - \triangle\text{ABD}$$
$$= \tfrac{1}{2}\text{CD} \times \text{AD} - \tfrac{1}{2}\text{BD} \times \text{AD}$$
$$= \tfrac{1}{2}(\text{CD} - \text{BD})\text{AD} = \tfrac{1}{2}\text{BC} \times \text{AD}$$
$$= \tfrac{1}{2}\text{base} \times \text{altitude}.$$

Hence we see that in all cases:

The area of a triangle is equal to one half the product of its base and altitude.

We may take any side as 'base,' remembering that 'altitude' means the perpendicular distance of that side from the opposite angular point.

Thus, in Fig. 19,

$$\triangle \mathsf{ABC} = \tfrac{1}{2}\mathsf{AC} \times \mathsf{BE} = \tfrac{1}{2}\mathsf{AB} \times \mathsf{CF},$$

where BE is perpendicular to AC and CF to AB.

EXAMPLES XIV. c.

Using the letters a, b, c to denote the sides opposite the angles A, B, C respectively in the triangle ABC, find the areas of the triangles, in Examples 1–4, from the data:

1. $a=6$ ft., $b=9$ ft., angle ACB = a right angle.

2. $a=1$ ft. 6 in., $c=2$ ft. 3 in., angle ABC = a right angle.

3. $a=5\frac{1}{2}$ chains, $b=3$ chains, angle ACB = a right angle.
Give your result as a decimal of an acre.

4. Base = 48 ft., altitude = 13 ft.
Give your result to the nearest sq. yard.

5. A triangular field has a base 330 yds. long and an altitude of 120 yds. A surveyor estimates its area as 4 acres. Find his error in sq. yds.

6. Find the side of a square field equal in area to a triangular field whose base is 152 yds. and altitude 76 yds.

7. A triangular field has an area of 1338 sq. yds. and a base 45 yds. long. Find its altitude to the nearest half-yd.

8. ABCD is a quadrilateral field. The diagonal BD is measured and found to be 99 yds. long. The perpendicular distances of A and C from DB are measured and found to be 45 yds. and 65 yds. respectively. Find the area of the field in acres.

Pythagoras' Theorem.

Fig. 20

133. If ABC is a triangle, right-angled at C, and a, b, c denote the lengths of its sides, it is found that $a^2+b^2=c^2$. This is known as Pythagoras' Theorem, and may be stated in words:

The square on the hypotenuse (the side opposite the right angle) of a right-angled triangle is equal to the sum of the squares on the other sides.

The converse is true, viz.:

If the square on one side of a triangle is equal to the sum of the squares on the other two sides, the angle contained by those two sides is a right angle.

Proof of Pythagoras' Theorem.

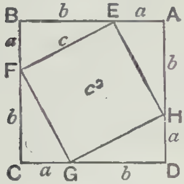


FIG. 21.

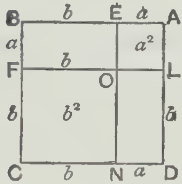


FIG. 22.

Take any square ABCD, and from its sides cut off lengths AE, BF, CG, DH, each equal to a suppose, as in Fig. 21. Join EF, FG, GH, HE. EFGH will be a square. Let $\mathrm{EF}=c$, so that the area of $\mathrm{EFGH}=c^2$.

Also $\mathrm{EB}=\mathrm{FC}=\mathrm{GD}=\mathrm{HA}=b$ suppose.

We see that the four triangles EBF, FCG, GDH, HAE, are all equal in area, and the area of each $=\frac{1}{2}ab$.

Hence the square ABCD = EFGH + the 4 triangles

$$=c^2+2ab. \qquad (1)$$

Again, divide the same square ABCD up in another manner, as in Fig. 22, drawing EON parallel to BC or AD, and FOL parallel to BA or CD.

Then the figs. BO and OD are rectangles, each of area ab.

Also EALO is the square on AE, and is equal to a^2, and FCNO is the square on FO or on BE, and is equal to b^2.

Hence the whole square ABCD $=a^2+b^2+$ rects. BO and OD

$$=a^2+b^2+2ab. \qquad (2)$$

$\therefore$ from (1), $\qquad a^2+b^2=c^2,$

or the sum of the squares on BE and BF = the square on EF.

Q.E.D.

134. Pythagoras' Theorem enables us to find the length of the diagonal of a square of given side.

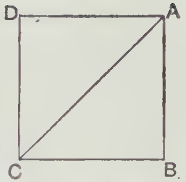


Fig. 23.

If ABCD is the square and $AB = a$, then

$$AC^2 = AB^2 + BC^2 = 2a^2;$$

$$\therefore\ AC = \sqrt{2}\,.\,a.$$

In a similar manner, if we know the sides of a rectangle, we can obtain the lengths of its diagonals.

If a, b denote the lengths of adjacent sides, the length of each diagonal $= \sqrt{a^2 + b^2}$. Thus we see that the diagonals of a rectangle are equal.

135. Important. The following three results are useful and should be committed to memory.

$$\sqrt{2} = 1{\cdot}41421, \qquad \sqrt{3} = 1{\cdot}73205, \qquad \sqrt{6} = 2{\cdot}44949.$$

The above are correct to five decimal places.

$$\frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}.$$

$$\frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2} = \frac{7}{2} \times 1{\cdot}41421 = 7 \times 0{\cdot}707105$$

$$= 4{\cdot}95 \text{ correct to 2 decimal places.}$$

$$\frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4}{3} \times 1{\cdot}73205 = 2{\cdot}3094\ldots.$$

Algebra tells us that $a^2 - b^2 = (a + b)(a - b)$.

This can often be usefully employed in numerical examples.

Thus $\qquad 5^2 - 3^2 = (5 + 3)(5 - 3) = 8 \times 2 = 16.$

$$99^2 - 1 = (99 + 1)(99 - 1) = 100 \times 98 = 9800.$$

$$99 \times 101 = (100 - 1)(100 + 1) = 100^2 - 1^2 = 10000 - 1 = 9999.$$

A triangle whose sides are 3, 4, 5 is right-angled, for

$$5^2 - 3^2 = (5 + 3)(5 - 3) = 4^2; \qquad \therefore\ 3^2 + 4^2 = 5^2.$$

In the same way it may be shown that

lengths of 5, 12, 13 form a **right-angled** triangle,

and „ 7, 24, 25 „ „ „.

EXAMPLES XIV. d.

1. Find, to the nearest hundredth of an inch, the length of a diagonal of a square whose side is 5 inches long.

2. The lengths of the sides of a rectangular field are 7 chains and 3 chains; find the length of a diagonal to the nearest link.

3. The diagonal of a square is 10 feet long; find the length of a side in feet correct to 3 decimal places. Also give the result to the nearest inch.

4. A rectangular flower border is 36 ft. long by 7 ft. wide. Find, to the nearest foot, the side of a square of equal area.

5. The diagonal of a square is 15 feet long. Find the length of a side to the nearest inch.

6. The diagonal of a square is 3 feet long: find its area.

Surds.

136. EXAMPLE 1. **Find the value of $\frac{1}{\sqrt{2}}$ correct to 4 decimal places.**

We first *rationalise* the denominator.

$$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{\sqrt{2}}{2}.$$

We now find the value of $\sqrt{2}$ as in Art. 130.

$$\therefore\ \frac{1}{\sqrt{2}}=\frac{1{\cdot}41421\ldots}{2}=\cdot70710\ldots$$

$$=\cdot7071 \text{ correct to 4 decimal places.}$$

Important. We know that $(a+b)(a-b)=a^2-b^2$.

$$\therefore\ (\sqrt{6}-2)(\sqrt{6}+2)=(\sqrt{6})^2-2^2=6-4=2.$$

$$(\sqrt{17}-3)(\sqrt{17}+3)=(\sqrt{17})^2-3^2=17-9=8.$$

EXAMPLE 2. **Find the value of $\frac{5}{\sqrt{2}-1}$ correct to 3 decimal places.**

We first rationalise the denominator, by multiplying numerator and denominator by $(\sqrt{2}+1)$.

$$\frac{5}{\sqrt{2}-1}=\frac{5(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{5(\sqrt{2}+1)}{2-1}=5(\sqrt{2}+1)$$
$$=5\times 2{\cdot}41421$$
$$=12{\cdot}071 \text{ correct to 3 decimal places.}$$

137. Important.

$(a+b)^2=a^2+2ab+b^2$ for all values of a and b.

$(\sqrt{x}+\sqrt{y})^2=x+2\sqrt{xy}+y$ „ „ x and y.

$(a-b)^2=a^2-2ab+b^2$ „ „ a and b.

$(\sqrt{x}-\sqrt{y})^2=x-2\sqrt{xy}+y$ „ „ x and y.

Thus $$(2-\sqrt{3})^2 = 4 - 4\sqrt{3} + 3 = 7 - 4\sqrt{3}.$$
$$(\sqrt{5}+\sqrt{2})^2 = 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}.$$

EXAMPLE. **Find the value of $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ correct to 3 decimal places.**

$$\frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{5-2\sqrt{5}+1}{5-1} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}.$$

We find that $\sqrt{5} = 2{\cdot}23607$ correct to 5 decimal places.

$$\therefore \text{ the given expression} = \frac{3 - 2{\cdot}23607}{2}$$
$$= \frac{{\cdot}76393}{2} = {\cdot}382 \text{ correct to 3 decimal places.}$$

EXAMPLES XIV. e.

[*In the following take* $\sqrt{2} = 1{\cdot}41421$, $\sqrt{3} = 1{\cdot}73205$, $\sqrt{5} = 2{\cdot}23607$, $\sqrt{6} = 2{\cdot}44949$, $\sqrt{7} = 2{\cdot}64575$ *correct to* 5 *decimal places.*]

Find the value of the following expressions correct to 3 decimal places:

1. $\frac{1}{\sqrt{3}}$. **2.** $\frac{1}{\sqrt{5}}$. **3.** $\frac{2}{\sqrt{7}}$. **4.** $\frac{1}{\sqrt{2}-1}$. **5.** $\frac{1}{\sqrt{5}-2}$.

6. $\frac{2}{\sqrt{3}-1}$. **7.** $\frac{4}{\sqrt{5}+2}$. **8.** $\frac{\sqrt{2}-1}{\sqrt{2}+1}$. **9.** $\frac{\sqrt{3}+1}{\sqrt{3}-1}$.

10. $\frac{2-\sqrt{3}}{2+\sqrt{3}}$. **11.** $\frac{\sqrt{5}+2}{\sqrt{5}-2}$. **12.** $\frac{3+\sqrt{7}}{3-\sqrt{7}}$. **13.** $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

14. $\frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{3}}$. **15.** $\frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$. **16.** $\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$. **17.** $\sqrt{\frac{\sqrt{3}+1}{\sqrt{3}-1}}$.

18. $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$. **19.** $\sqrt{\frac{\sqrt{6}-2}{\sqrt{6}+2}}$. **20.** $\sqrt{\frac{5-\sqrt{3}}{5+\sqrt{3}}}$.

XV. PRACTICE.

Simple Practice.

138. PRACTICE is a method of finding the value of any number of things, or of any quantity, by means of *Aliquot Parts.*

An *Aliquot Part* of a quantity is a *measure* or *exact divisor* of it.

EXAMPLE 1. **Find the cost of 249 things at £2. 13s. 9d. each.**

First Method (using decimals).

	£.		
	249	= the cost at £1 each.	
	2		
	498	= ,,	£2 each.
10*s*. = $\frac{1}{2}$ of £1	124·5	= ,,	10*s*. each.
2*s*. 6*d*. = $\frac{1}{4}$ of 10*s*.	31·125	= ,,	2*s*. 6*d*. each.
1*s*. 3*d*. = $\frac{1}{2}$ of 2*s*. 6*d*.	15·5625	= ,,	1*s*. 3*d*. each.
	£669·1875	= ,,	£2. 13*s*. 9*d*. each.
	20		
	s. 3·75$\cancel{0}\cancel{0}$		
	12		
	d. 9·00		

∴ the cost required is £669. 3*s*. 9*d*.

[The value of £·1875 in shillings and pence might be written down "at sight."]

Second Method (without decimals).

	£.	*s*.	*d*.		
	249 .	0 .	0	= the cost at £1 each.	
			2		
	498 .	0 .	0	= ,,	£2 each.
10*s*. = $\frac{1}{2}$ of £1	124 .	10 .	0	= ,,	10*s*. each.
2*s*. 6*d*. = $\frac{1}{4}$ of 10*s*.	31 .	2 .	6	= ,,	2*s*. 6*d*. each.
1*s*. 3*d*. = $\frac{1}{2}$ of 2*s*. 6*d*.	15 .	11 .	3	= ,,	1*s*. 3*d*. each.
	669 .	3 .	9	= ,,	£2. 13*s*. 9*d*. each.

EXAMPLE 2. **Find the cost of 137 things at £3. 15s. 9$\frac{1}{2}$d. each.**

First Method (using decimals).

	£.		
	137	= the cost at £1 each.	
	3		
	411	= ,,	£3 ,,
10*s*. = $\frac{1}{2}$ of £1	68·5	= ,,	10*s*. ,,
5*s*. = $\frac{1}{2}$ of 10*s*.	34·25	= ,,	5*s*. ,,
6*d*. = $\frac{1}{10}$ of 5*s*.	3·425	= ,,	6*d*. ,,
3*d*. = $\frac{1}{2}$ of 6*d*.	1·7125	= ,,	3*d*. ,,
$\frac{1}{2}$*d*. = $\frac{1}{6}$ of 3*d*.	·2854	= ,,	$\frac{1}{2}$*d*. ,, ...(*a*).
	£519·1729	= ,,	£3. 15*s*. 9$\frac{1}{2}$*d*. each.
	20		
	s. 3·458$\cancel{0}$		
	12		
	d. 5·496		

∴ **£519.** 3*s*. 5$\frac{1}{2}$*d*. is the required cost.

NOTE.—If the line (a) had been continued, it would have been ·28541666 ....

The required value is then £519·17291666 ...

$$\begin{array}{r} 519\cdot17291666\ldots \\ 20 \\ \hline s.\ 3\cdot4583333\ldots \\ 12 \\ \hline d.\ 5\cdot4999999\ldots = 5\cdot5 \text{ (see Art. 109).} \end{array}$$

∴ £519. 3*s*. $5\frac{1}{2}d$. is the *exact* cost.

Second Method (without decimals).

	£.	s.	d.		
	137 .	0 .	0	= the cost at £1 each.	
			3		
	411 .	0 .	0	= ,,	£3 ,,
10*s*. = $\frac{1}{2}$ of £1	68 .	10 .	0	= ,,	10*s*. ,,
5*s*. = $\frac{1}{2}$ of 10*s*.	34 .	5 .	0	= ,,	5*s*. ,,
6*d*. = $\frac{1}{10}$ of 5*s*.	3 .	8 .	6	= ,,	6*d*. ,,
3*d*. = $\frac{1}{2}$ of 6*d*.	1 .	14 .	3	= ,,	3*d*. ,,
$\frac{1}{2}d$. = $\frac{1}{6}$ of 3*d*.		5 .	$8\frac{1}{2}$	= ,,	$\frac{1}{2}d$. ,,
	£519 .	3 .	$5\frac{1}{2}$	= ,,	£3. 15*s*. $9\frac{1}{2}d$. each.

139. Care must be taken to choose suitable aliquot parts.

In finding the cost of a number of things at £3. 15*s*. $3\frac{1}{2}d$. each,

we might take in the decimal method:		but this is better if decimals are not used:
10*s*. = $\frac{1}{2}$ of £1		10*s*. = $\frac{1}{2}$ of £1.
5*s*. = $\frac{1}{2}$ of 10*s*.		4*s*. = $\frac{1}{5}$ of £1.
3*d*. = $\frac{1}{20}$ of 5*s*.		1*s*. = $\frac{1}{4}$ of 4*s*.
$\frac{1}{2}d$. = $\frac{1}{6}$ of 3*d*.;		3*d*. = $\frac{1}{4}$ of 1*s*.
		$\frac{1}{2}d$. = $\frac{1}{6}$ of 3*d*.

EXAMPLE 1. **Find the cost of 927 things at £3. 15s. $3\frac{1}{2}$d. each.**

	£.			
	927	= the cost at £1 each.		................(a)
	3			
	2781	= ,,	£3 ,,	
10*s*. = $\frac{1}{2}$ of £1	463·5	= ,,	10*s*. ,,	
4*s*. = $\frac{1}{5}$ of £1	185·4	= ,,	4*s*. ,,	....................(b)
	etc.			

N.B.—To obtain line (b), we divide line (a) [£927] by 5.

EXAMPLE 2. **Find the cost of $6005\frac{2}{11}$ things at 18s. $3\frac{3}{4}$d. each.**

$$6005\tfrac{2}{11} = 6005\cdot1818\ldots.$$

We shall obtain the cost to the nearest farthing if we work to four decimal places, *correcting* the last decimal place in each case.

	£.		
	6005·1818 = the cost at £1 each.		
10s. = $\frac{1}{2}$ of £1	3002·5909 =	,,	10s. each.
5s. = $\frac{1}{2}$ of 10s.	1501·2955 =	,,	5s. each.
2s. 6d. = $\frac{1}{2}$ of 5s.	750·6478 =	,,	2s. 6d. each.
6d. = $\frac{1}{5}$ of 2s. 6d.	150·1296 =	,,	6d. each.
3d. = $\frac{1}{2}$ of 6d.	75·0648 =	,,	3d. each.
$\frac{3}{4}$d. = $\frac{1}{4}$ of 3d.;	18·7662 =	,,	$\frac{3}{4}$d. each.
	£5498·4948 =	,,	18s. 3$\frac{3}{4}$d. each.
	20		
	s. 9·8960		
	12		
	d. 10·752		

∴ £5498. 9s. 10$\frac{3}{4}$d. is the cost to the nearest farthing.

EXAMPLES XV. a. (*Oral.*)

Read off the value of

1. 96 at 6*d.* each.	**2.** 97 at 6*d.* each.	**3.** 99 at 6*d.* each.
4. 135 ,, 6*d.* ,,	**5.** 84 ,, 3*d.* ,,	**6.** 85 ,, 3*d.* ,,
7. 185 ,, 3*d.* ,,	**8.** 262 ,, 3*d.* ,,	**9.** 100 ,, 1*s.* ,,
10. 150 ,, 1*s.* ,,	**11.** 639 ,, 1*s.* ,,	**12.** 70 ,, 2*s.* ,,
13. 75 ,, 2*s.* ,,	**14.** 71 ,, 2*s.* ,,	**15.** 79 ,, 2*s.* ,,
16. 92 ,, 5*s.* ,,	**17.** 126 ,, 5*s.* ,,	**18.** 319 ,, 5*s.* ,,
19. 128 ,, 2*s.* 6*d.* ,,	**20.** 129 ,, 2*s.* 6*d.* ,,	**21.** 314 ,, 2*s.* 6*d.* ,,
22. 101 ,, 2*s.* 6*d.* ,,	**23.** 407 ,, 2*s.* 6*d.* ,,	**24.** 315 ,, 2*s.* 6*d.* ,,
25. 156 ,, 6*s.* 8*d.* ,,	**26.** 322 ,, 6*s.* 8*d.* ,,	**27.** 134 ,, 6*s.* 8*d.* ,,
28. 324 ,, 3*s.* 4*d.* ,,	**29.** 131 ,, 3*s.* 4*d.* ,,	**30.** 519 ,, 3*s.* 4*d.* ,,
31. 118 ,, 3*s.* 4*d.* ,,	**32.** 246 ,, 13*s.* 4*d.* ,,	**33.** 69 ,, 13*s.* 4*d.* ,,
34. 73 ,, 13*s.* 4*d.* ,,	**35.** 960 ,, 1*s.* 8*d.* ,,	**36.** 961 ,, 1*s.* 8*d.* ,,
37. 158 ,, 1*s.* 8*d.* ,,	**38.** 185 ,, 1*s.* 8*d.* ,,	**39.** 139 ,, 1*s.* 8*d.* ,,
40. 119 ,, 1*s.* 8*d.* ,,		

Read off suitable aliquot parts to take in calculating the cost of a number of things at

41. 17*s.* 6*d.* each.	**42.** 15*s.* 4*d.* each.	**43.** 12*s.* 3*d.* each.
44. 8*s.* 9*d.* ,,	**45.** 9*s.* 3*d.* ,,	**46.** 13*s.* 7*d.* ,,
47. 4*s.* 2*d.* ,,	**48.** 15*s.* 3$\frac{1}{2}$*d.* ,,	**49.** 14*s.* 4$\frac{3}{4}$*d.* ,,
50. 13*s.* 4*d.* ,,	**51.** 13*s.* 4$\frac{1}{2}$*d.* ,,	**52.** 4*s.* 11$\frac{1}{4}$*d.* ,,
53. 17*s.* 9$\frac{1}{2}$*d.* ,,	**54.** 17*s.* 8$\frac{1}{2}$*d.* ,,	**55.** 10*s.* 1$\frac{1}{2}$*d.* ,,
56. 4*s.* 5$\frac{3}{4}$*d.* ,,		

EXAMPLES XV. b.

[*Revise each example before proceeding to the next. Give a rough check wherever you can.*]

Find the value of

1.	39 articles at	£1. 2*s*. 6*d*.	each.	**2.**	97 articles at	£2. 5*s*.	each.
3.	123 ,,	£1. 17*s*. 6*d*.	,,	**4.**	227 ,,	£1. 12*s*. 6*d*.	,,
5.	94 ,,	£1. 6*s*. 8*d*.	,,	**6.**	197 ,,	£3. 4*s*. 9*d*.	,,
7.	225 ,,	£1. 15*s*. 8½*d*.	,,	**8.**	340 ,,	£1. 8*s*. 6*d*.	,,
9.	109 ,,	£7. 2*s*. 6¾*d*.	,,	**10.**	1000 ,,	£12. 7*s*. 9*d*.	,,
11.	624 ,,	£15. 16*s*. 8*d*.	,,	**12.**	309 ,,	£9. 9*s*. 2*d*.	,,
13.	1471 ,,	£11. 4*s*. 7½*d*.	,,	**14.**	907 ,,	£51. 14*s*. 3*d*.	,,
15.	2025 ,,	£25. 11*s*. 8*d*.	,,	**16.**	972 ,,	£75. 3*s*. 6½*d*.	,,
17.	629 ,,	£17. 13*s*. 4½*d*.	,,	**18.**	862 ,,	£16. 14*s*. 3½*d*.	,,
19.	307 ,,	£9. 14*s*. 5½*d*.	,,	**20.**	1749 ,,	£18. 9*s*. 6½*d*.	,,
21.	608 ,,	£32. 12*s*. 4¾*d*.	,,	**22.**	763 ,,	£24. 19*s*. 2¾*d*.	,,
23.	354 ,,	£35. 13*s*. 4*d*.	,,	**24.**	921 ,,	£28. 13*s*. 9¼*d*.	,,
25.	2066 ,,	£14. 7*s*. 9*d*.	,,	**26.**	323 ,,	£5. 10*s*. 4¾*d*.	,,

[In the following examples, calculate the result *to the nearest penny.*]

27.	$452\frac{1}{4}$ articles at	£4. 3*s*. 1½*d*.	ea.	**28.**	$693\frac{2}{3}$ articles at	£7. 17*s*. 7½*d*.	ea.
29.	$104\frac{5}{8}$,,	£3. 10*s*. 5*d*.	,,	**30.**	$437\frac{7}{15}$,,	£9. 0*s*. 10½*d*.	,,
31.	$609\frac{4}{7}$,,	£4. 2*s*. 2*d*.	,,	**32.**	$531\frac{13}{32}$,,	£11. 3*s*. 1½*d*.	,,

Use the 'Practice' method, *i.e.* the method of 'aliquot parts,' to find the value of

33. £14. 17*s*. 10½*d*. × 1137. **34.** £3. 4*s*. 9*d*. × 394.

35. £7. 2*s*. 6¾*d*. × 218. **36.** £22. 9*s*. 3*d*. × 1471.

37. £25. 17*s*. 1½*d*. × 907. **38.** £9. 14*s*. 5½*d*. × 614.

Compound Practice,

140. *i.e.* when the value of a compound quantity is to be found by taking aliquot parts.

EXAMPLE 1. **Find, to the nearest penny, the cost of 3 cwt. 1 qr. 17 lb. at £4. 13s. 3½d. per cwt.**

First Method (using decimals).

$$\begin{array}{r|l} 12 & 3{\cdot}5d. \\ 20 & 13{\cdot}2917s. \quad \text{(correct to 4 decimal places)} \\ & 4{\cdot}6646£. \quad (\quad ,, \quad ,, \quad) \end{array}$$

	£.		
	4·6646	=	the cost of 1 cwt.
	3		
	13·9938	=	,, 3 cwt.
1 qr. = $\frac{1}{4}$ of 1 cwt.	1·1662	=	,, 1 qr.
14 lb. = $\frac{1}{2}$ of 1 qr.	·5831	=	,, 14 lb.
2 lb. = $\frac{1}{7}$ of 14 lb.	·0833	=	,, 2 lb.
1 lb. = $\frac{1}{2}$ of 2 lb.	·0417	=	,, 1 lb.
	£15·8681	=	the cost required.
	20		
	s. 17·3620		
	12		
	d. 4·344		

∴ £15. 17*s.* 4*d.* is the required cost.

Second Method (working the pence to two decimal places).

	£.	*s.*	*d.*		
	4 .	13 .	3·5	=	the cost of 1 cwt.
			3		
	13 .	19 .	10·5	=	,, 3 cwt.
1 qr. = $\frac{1}{4}$ of 1 cwt.	1 .	3 .	3·88	=	,, 1 qr.
14 lb. = $\frac{1}{2}$ of 1 cwt.		11 .	7·94	=	,, 14 lb.
2 lb. = $\frac{1}{7}$ of 14 lb.		1 .	7·99	=	,, 2 lb.
1 lb. = $\frac{1}{2}$ of 2 lb.			9·99	=	,, 1 lb.
	£15 .	17 .	4·30	=	the required cost

= £15. 17*s.* 4*d.* to the nearest penny.

EXAMPLE 2. **Rates are levied at 4s. 9½d. in the £. Find the amount payable on £372. 15s. to the nearest penny.**

	£.				
	372·75				
4*s.* = $\frac{1}{5}$ of £1	74·55	= the amount payable at			4*s.*
6*d.* = $\frac{1}{8}$ of 4*s.*	9·3188	=	,,	,,	6*d.*
3*d.* = $\frac{1}{2}$ of 6*d.*	etc.	=	,,	,,	3*d.*
$\frac{1}{2}$*d.* = $\frac{1}{6}$ of 3*d.*		=	,,	,,	$\frac{1}{2}$*d.*
		= the amount required.			

EXAMPLES XV. c.

[*Revise each example before proceeding to the next. Give a rough check wherever you can. In each case calculate the cost to the nearest penny.*]

Find the cost of

1. 5 cwt. 2 qrs. 14 lb. at £2. 5*s.* 6*d.* per cwt.

2. 7 cwt. 3 qrs. 7 lb. at £1. 10*s.* 6*d.* per cwt.

3. 3 tons 12 cwt. 1 qr. at £4. 5*s.* 3*d.* per ton.

Find the cost of

4. 7 lb. 5 oz. 12 dwt. at £3. 10*s.* 6*d.* per lb.

5. 9 lb. 4 oz. 1 dwt. at £7. 14*s.* 4*d.* per lb.

6. 15 oz. 6 dwt. 17 grs. at 5*s.* 10*d.* per oz.

7. 13 acres 3 ro. 25 poles at £52. 10*s.* per acre.

8. 9 yds. 2 ft. 10 in. at 5*s.* 7½*d.* per yard.

9. Find the amount of the rate payable on £625. 10*s.* at 4*s.* 6½*d.* in the £.

10. A bankrupt pays 13*s.* 7½*d.* in the £. What is the dividend on a debt of £326. 14*s.* 9*d.* to the nearest penny?

11. Given that 1 yard = 0·9144 metre, find the length of 9 yds. 2 ft. 4 in. to the nearest centimetre.

12. Given that 1 ton = 1016 kilos., express 7 cwt. 2 qrs. 21 lb. in kilogrammes.

13. Find, to the nearest penny, the cost of 13 cwt. 2 qrs. 16 lb. at £3. 7*s.* 6*d.* per cwt.

14. Find, to the nearest penny, the cost of 9 lb. 10 oz. 16 dwt. at £3. 11*s.* 3*d.* per lb.

15. A bankrupt pays 5*s.* 10*d.* in the pound; find the dividend on a debt of £61. 10*s.*

16. Find the cost of 324 ac. 6 sq. chains at £41. 15*s.* per acre.

17. A bankrupt, who owed £5265. 10*s.* 6*d*, paid 15*s.* 4*d.* in the £; find the amount of his assets to the nearest pound.

18. Find the cost of laying a light railway, 5 miles 5¼ furlongs long, at £1050 per mile.

19. What is the rent, to the nearest shilling, of a farm of 423 acres, 3 roods, 26 perches, at £1. 15*s.* per acre?

20. A company with £1 shares pays a dividend of 3*s.* 2½*d.* on each share. Find the total dividend paid on the full capital of £450,000.

XVI. REVISION PAPERS.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

XVI. a.

1. Multiply £37. 16*s.* 3*d.* by 250.

2. A painter receives 15 francs 30 centimes for painting 102 letters; what is the cost of painting each letter?

3. Simplify (1) $\frac{6}{77}+\frac{9}{91}-\frac{8}{143}$. (2) $\frac{2\frac{1}{3}\times 1\frac{1}{7}\times 1\frac{1}{4}}{1\frac{2}{3}+2\frac{1}{4}}$.

4. Multiply 32·024 by ·025, and divide ·0182 by ·013.

5. Reduce £2. 14*s.* 4½*d.* to the decimal of £1; and find the value of $\frac{1}{7}$ of 4*s.* 1*d.* + $\frac{1}{3}$ of 7*s.* 3*d.* + 14·625*s.*

6. A garden is 72 yds. long by 31 yds. wide. What is the cost of fencing it all round at 7*d.* a foot, allowing 7*s.* 6*d.* extra for a gate?

7. Find the cost of 647 things at £3. 14*s.* 6*d.* each.

XVI. b.

1. Find the L.C.M. of 64, 54, 27 and 120.

2. If 7 kettles cost 9*s.* 7½*d.*, find the cost of 11 kettles.

3. Simplify (1) $2\frac{3}{7}-3\frac{1}{9}+2\frac{3}{5}+\frac{26}{315}$. (2) $\frac{7\frac{2}{11}-\frac{8}{9}}{2\frac{1}{3}+1\frac{10}{11}}-\frac{11}{15}$.

4. (1) Simplify $(1·4-·362)\div(·31+·123-·0005)$.
   (2) Find the value of £·8759 to the nearest farthing.

5. If 80½ tons of coal cost £79. 10*s.*, find the cost per ton to the nearest penny.

6. A bankrupt pays 4*s.* 6*d.* in the £; how much should I receive if he owed me £35. 3*s.* 4*d.*?

7. Of a regiment, $\frac{1}{7}$ were invalided, $\frac{1}{13}$ killed and $\frac{1}{11}$ wounded. If 690 men were left, how many men were there originally in the regiment?

XVI. c.

1. Find two numbers between 900,000 and 1,000,000 which are exactly divisible by 37,259.

2. If 62 metres 50 centimetres of silk cost 293 francs 75 centimes, find the price of a metre.

3. Simplify (1) $3-\frac{3}{20}-\frac{4}{15}-\frac{7}{12}$. (2) $\frac{7\frac{51}{56}-3\frac{23}{44}-2\frac{12}{77}}{1\frac{1}{14}\div\frac{8}{15}}$.

4. If a set of 9 hens lay 1423 eggs, and another set of 13 hens lay 1924 eggs in the same time, which set on the average are the better layers?

5. Multiply ·0204 by 2·06, and divide 4·0034 by ·037.

6. Reduce 19*s.* 7½*d.* to the decimal of £1. 2*s.* 6*d.* correct to five decimal places.

7. The fare for a certain railway journey is 18 francs 70 centimes; while for another journey, 87 kilometres longer than the former, the fare is 24 francs 20 centimes. Find the length of each journey, the rate per kilometre being the same for both.

XVI. d.

1. Find the three numbers between 2500 and 3000 which are divisible by all the numbers 21, 24, 28.

2. Simplify (1) $2\frac{13}{15} - \frac{11}{35} + 3\frac{6}{7} - 6\frac{2}{5}$. (2) $\dfrac{8\frac{1}{3} - 6\frac{3}{4}}{1\frac{6}{7} + \frac{5}{9}} \div (10 - \frac{1}{40})$.

3. Divide 429·55 by ·01375, and express £2. 13*s.* $4\frac{1}{2}$*d.* as a decimal of £4.

4. A took $\frac{1}{3}$ of a sum of money, B took $\frac{1}{4}$ of the remainder and C took $\frac{1}{5}$ of what was then left. After this, £25 remained; find the sum of money.

5. If a man buys eggs at 18 pence a score, and sells them at the rate of 8 for a shilling, how much profit does he make on 120 eggs?

6. A rectangular field is 160 yds. long and 77 yds. wide, and costs £112. How much is this per acre?

7. Part of the site of Christ's Hospital, 67,680 sq. ft. in area, was sold for £238,781. Find the price per acre to the nearest pound.

XVI. e.

1. What weight of copper is worth 723 francs 69 centimes if 1 kilogramme is worth 1 franc 29 centimes?

2. Simplify (1) $2\frac{1}{12} \div 5\frac{5}{8} + 1\frac{7}{11} \times 2\frac{4}{9} - 2\frac{1}{7} \times 1\frac{3}{25}$.

(2) $\dfrac{3\frac{1}{2}}{4\frac{2}{3}} - \dfrac{2\frac{1}{8}}{1\frac{1}{2}} + \dfrac{1\frac{2}{3}}{2\frac{1}{2}}$.

3. If $\frac{1}{10}$ of a ton of metal is worth £68, what is the value of $5\frac{1}{4}$ tons?

4. Multiply 5·621 by $3\frac{1}{7}$, and find the value of ·05836 of £2 to the nearest penny.

5. On an income of £526 the tax is £21. 18*s.* 4*d.*; what is the tax on each £?

6. How many men will earn £37. 16*s.* in the same time that 13 men earn £35. 2*s.*, if all the men are paid at the same rate?

7. A square field is 10 acres in area; find the length of one of its sides.

Find, to the nearest yard, the length of one side of a square field whose area is one acre.

XVI. f.

1. Find the value of $\frac{4}{15}$ of £1. 19*s.* $8\frac{1}{4}$*d.*

2. Simplify the expression $\dfrac{\cdot052 \times 1\cdot87 \times \cdot0021}{3\cdot5 \times 6\cdot63 \times 1\cdot1}$.

3. Find the value of $2\frac{1}{7}$ of $3\frac{1}{9}$ of 4 lb. 8 oz. 10 dwt. 12 gr. Troy.

4. Reduce $\frac{3}{20}$ of 4 guineas to the fraction of £7, and £3. 17*s.* $2\frac{1}{2}$*d.* to the decimal of £4 to eight places of decimals.

5. By selling a horse for £72, a man lost one quarter of what he gave for it. How much did he give for it?

6. A man buys 10 lb. of tea at 1*s.* 8*d.* per lb., and 12 lb. of another kind at 1*s.* 10*d.* per lb. How much does he spend?

If he mixes the teas together and sells the mixture at 2*s.* per lb., how much does he gain?

7. Find the cost of a carpet for a room 24 ft. long and 15 ft. wide at 3*s.* 3*d.* a square yard.

XVI. g.

1. A lath is 2·31 yds. long. Express its length in metres and decimals of a metre, 1 yard being equal to $\frac{32}{35}$ metres.

2. For £20 borrowed now I must repay £21. 5*s.* a year hence. What must I repay a year hence for £625 borrowed now at the same rate?

3. Wall paper is sold by the piece of 12 yds. long and 20 inches broad. What area of wall will a piece cover?

4. A motorist travels 85 miles in $4\frac{1}{4}$ hours. If he maintains the same speed, how long (to the nearest hour) will he take for 119 miles?

5. A grocer buys 9 lb. of tea at 1*s.* 1*d.* per lb., and 5 lb. at 1*s.* 7*d.* How much does he spend? If he mixes the two kinds and sells the mixture so as to gain 3*d.* on every shilling he spends, at what price per lb. (to the nearest penny) must he sell the mixture?

6. A man spent $\frac{2}{3}$ of his income on board and lodging, $\frac{1}{6}$ on dress and $\frac{1}{10}$ on other things. If he then had £21. 10*s.* left, what was his income?

7. An equal number of half-sovereigns, half-crowns, florins and sixpences amount to £45. How many coins of each kind are there?

XVI. h.

1. A traveller walks $\frac{1}{7}$ of his journey, bicycles $\frac{1}{3}$, and then finds he has 22 miles more to travel. What is the length of the journey?

2. Simplify (1) $3\frac{1}{4}-1\frac{2}{3}\div 2\frac{1}{7}+1\frac{1}{8}-1\frac{4}{9}\times 2\frac{1}{26}$.

(2) $(\frac{7}{8}+\frac{2}{5})\div(\frac{2}{7}+\frac{1}{5})\times(\frac{1}{2}-\frac{1}{3})$.

3. Find, to the nearest penny, the cost of 18,255 cu. ft. of gas at 2*s.* 4*d.* per 1000 cu. ft.

4. A creditor receives £19. 9*s.* 6*d.* from a bankrupt who is paying 6*s.* 4*d.* in the £. What is the amount of the debt?

5. A farmer bought 11 oxen at £11. 17*s.* 6*d.* each. He sold 7 at £12. 10*s.* each, the rest at £12 each. How much profit did he make?

6. (1) Find the value of (correct to 4 decimal places) $\frac{\cdot 3115}{\cdot 0245}$ without using long division.

(2) Find the value of ·625 of £16. 13*s.* 4*d.*

7. If 61 feet of wire cost 12*s.* $8\frac{1}{2}$*d.*, how many feet can be bought for £2. 0*s.* $2\frac{1}{2}$*d.*?

XVI. k.

1. Two men have strides 31 and 32 inches long. How many more strides does the one take than the other in walking 4 miles 896 yds.?

2. Which is the greater, and by how much, $\frac{2}{9}$ of a gallon or $\frac{1}{8}$ of seven quarts? Give your result as a fraction of a quart.

3. Find the value of ·2658 of £2 to the nearest penny.

4. Reduce £9. 7*s.* 3*d.* to the decimal of £39. 1*s.* 3*d.* [Without using long division, if you can.]

5. Three wheels take respectively 14, 13, and 12 minutes for a revolution. If they all start at the same instant, after how many complete minutes will they all have first made complete revolutions at the same instant? How many revolutions has each wheel then made?

6. The legal weight of a penny is $\frac{1}{3}$ of an ounce avoirdupois. A man sold his bicycle, which weighed 36 lb., for its equivalent weight in legal pence; how much did he sell it for?

7. By selling a cow for £12. 15*s.* a man lost $\frac{1}{7}$ of what he gave for it. How much did he give for it?

XVI. l.

1. A florist buys flowers at the rate of 1*s.* for 14; he sells them at 1*s.* 5*d.* a dozen. What fraction of a penny profit does he make on each flower?

What money profit does he make on the expenditure of 1*s.*?

2. Work as clearly as possible, by the shortest method you can think of, the following:

(*a*) Find the cost of 2400 articles at 19*s.* $11\frac{1}{2}$*d.* each.

(*b*) Find the cost of 8250 bricks at 10*s.* 6*d.* a thousand.

3. Divide ·03317 by ·31, and reduce £3. 4*s.* $7\frac{1}{2}$*d.* to the decimal of £5.

4. When $\frac{16}{23}$ of a lath is cut off, 1 ft. 9 in. remain. What is the length of the lath?

5. If 14 horses plough a field in 4 days, how long will 16 horses take to plough the same field?

6. A rectangular field whose area is 3 sq. chains is 132 feet long. What is its breadth? What would it cost at £74. 10*s.* an acre?

7. A takes $\frac{1}{5}$ of a sum of money, B takes $\frac{1}{3}$ of what is left, and C, who has the rest, gets £42. Find the sum of money.

XVI. m.

1. Find the least number which when multiplied by 11,340 gives a perfect square for the product.

2. A person bought 2 tons of sugar at $2\frac{1}{4}$*d.* per lb. Of the amount so purchased 6 cwt. 84 lb. became damaged. He sold this portion at

$1\frac{1}{4}d.$ per lb., and the rest at $3d.$ per lb. What did he gain or lose by the transaction?

3. If a number is divided by 240, by using the factors 6, 8 and 5, and the respective remainders are 3, 1 and 2, find the complete remainder.

4. If three bicyclists complete a circular course in 54, 63, 72 seconds starting together, in what time after the start will they first be all together at the starting post, provided that they go on riding at the same rates?

5. If a metre is taken to be 39·37011 inches, find, correct to the nearest inch, the difference between 10 miles and 16 kilometres.

6. Show that the product of any four consecutive integers is divisible by 24 without remainder.

7. Evaluate, with as little work as possible,

$$3{\cdot}1416 \times 2423{\cdot}76 \times 0{\cdot}0063425, \text{ to the nearest integer.}$$

XVI. n.

1. Find the difference between ·0024 of a day and 3·6 minutes to the nearest half second.

2. A took $\frac{2}{3}$ of £325. 5*s.* and B took $\frac{3}{4}$ of what was left. What sum of money remained?

3. Find the value of ·036 of £3. 6*s.* 8*d.* to the nearest penny.

4. After paying a tax at the rate of 10*d.* in the £ a man had £403. 9*s.* 2*d.* per annum left. What was his total income?

5. What is the least sum of money which contains 17*s.* 6*d.*, 22*s.* 6*d.* and 13*s.* 4*d.* each an exact number of times?

6. Find the square root of 213444 by using factors.
What is the square root of 21·3444?

7. I wish to cover the floor of a room 15 ft. 6 in. by 14 ft. 9 in. with carpet 20 inches wide. How much must I buy, if the upholsterer refuses to sell fractions of a yard?

XVI. o.

1. Find the value of $\frac{3}{8}$ of a guinea $+ 1\frac{2}{5}$ of $7s. + \frac{1}{3}$ of 4*s.* 6*d.* to the nearest penny.

2. From a piece of cloth 43·02 metres long, equal pieces 65 cm. long are cut. How many such pieces are cut, and what length of cloth is left?

3. Find the cost, to the nearest penny, of 235 things at £7. 13*s.* $4\frac{1}{2}d.$ each.

4. A bankrupt pays 8*s.* 6*d.* in the £; how much does he pay on a debt of £63. 8*s.* 4*d.*?

5. A rectangular wood is 637 yds. long and 52 yds. wide; find the side of a square piece of land of the same area.

6. A father left a sum of money to his son, who, when he died, left $1\frac{1}{2}$ times as much to his son, and this son, who again left $1\frac{1}{2}$ times as much as he received, left £39000. What sum of money was originally left?

7. Show that $\sqrt{8}$ lies between $\frac{17}{6}$ and $\frac{82}{29}$.

XVI. p.

1. Extract the square root of $237\frac{4}{25}$.

2. If one metre is equal to 39·3708 inches, find, correct to four decimal places, the number of square yards in one square metre.

3. Find the value of £21. 17*s.* 6*d.* in francs and centimes, exchange being reckoned at £1 = 25 francs 36 centimes.

4. By selling sugar at £38 a ton, a man made a profit of one quarter of what he bought it for. How much did he give for it per ton?

5. Find, to the nearest penny, the cost of 3 tons 7 cwt. 3 qrs. of coal at £2. 2*s.* 6*d.* per ton.

6. An open cistern is 4 ft. 9 in. long, 3 ft. 9 in. wide and 2 ft. 6 in. deep; find the cost of lining the whole of the inside with lead at 4*d.* a square foot.

7. A number of men were drawn up in a hollow square four deep with 40 men in the front rank. They were afterwards arranged in a solid square. Find the number of men in its front rank.

XVI. q.

1. A ton of sugar costs £30. 6*s.* 8*d.* How much is this per lb.?

2. A block of metal 6 ft. long, 11 in. wide and 9 in. thick is rolled out into a straight rod, the area of whose section is 6 sq. inches. How long is the rod?

3. Six bells toll at intervals of 3, 5, 9, 15, 24 and 30 seconds. If they began all together, how often would they all toll together again in 4 hours?

4. Find the value of $\dfrac{1}{\sqrt{3}-1}$ correct to two decimal places.

5. Given that 1 cm. = 0·0328 foot, find, correct to 3 decimal places, the number of sq. inches in 10 sq. cm.

6. A square field of $2\frac{1}{2}$ acres is fenced round. Find the cost of the fence at 2*s.* 1*d.* per linear yard.

7. If £15. 0*s.* $2\frac{1}{2}$*d.* is the income tax on £720. 10*s.*, what is the tax on £850. 4*s.*?

XVI. r.

1. A man is required to divide by 3·2. Instead of doing this, he obtains the correct result by multiplying by a certain decimal fraction. Show how to find the decimal fraction.

2. State clearly why the answers suggested to the following sums are obviously wrong:

(*a*) 7643×9009. *Answer*: 68856787.

(*b*) Simplify $\dfrac{\frac{1}{10} \text{ of 1 kilometre} - 11 \text{ metres} + 320 \text{ centimetres}}{\frac{1}{3} \text{ of 35 decimetres} + \frac{1}{5} \text{ of 7 metres}}$.

Answer: 348 centimetres.

3. The profits of a business are £4875. One of the partners finds that 0·24 of his share of the profits is worth £780. What fraction of the profits belongs to him?

4. If 3 ducks are worth 4 chickens, and a pair of chickens cost 5*s.* 6*d.*, find the cost of 7 ducks.

5. Find the cost of 7 tons 3 cwt. 3 qrs. of coal at 24*s.* per ton.

6. In one year a man lost one-quarter of his money, and in the second year he lost one-eighth of the remainder, and then found that he had £777 left. How much had he at first?

7. A concert manager reckons that each person requires a space 3 ft. 6 in. by 1 ft. 9 in. What area must he have at his disposal in order to accommodate 600 people?

XVI. s.

1. A bale of cotton contains 480 lb. Find the cost of 125 bales of cotton at 5·47*d.* per lb.

2. Find, correct to five decimal places, the value of

$$401{\cdot}8379 \times {\cdot}081246.$$

3. A boat's crew rows 33 strokes a minute and travels 29 ft. per stroke; find, to the nearest second, the time they take to row a mile.

4. If 37 articles cost £3. 18*s.* $7\frac{1}{2}$*d.*, how many can be bought for £10. 4*s.*?

5. Find, correct to 5 decimal places, the value of

$$\frac{1}{3 \times 5} + \frac{1}{3 \times 5 \times 7} + \frac{1}{3 \times 5 \times 7 \times 9}.$$

6. A rectangular area is 25 m. 2 dm. 8 cm. long, and 1 m. 5 dm. 8 cm. wide. Find the side of a square of equal area.

7. A town increases at the rate of 25 per thousand each year. If the population is 15680 at the beginning of a year, what is the population at the end of that year?

What will the population be at the end of the next year? Neglect fractions.

XVI. t.

1. How many pieces each 1·4 in. long can be cut from a rod 9·67 ft. long? Give the remainder in inches and decimal of an inch.

2. A man buys 21 shares in a company for £21. 12*s.* 6*d.* each, and has to give the broker, who obtains them for him, 1*s.* 6*d.* on each share. How much does he spend?

3. Find the square root of 3·891 correct to two decimal places.

4. The rent of an estate is £660 per annum, but the owner has to spend $\frac{1}{8}$ of the rent in repairs and 3*s.* in the £ on rates and taxes. What is his net income?

5. What is the weight of 2 yds. 1 ft. 9 in. of wire at 1 lb. 3 oz. 8 drs. per yard?

6. Express $\frac{2}{3}$ of $1\frac{5}{9}$ of a mile in terms of a metre, supposing 32 metres = 35 yards. Give your result to the nearest tenth of a metre.

7. A picture frame 1 m. 5 cm. long and 74 cm. broad is made of wood 6 cm. wide. How many sq. cm. of wood does it contain? Give a rough check.

XVI. u.

1. Find (using as few figures as possible) the value of

(*a*) 34·675 × ·125. (*b*) 3·24 ÷ ·625.

2. A man plants a number of trees evenly in rows, allowing 45 sq. yds. for each tree; find the distance between each two trees to the nearest foot.

3. If 3 metres = 118 inches, find the number of metres in a mile. Give your result to the nearest metre.

4. To each boy in a class a man gave as many farthings as there were boys in the class. If the man spent £1. 4*s.* 1*d.*, how many boys were there in the class?

5. Add $5\frac{1}{2}$ cwt. to 3·125 qrs., and reduce the result to the decimal of a ton.

6. Find the value of 319 cwt. 3 qrs. 14 lb. at £2. 12*s.* 6*d.* per cwt.

7. Find the value of $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{2}-1}$ correct to two places of decimals.

XVI. v.

1. Give reasons why the product of any three consecutive integers is divisible by 6 without remainder.

2. Find as a decimal, correct to 6 places, the value of

$$\frac{1}{2}+\frac{1}{2\times4}+\frac{1}{2\times4\times6}+\frac{1}{2\times4\times6\times8}+\frac{1}{2\times4\times6\times8\times10}.$$

3. A man with a capital of £2570 increases it each year by one-fifth. How much to the nearest £ has he at the end of three years?

4. A man buys eggs at 10*d.* a dozen, and sells them at 1*d.* each. Find his gain (1) on 300 eggs, (2) on £1.

5. Reduce £9. 17*s*. $4\frac{1}{2}$*d*. to a decimal of £1.

A new system of coinage has been proposed in which 10 new farthings = 1 cent, 10 cents = 1 florin, 10 florins = £1. Express the above sum in the proposed system.

6. Find the cost of 345 oz. of gold at £3. 17*s*. $10\frac{1}{2}$*d*. per oz.

7. How far does a man go in running twice round a square field 10 acres in area?

XVII. PROBLEMS ON AREAS.

141. In finding areas of walls or floors of rooms, it will generally be found advisable to work in sq. feet, and fractions of a sq. foot.

Example 1. **Find the cost of papering the walls of a room 16 ft. long, 13 ft. 4 in. wide and 11 ft. 6 in. high, with paper 20 in. wide, at 15 pence a yard.**

The perimeter of the room $=2\times16+2\times13\frac{1}{3}$

$=2\times29\frac{1}{3}=2\times\frac{88}{3}$ feet.

$\therefore$ the area of the walls $=2\times\frac{88}{3}\times\frac{23}{2}$ sq. ft. (area of walls = perimeter × height)

$=\frac{88\times23}{3}$ sq. ft.

$\therefore$ the length of paper required $=\frac{88\times23}{3}\times\frac{12}{20}$ ft. (length = area ÷ breadth)

$=\frac{88\times23}{5}$ feet.

$\therefore$ the cost of the paper $=\frac{88\times23}{5}\times5$ pence

$=88\times23$ pence

$=11\times184$ pence

$=2024$ pence

$=168$*s*. 8*d*.

$=$£8. 8*s*. 8*d*.

Example 2. **A quadrangle 96 ft. long, 52 ft. 6 in. wide, has a path of width 5 ft. 6 in. running inside it and all round it. The part inside the path is turfed with turf costing 8d. a sq. yd. Find the area of the path and the cost of the turf.**

The whole area $=96\times52\frac{1}{2}=12\times420=5040$ sq. ft.

The area of the turf $=85\times41\frac{1}{2}=17\times207\frac{1}{2}=3527\frac{1}{2}$ sq. ft.

The area of the path = the difference $=1512\frac{1}{2}$ sq. ft.

The cost of turfing $3527\frac{1}{2}$ sq. ft. at $\frac{8}{9}$*d*. per sq. ft.

$=3527\frac{1}{2}\times\frac{8}{9}$ pence

$=\frac{28220}{9}$ pence

$=3136$*d*. to the nearest penny

$=261$*s*. 4*d*.

$=$£13. 1*s*. 4*d*.

EXAMPLE 3. **A plan of a rectangular room is drawn to the scale of one-tenth of 1 in. to 1 foot,** *i.e.* **one tenth of 1 in. on the plan represents a length of 1 foot. What is the area of the room if its plan is 1·3 in. by 2·5 in.?**

1·3 in. represents $1{\cdot}3 \times 10$, *i.e.* 13 feet,

2·5 in. ,, $2{\cdot}5 \times 10$, *i.e.* 25 ,,

$\therefore$ the area of the room is $13 \times 25 = \frac{1300}{4} = 325$ sq. ft.

EXAMPLE 4. **A rectangular area of 1368 sq. ft. is represented on a plan drawn to a scale of one-tenth of 1 in. to 1 ft. What is the length of the rectangle, if its width on the plan is 2·4 in. ?**

2·4 in. represents $2{\cdot}4 \times 10$, *i.e.* 24 feet.

$\therefore$ the length of the rectangle $= \frac{1368}{24} = \frac{342}{6} = 57$ feet.

EXAMPLES XVII. a.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

[*All the areas in the following examples may be taken as rectangular.*]

1. Find the area of a room which is 15 ft. 9 in. long and 11 ft. 4 in. wide.

2. A rectangular field is 10 chains long and 9 chains 40 links wide. Find its area in acres and fraction of an acre.

3. A wall is 30 ft. long and 12 ft. 6 in. high. Find the cost of painting one face at 2*s.* 6*d.* per square yard.

4. I pay a man at the rate of 7 shillings an acre for mowing my paddock, which is 165 yds. long and 48 yds. wide. How much does he earn if I count odd pence as a shilling ?

5. A room is 19 ft. 6 in. long, 13 ft. 4 in. wide and 11 ft. high. Find the area of its walls to the nearest sq. yard. What is the cost of painting the walls if the charge is half-a crown per sq. yard and fraction of a sq. yard ?

6. The seating area of a concert hall is 63 ft. 6 in. long and 40 ft. wide. How many people will it accommodate if it is calculated that each person requires 6 sq. ft.

7. The area of a rectangular field is 2 ac. 1 ro. 4 sq. poles, and it is 91 yards wide. Find its length

8. The area of a square lawn is one-tenth of an acre. What is the length of its side.

9. The length of a hall is three times its breadth and its area is 252 sq. yds. 108 sq. in. Find its length and breadth.

10. What length of carpet 30 in. wide will cover a room 15 ft. wide and 26 ft. 4 in. long ? What is the cost of the carpet at 3*s.* 6*d.* per yard length ?

11. A hall 7 ft. 6 in. wide is paved with tiles 9 inches square. How many rows of tiles will there be? How many tiles must be bought to cover the hall if it is 20 ft. long, assuming that a tile cannot be cut without wasting the part cut off?

12. Find the area of the hall shown in the accompanying figure, the dimensions being given in feet.

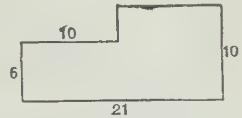


FIG. 24.

13. A rectangular lawn-tennis court 30 yds. long and 17 yds. wide has a path 3 feet wide round and outside it. Find the area of the path.

14. A field is 12 chains long and 7 chains 40 links wide. Find its area in acres, roods and sq. poles to the nearest sq. pole.

15. Find the rent of a rectangular field 165 yds. by 88 yds. at 25 shillings an acre.

16. Find the cost of paving a square whose side is 25 feet long at the rate of 16 pence per sq. yard. Give your result to the nearest penny.

17. How many sheets of paper 10 in. by 8 in. would it take to cover an acre of ground?

18. I have two rectangular sheets of paper, one 1 ft. 2 in. by 1 ft. 8 in., and the other 1 ft. 3 in. by 1 ft. 11 in. I cut them up in a suitable manner and fit the pieces together (without waste) and form a square. Find the length of a side of the square.

19. Taking a foot as equal to 30·5 cm., find in sq. metres and centimetres the area of a room 16 ft. by 12 ft.

20. A room is 6·4 metres long and 4·5 metres wide. Find its area in sq. feet to the nearest hundredth, taking 1 metre = 3·28 ft.

21. A writing table 2 ft. 6 in. by 5 ft. is covered with leather at 1*s.* per sq. foot, with the exception of an edge $2\frac{1}{2}$ in. wide all round it. Find the cost of the leather to the nearest penny.

22. A lawn 84 ft. 6 in. by 50 ft. has a path 5 ft. 3 in. wide running all round it. Find the cost, to the nearest penny, of making the path at 10*d.* a sq. yard.

23. Find the cost of painting the walls of a room 13 ft. 4 in. wide, 17 ft. 6 in. long and 11 ft. high, at 4*s.* 6*d* a sq. yard.

24. A carpet 2 ft. 3 in. wide costs 3*s.* 6*d.* a yard. Find the expense of carpeting the floor of a room 11 ft. 6 in. by 15 ft. 9 in.

25. A room is 19 ft. long and 13 ft. 4 in. wide. Find, to the nearest penny, the cost of staining a border all round it 2 ft. wide, at 2*d.* a sq. foot.

26. A man plants 6 rows of gooseberry bushes, 13 in a row, and 4 ft. apart each way. If he leaves a border 2 ft. 6 in. wide all round them, what is the area of the ground?

27. The floor of a room 19 ft. 6 in. long and 13 ft. 4 in. wide is covered with carpet 27 in. wide. What area of carpet is wasted if the strips are laid parallel to the length?

28. In the previous question, what is the area of the waste if the strips are laid the other way?

A 'piece' of wall-paper is a strip 12 yds. long, generally about 2 ft. 3 in. wide. Complete pieces only are sold.

29. Find the cost of papering a room 30 ft. long, 17 ft. 6 in. wide and 12 ft. high, with paper 2 ft. 3 in. wide, at 2*s.* 3*d.* a piece.

30. A room 24 ft. 6 in. long, 16 ft. 8 in. wide and 11 ft. high, has one doorway 7 ft. 6 in. by 3 ft. 6 in., and two windows each 8 ft. by 4 ft. How many pieces of wall-paper 2 ft. 3 in. wide must be bought to paper its walls?

31. A cistern, without a lid, is 4 ft. square at the bottom and 3 ft. 9 in. deep; inside measurements. Find the cost of painting it inside at $2\frac{1}{2}d.$ per sq. foot.

32. Find the cost of painting the walls of a room 19 ft. 6 in. long, 15 ft. 3 in. wide and 10 ft. 6 in. high, at 4*s.* per sq. yard, allowing 96 sq. ft. for door, windows and fireplace.

33. The walls and ceiling of a room 6·5 m. long, 5·2 m. wide and 3·4 m. high are papered with paper 7·2 dm. wide. Find the length of paper required, if no length less than a metre can be bought.

34. A closed box measures 2 ft. 3 in. by 1 ft. 6 in. by 1 ft. 3 in. deep (all external dimensions); find the cost, to the nearest penny, of painting the top and sides at $3\frac{1}{2}d.$ per sq. foot.

35. Find the total area of the inside of the box in the preceding question if it is made of wood $\frac{1}{2}$ in. thick.

Further Examples on Volume, etc.

142. Example 1. **A cistern, whose base is square and depth 18 inches, just contains 750 lb. wt. of water. Find, to the nearest inch, the length of an edge of the base. A cubic foot of water may be taken as weighing 1000 oz., *i.e.* 62·5 lb.**

The cubic content of the cistern $=\dfrac{750\times 16}{1000}$ c. ft. $=12$ c. ft.

Hence, if x in. $=$ the length required,

$$18x^2 = \text{the vol. in c. inches} = 12\times 1728.$$

$$\therefore\ x^2=\tfrac{2}{3}\times 1728 = 576\times 2.$$

$$\therefore\ x = 24\sqrt{2}$$

$$= 24\times 1{\cdot}41421 = 4\times 8{\cdot}48526$$

$$= 34 \text{ in. to the nearest inch.}$$

EXAMPLE 2. An open brick cistern whose walls and base are $4\frac{1}{2}$ inches thick contains 500 gallons. If the outside measurements of its base are 4 ft. 6 in. and 5 ft. 6 in., what is its inside depth to the nearest inch? A cubic foot of water contains 6·25 gallons.

$$6{\cdot}25 \text{ gallons occupy } 1 \text{ c. ft.}$$

$$\therefore\ 1 \text{ gallon occupies } \frac{1}{6{\cdot}25} \text{ c. ft.}$$

$$\therefore\ 500 \text{ gallons occupy } \frac{500}{6{\cdot}25}=\frac{50000}{625}=80 \text{ c. ft.}$$

The inside width $=3$ ft. 9 in. $=3\frac{3}{4}$ ft.

,, ,, length $=4$ ft. 9 in. $=4\frac{3}{4}$ ft.

Hence, if x ft. be the depth,

$$x\times\tfrac{15}{4}\times\tfrac{19}{4}=\text{the volume in c. ft.}=80.$$

$$\therefore\ x=\tfrac{80\times16}{15\times19}=\tfrac{256}{57} \text{ ft.}$$

```
57 ) 256·00 ( 4·49 ft.
     228          12
      280       5·88 in.
      228
       520
```

$\therefore$ the required depth $=4$ ft. 6 in. to the nearest inch.

EXAMPLE 3. A cistern with an outflow pipe is full of water. A stone is placed in it, causing an overflow which is found to weigh 59 lb. Find the volume of the stone to the nearest cubic inch. A cubic foot of water weighs 62·5 lb.

The volume of the stone $=$ the volume of the water displaced by it

$$=\frac{59}{62{\cdot}5} \text{ c. ft.}=\frac{59\times8}{62{\cdot}5\times8}$$

$$=\tfrac{472}{500}=\cdot944 \text{ c. ft.}$$

$$=11{\cdot}328\times12^2 \text{ c. in.}$$

$$=135{\cdot}936\times12 \text{ c. in.}$$

$$=1631 \text{ c. in. to the nearest c. in.}$$

EXAMPLES XVII. b.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

[*In the following examples it may be assumed that* 1 *c. ft. of water weighs* 62·5 *lb.* (=1000 *oz.*), *and contains* 6·25 *gallons.*]

1. Find the volume of a cube whose edge is 8 feet.

2. Find its surface.

3. A cuboid measures 3 ft. by 4 ft. by 12 ft.; find its volume and surface.

4. A cuboid is 246·5 cm. in length, 28·2 in breadth and 15·6 in depth. Find its volume in cub. cm.

Express it also in cub. metres.

5. Find the cost of painting the surface of a cube, whose edge is 7 ft., at a shilling a square yard.

6. The length of a room is 60 feet, width 40, height 16. How many persons will it hold if each is allowed 480 cub. ft. of air?

7. If the ceiling were raised 4 ft., how many would the room hold?

8. If the original dimensions were all halved, how many would the room hold?

9. A rectangular block of stone near Baalbec measured 70 ft. in length, 14 ft. in breadth and 14 ft. 5 in. in depth; what was its volume?

10. A rectangular tank is 24 feet long, 21 feet wide and 7 feet deep; what did it cost to dig at $7\frac{1}{2}d.$ per cub. yard?

11. A brick measures 9 in. by 4 in. by $3\frac{1}{2}$ in.; find its volume and surface.

12. How many litres can be contained by a cistern 3 m. 5 dm. long, 2 m. 4 dm. wide and 2 m. deep?

13. A cistern measures 4 ft. 8 in. by 3 ft. 6 in. by 2 ft. 9 in. deep; what volume of liquid will it contain, and what will be the cost of lining its sides and bottom with sheet lead weighing 12 lb. to the sq. ft. at £2 per cwt.?

14. A cubical cistern will contain 729 litres. Find the area of its base.

15. A rectangular box whose external dimensions, including the lid, are 32, 21, 16 cms. is made of wood 5 mm. in thickness; what is the volume of wood in it?

16. What number of 4-inch cubes can be cut from a 1 foot cube?

17. Half a cwt. of water is poured into a vessel whose base is a sq. ft.; how deep must the vessel be to contain it?

18. A log of timber has a rectangular section 18 in. by 14 in. The length is 10 feet. Find what length remains when $2\frac{1}{2}$ cub. ft. have been cut off one end.

19. If 195 c. ft. of sand be thrown into a tank 12 ft. long and $3\frac{1}{2}$ ft. wide, find, to the nearest inch, how much the water will rise.

20. Find the weight of water in a rectangular cistern whose internal length, width and depth are 6 ft. 6 in., 4 ft. 3 in., 4 ft.

21. If 44·2 cub. ft. are drawn off this cistern, how much is the surface lowered?

22. A rectangular brick-kiln measures 27 ft. in length, 15 ft. in breadth and 11 ft. 3 in. in height. How many bricks measuring 9 in. by $4\frac{1}{2}$ by 3 will it contain without gaps between them?

23. A room whose length is twice its width and whose height is $10\frac{1}{2}$ ft. costs £1. 14*s.* $1\frac{1}{2}d.$ to paper at $4\frac{1}{2}d.$ per sq. yard. What are the dimensions of the floor?

24. In the course of a day $\frac{1}{2}$ in. of rain fell. Find the weight which fell upon 1 acre to the nearest ton.

25. In a floodgate is a rectangular opening 25 cm. by 8 cm.; water flows through this at 15 kilometres an hour. How many litres flow through in a minute?

26. A cistern 10 ft. long and 5 ft. wide will hold 1000 gallons. Find its depth.

27. When 100 gallons of water are put into a tank on a square base, the depth of water is 5 inches. Find, to the nearest inch, the length of an edge of the base of the tank.

28. Given that lead weighs 11·4 times as much as an equal volume of water, find the weight, to the nearest lb. of a sheet of lead, 6 ft. long, 4 ft. 6 in. wide and a quarter of an inch thick.

29. A plank of timber is 7 ft. long, 21 inches wide and 3 inches thick. If 2 cubic feet are cut from one end, what length is left?

30. The estimated area of a section of a stream at right angles to the current is 30 sq. ft. If the stream runs at the rate of 4 miles an hour, find the weight of water, to the nearest ton, which passes a fixed point in a day.

31. A builder buys bricks by the thousand. How many thousand must he buy in order to build a wall 176 ft. long, 6 ft. high and 18 in. thick? A brick is 9 in. by $4\frac{1}{2}$ in. by 3 in.

32. The external dimensions of a closed iron safe are 2 ft. 4 in., 2 ft., 1 ft. 4 in. If it is made of iron 2 in. thick, find the volume of iron used, and its weight if 1 cub. in. of iron weighs 5 oz.

33. A man, immersed in a cistern originally full of water, causes an overflow which is found to weigh 12 stone 4 lb. Find the cubic content of the man in c. ft. to two decimal places.

34. A cistern whose internal dimensions are 4 ft. 6 in. by 3 ft. by 2 ft. deep, has water in it to a depth of 16 inches. How many inabsorbent bricks can be put in it without causing any overflow of water, each brick being 9 in. by $4\frac{1}{2}$ in. by 3 in.?

35. From the corners of a rectangular piece of cardboard, 18 inches by 12 inches, I cut out squares of 3 inch sides, and folding up the sides, form a box without a lid. Find the cubic content of the box.

36. A wall 50 yds. long, 6 ft. high and 18 in. thick is built with bricks which measure 9 in. by $4\frac{1}{2}$ in. by 3 in. How many bricks are used?

Circles.

143. If we look at a garden roller, we see that any section of it at right angles to the axis about which the roller turns is a circle, and all circular sections are of the same size. Such a solid is called a **right circular cylinder**, or shortly a **right cylinder**.

A round ruler is an example.

To measure the diameter of a circular coin, a right cylinder, or any object with a circular section.

Place two set-squares against the edge of a graduated ruler with the object between them, and read off the length of the diameter as shown in the figure.

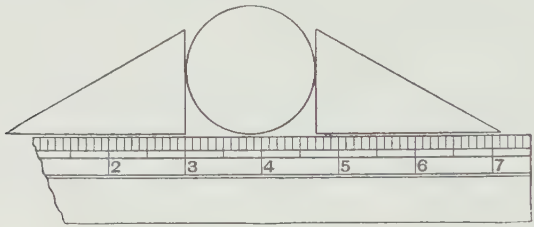


FIG. 25.

To measure the circumference of a right cylinder.

Wrap a piece of paper evenly round the cylinder until the edges overlap, and prick through the paper where it is double. Unroll the paper and, spreading it out flat, measure the distance between the punctures.

This gives the length of the circumference of the cylinder. We might also wrap a piece of thread round the cylinder and cut the thread with a sharp knife where it is double. The length of the thread cut off will give us the circumference.

To find the circumference of a circular coin, place a small wet spot of ink on its edge, and taking care not to let the coin slip, roll it along a straight line on paper. The distance between the ink-marks will give the length of the circumference.

144. The fraction $\dfrac{\text{circumference of a circle}}{\text{its diameter}}$ is constant.

Take any cylindrical object, and measure carefully its diameter and its circumference.

Divide the length of the circumference by the length of the diameter. If we do this with several such objects, we shall find that the result is always the same, viz., a little more than 3, roughly $3\frac{1}{7}$.

Thus we see that the value of the fraction

$$\frac{\text{circumference of a circle}}{\text{its diameter}} \text{ is constant.}$$

This constant is denoted by π (a Greek letter called pi).

Its exact value cannot be found, but a useful working value (correct to 4 decimal places) is 3·1416. $3\frac{1}{7}$ can be used when we do not require a very accurate result. $3\frac{1}{7} = 3{\cdot}1428\ldots$ and is therefore only correct to two decimal places.

Very useful. $\frac{1}{\pi} = {\cdot}31831$ correct to five decimal places.

From the above we see that if C = the circumference of a circle and r its radius,

$$\frac{C}{2r} = \pi.$$

Therefore $$C = 2\pi r.$$

145. Any plane (flat) figure bounded by more than four straight lines is called **a polygon.**

The area of a circle $= \pi r^2$, *where* r *is its radius.*

Let the circumference of the circle, centre O, be divided into any number (n) of equal parts, AB, BC, ... Join AB, BC ..., OA, OB ..., and draw ON at right angles to AB.

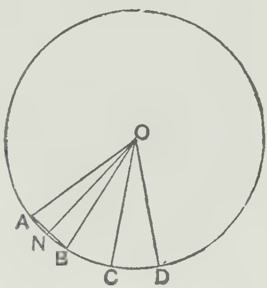


FIG. 26.

Area of the $\triangle AOB = \frac{1}{2} AB \,.\, ON$;

$\therefore$ the area of the polygon $= \frac{n}{2} \,.\, AB \,.\, ON$

$$= \frac{ON}{2} \times \text{perimeter of polygon}$$

($n \,.\, AB =$ perimeter of polygon).

Now let the number of sides, n, be increased indefinitely.

Ultimately, when n is infinitely large, the perimeter of the polygon becomes the circumference of the circle, and ON becomes equal to the radius of the circle;

$\therefore$ the area of the circle $= \frac{r}{2} \times$ circumference of circle

$$= \frac{r}{2} \times 2\pi r = \pi r^2.$$

EXAMPLES XVII. c.

[*Unless otherwise directed, take* $\pi = 3{\cdot}1416$ *and* $\frac{1}{\pi} = {\cdot}31831$. *Revise each example before proceeding to the next, and show up all the working.*]

1. The diameter of a half-penny is 1 inch. Find the length of its circumference to the nearest hundredth of an inch.

2. A gardener paces round the edge of a circular flower-bed and finds that its circumference is 19 yds. Find its radius to the nearest yard.

3. How many 6 ft. hurdles must be used to fence in a circular piece of ground of 31 yds. radius?

4. How many revolutions will a bicycle wheel of 28 in. diameter make in travelling one mile? (Take $\pi = \frac{22}{7}$.)

5. The minute hand of a watch is ·8 in. long. How far, to the nearest foot, will its extremity travel in a day?

6. Given that light will travel round the earth $7\frac{1}{2}$ times in a second, calculate the speed of light in miles per second. Give your result to the nearest thousand miles. (Radius of the earth = 4000 miles.)

7. A pony works a pump by walking in a circle and so turning a capstan whose arm is 20 feet long. How far, to the nearest mile, does the pony walk in doing 250 revolutions of the capstan?

8. A garden roller is 24 in. in diameter and 27 in. wide. What area does it roll in one revolution? (Result to the nearest sq. ft.)

9. The diameter of a half-penny is one inch. Find its area to the nearest hundredth of a square inch.

10. Find, to the nearest sq. foot, the area of a circular flower-bed of 9 ft. diameter.

11. A section of a trunk of a tree is 3 ft. in diameter. Assuming it is circular, find its area to the nearest square inch.

12. A circle encloses 480 sq. yds.; find its radius to the nearest foot.

13. A circle has an area of 25 sq. ft.; find its circumference, taking $\sqrt{\pi} = 1{\cdot}77$.

14. Given that the area of a half-crown is 1·2275 sq. in., find its radius in inches to 2 decimal places.

15. A flower-bed is in the form of a rectangle with semi-circles added on at the ends. If its straight sides are 15 ft. long and its width 6 ft., find its area to the nearest square foot.

16. A circle has a circumference of 14 ft. Find its area to the nearest hundredth of a square foot.

XVIII. AVERAGES.

146. WHEN a train travels 40 miles from station to station in 2 hours, its **average** pace is $\frac{40}{2}$ $(=20)$ miles per hour. It starts from the first station and stops at the last, and perhaps at intermediate stations, so that it does not travel at this rate the whole time.

In other words, if the train had travelled throughout at the uniform speed of 20 miles an hour, it would have travelled the same distance in the same time.

If 100 sheep are bought for £175, the **average** price of each sheep $=\frac{175}{100}=$ £1·75. The sheep are not necessarily of equal value. Some are probably worth more than others. If the sheep were of equal value, the value of each sheep would be £1·75.

I walk a distance of 24 miles in 8 hours, stopping for lunch, etc. My **average** pace is $\frac{24}{8}$, *i.e.* 3, miles per hour. Whilst walking I must have often moved faster than this, for some time was consumed at lunch.

Three boys are respectively 16, 17 and 18 years old.

Their **average** age $=\frac{16+17+18}{3}=\frac{51}{3}=17$ years.

40 acres of land (some good, some poor soil) are bought for £2000. The **average** price per acre $=\frac{2000}{40}=$ £50.

We thus arrive at the following definition:

The **average** of any quantity of numbers is their **sum** divided by their **number**.

EXAMPLE 1. **The average of 13, 15, 17, 19**

$$=\frac{13+15+17+19}{4}=\frac{64}{4}=16.$$

EXAMPLE 2. **On 5 consecutive days the maximum temperature was 37°, 41°, 62°, 49°, 48°.**

The average maximum temperature for the 5 days

$$=\frac{37+41+62+49+48}{5}=\frac{237}{5}=47\cdot4 \text{ degrees.}$$

EXAMPLE 3. **5400 people inhabit 1200 houses.**

The average number in a household $=\frac{5400}{1200}=4\cdot5$.

EXAMPLE 4. **I buy 3 bottles of wine at 3s. 2d. a bottle and 5 bottles at 4s. 9d. What is the average price per bottle?**

3 bottles at 3*s.* 2*d.* cost 9*s.* 6*d.*
5 ,, 4*s.* 9*d.* ,, 23*s.* 9*d.*
∴ the 8 bottles cost 33*s.* 3*d.*

∴ the average price per bottle $= \frac{33s.\ 3d.}{8} = 4s.\ 2d.$ to the nearest penny.

EXAMPLES XVIII.

Find the average of the following numbers:

1. 19, 37, 84, 44.
2. 13, 71, 82, 103, 726.
3. 51, 82, 7, 17, 89, 90.
4. 131, 72, 109, 89, 42.
5. 14·7, 13·2, 11·3, 10·9, 7·6.
6. $3\frac{1}{2}, 4\frac{1}{4}, \frac{1}{5}, 2\frac{1}{8}$.

7. The ages of 6 boys are 14 yrs. 11 mo., 11 yrs. 2 mo., 12 yrs., 15 yrs. 7 mo., 16 yrs. 9 mo., 15 yrs. 8 mo.; what is their average age to the nearest month?

8. A man walks 17, 19, 21, 25, 18, 21 miles on 6 successive days; what is his average daily walk?

9. The crew of a four-oared boat weigh respectively 10 st. 7 lb., 11 st. 2 lb., 11 st. 7 lb. and 10 st. 8 lb. What is their average weight?

10. Of six men, one weighs 13 st., two each weigh 11 st. 6 lb. and the three others 12 st. 7 lb. each. What is their average weight to the nearest lb.?

11. The rain-fall from Jan. to June 1908 at Greenwich was 1·49, 1·45, 2·21, 2·1, 1·52, 2·07 inches. What was the average per month to the nearest tenth of an inch?

12. In 11 innings at cricket a man made totals of 55 runs in the first 3 innings, 85 in the next 4 and 32 in the last 4. What was his average per innings?

13. In 14 cricket matches, a man had 9 innings, making an average of 11 in the first 5 innings and an average of 13 in the last 4. What was his average (1) per innings, (2) per match?

14. A train runs 455 yds. in the first 3 minutes, 1372 yds. in the next 4 and 3680 yds. in the next 5. What is its average speed in miles per hour (to the nearest half-mile) during the whole 12 minutes?

15. A man mixes 15 lb. of tea at 1*s.* 10*d.* per lb., 12 lb. at 2*s.* 6*d.* and 13 lb. at 1*s.* 8*d.* What is the value of the mixture per lb.?

16. A grocer mixes 6 lb. of tea at 2*s.* 6*d.* a lb., 5 lb. at 1*s.* 9*d.* and 4 lb. at 1*s.* 3*d.*, and sells the mixture at 2*s.* 3*d.* per lb. How much does he gain per lb. of the mixture?

17. For the British Islands the value of the average imports for consecutive years were £528,391,274, £542,600,289, £551,038,628, £565,019,917, £607,888,500, £645,807,942. Find the average annual value for these years to the nearest thousand pounds.

18. The number of scholars at a school for six consecutive days was 157, 149, 153, 159, 162, 160. Find the average daily attendance to the nearest whole number.

If there were 180 names on the books, what was the daily average per cent., *i.e.* per 100 scholars, to the nearest whole number?

19. The increase, or decrease, in the Civil Service Estimates for each of the years from 1900 to 1905 was as follows, in £:

+659,143, +791,312, +3,309,618, −19,369, +1,063,551, −545,206.

Find the average annual increase for this period to the nearest £.

20. The gross Navy Estimates increased from £8,969,969 in 1870 to £33,942,003 in 1908. Find the average annual increase to the nearest £1000.

21. A man with a household of 9 persons estimates that he consumes 185,000 gallons of water in a year. His neighbour, with a household of 4, estimates his annual consumption at 38,000 gallons Each of them pays £3 4*s.* a year for his water supply. Calculate, for each household, the daily average consumption per head (to the nearest gallon) and the cost per 1000 gallons (to the nearest penny).

22. One man's expenses from 1st March to 25th July, both inclusive, were £57 5*s.* 2*d* ; another's were £82 13*s.* 5*d.* from 18th May to 5th December, both inclusive. Find which of these is the higher rate of expenditure

23. The average temperature for Monday, Tuesday, and Wednesday, was 53°. The average for Tuesday, Wednesday and Thursday was 56°, that for Thursday being 60°. What was the temperature on Monday?

XIX. UNITARY METHOD. SHORTENED.

147. AFTER some experience with the Unitary Method, the work may be considerably shortened, with the help of mental reasoning. (Compare page 109.)

EXAMPLE. 1. **Find the cost of 11 sheep if 7 sheep cost £13. 2s. 6d.**

Preliminary Statement. 7 sheep cost £13. 2*s.* 6*d.*

11 ,, how much?

The required cost $= £13.\ 2s.\ 6d. \times \frac{11}{7}$

$= £1.\ 17s.\ 6d. \times 11$

$= £20.\ 12s.\ 6d.$

The Reasoning is as follows: The *more* sheep there are, the *more* they cost.

11 is *more* than 7.

∴ **the number by which £13. 2*s.* 6*d.* is to be multiplied must be the *greater*, viz. the 11.**

EXAMPLE 2. **How long will 21 men take to do a piece of work which 15 men can do in 7 days?**

Preliminary Statement. 15 men do the work in 7 days.
21 ,, ,, how many days?

The required number of days $=7\times\frac{15}{21}=5$.

The Reasoning. The *more* men there are, the *fewer* days they take to do the work.

21 is *more* than 15. $\therefore$ the answer is *less* than 7 days.

$\therefore$ we must multiply 7 days by the *smaller* number of men (15) and divide by the *greater* number (21).

EXAMPLE 3. **If 5 boys can do the work of 3 men, how long will 7 boys and one man take to do a piece of work which occupies 3 boys and 2 men for 26 days?**

1 boy is equivalent to $\frac{3}{5}$ of a man.

$\therefore$ 7 boys and one man are equivalent to $\left(\frac{21}{5}+1\right)$ men
$=\frac{26}{5}$ men.

Also 3 boys and 2 men are equivalent to $\left(\frac{9}{5}+2\right)$ men
$=\frac{19}{5}$ men.

Preliminary Statement. $\frac{19}{5}$ men take 26 days to do the work.
$\frac{26}{5}$,, how many days to do the work?

The required number of days $=26\times\dfrac{\frac{19}{5}}{\frac{26}{5}}=19$.

[The reasoning is similar to that in Example 2.]

EXAMPLES XIX. a.

[*Use the* shortened form *of the Unitary Method to solve the first* 10 *problems, and when you have done all the work, write out carefully the reasoning you have employed. Always make a* Preliminary Statement. *Show up all the working and avoid side sums.*]

1. If the railway fare for 71 miles is 11*s.* 10*d.*, what is the fare for 51 miles?

2. If I give 5*s.* for 72 eggs, how much do I give for 96 eggs?

3. If 14 men do a certain amount of work in 12 days, how many men will do the same amount in 21 days?

4. Find the cost of 60,000 oranges at 4 shillings a gross.

5. If 3 men plough a field in 20 days, how many days will 12 men take to do it?

6. How much will 58 men earn in a week, if 87 men earn £96. 15*s.* 6*d.* in that time?

7. If five-eighths of a ship's cargo is worth £3060, what is the value of two-thirds of it?

8. If a number of men do five-sixteenths of a piece of work in 10 days, how long will they take to do the rest?

9. If 14 men do a piece of work in 23 days, how many men must be engaged if the work is to be done in 6 days?

10. If a number of men do three-sevenths of a piece of work in 39 days, what fraction of the work will they do in 9 days?

11. If $4\frac{2}{5}$ lb. cost 2*s*. $3\frac{1}{2}d.$, what is the cost of five-sevenths of a cwt.?

12. If 1 kilogramme = 2·2046 lb., express one ton in kilogrammes to the nearest kilogramme.

13. If 23 men do seven-eighths of a piece of work in a week and then leave it, how many men must be engaged to finish the work in another week?

14. If an oz. of gold is worth 4 guineas, what is the value of 15 grains?

15. If 50 guineas is the rent of a house worth £840, what should be the rent, to the nearest £, of one which is worth £2500?

16. If a bankrupt pays 6*s*. 3*d*. in the £, what does a creditor receive, to the nearest penny, for a debt of £645. 11*s*. 9*d*.?

17. Income-tax is at 1*s*. 2*d*. in the £, but tax is only paid on the excess of an income over £150. What does a man pay whose income is £625. 10*s*.?

18. When a man has paid an income-tax at 10*d*. in the £, he finds he has £301. 17*s*. 6*d*. per annum left. What will he have left when he pays 1*s*. 4*d*. in the £?

19. A typist can copy 40 words a minute and charges 10*d*. for copying 1000 words; how much does she earn per day of 8 hours?

20. If 18 sheep or 30 lambs eat 39 bushels of turnips, how many bushels will 45 sheep and 35 lambs consume in the same time?

21. If a stock-broker charges 2*s*. 6*d*. for every £100 worth of stock which he sells for me, how much, to the nearest penny, does he charge for selling stock worth £1461?

22. If $\frac{3}{5}$ of a share is worth £23. 7*s*. 3*d*., what is the value of $2\frac{2}{3}$ shares?

23. If 4 cwt. 10 lb. of sugar cost £6. 13*s*. 7*d*., what will be the cost of 2 cwt. 1 qr. 17 lb.?

24. If in sawing up 350 c. ft. of timber 14 c. ft. is wasted, what will be the waste in 750 c. ft.?

25. A bankrupt's assets are £564. 4*s*. 10*d*. and his liabilities £765. 3*s*. 2*d*.; how much, to the nearest penny, does he pay in the £?

26. Income-tax stands at 10*d*. in the £, no tax being charged on the first £150; find what tax a man pays whose income is £847. 10*s*.

27. If 5 boys do the work of 2 men, and 7 men earn £6. 17*s*. 1*d*. in a week, what does a boy earn per week?

28. After paying income-tax at 9*d*. in the £, a man has £830. 12*s*. 9*d*. left; what was his income?

29. If 16*s.* is the railway fare for 128 miles, how far can I travel for 13*s.* 3*d.*?

30. If $\frac{3}{8}$ of a ship's cargo is worth £762, what fraction of the cargo does a man get for £635?

31. If 10 men do $\frac{1}{5}$ of a piece of work in 7 days, how many more men must be engaged if the work is to be done in 5 days more?

32. A parish has to pay rates on £2765, and it is required to raise a rate of £98. How much in the £ must be levied to raise this sum?

33. 7 men can do a piece of work in 10 days, but after 2 days 3 of the men strike. How long will the other men take to finish the work?

34. If the town rates on a house whose rental is £52 amount to £11. 5*s.* 4*d.*, what rates will a house of £102. 10*s.* rental be liable for?

35. If a man buys oranges at 9*d.* a dozen, what must he sell 100 for in order to make a profit of 3*s.*?

36. A man bought 90 oranges at 7*d.* a dozen and sold them at 9*d.* a dozen; what profit did he make?

37. A dealer bought eggs at 6 for 5*d.* and sold them at 5 for 6*d.*: what profit did he make on (1) 300 eggs, (2) 300 pence?

38. After paying income-tax at 1*s.* 2*d.* in the £, a man had £598. 8*s.* 7*d.* left; what income had he at first?

39. When the income-tax was at 8*d.* in the £, no tax being charged on the first £150, a man found that he had £609. 3*s.* 4*d.* left when he had paid the tax. How much had he at first?

40. In an examination paper which was marked to a maximum of 75, a boy got 31 marks. How many to the nearest integer would he have obtained if the maximum had been 100?

41. A bankrupt pays a dividend of 7*s.* 9*d.* in the £, what is a creditor's loss on a debt of £75. 10*s.*?

42. If 5 boys do the same amount of work in an hour as 3 men, which would do the most in an hour, 34 boys or 20 men?

43. A printer finds that it costs him £35 to print 500 copies of a book. What must he sell each book for in order that he may make a profit of at least 6*d.* a copy?

44. Find the income-tax on £654 at 1*s.* 1*d.* in the £.

45. A printer prints 500 copies of a book at a cost of £53. 4*s.* 6*d.* and sells them for £73. 0*s.* 4*d.* How much profit does he make on each copy?

46. If 14 lb. of apples cost 4*s.* 10*d.*, how many lb. must be sold for 21*s.* in order to make a profit of 1*d.* per lb.?

47. If 3 men eat as much as 4 boys, and the cost of feeding 6 boys for a week is 54*s.*, how much will it cost per week to feed 5 men and 5 boys?

48. If eggs are bought at 2*s.* a score, for how much must 340 eggs be sold in order to make a profit of 14*s.* 6*d.*?

49. A man insures his life for £500, at a premium of £3. 12*s.* 4*d.* per annum for every £100. What premium per annum does he pay?

50. A pipe can fill a tank in 6 hours; what fraction of the tank does it fill in an hour?

A second pipe can fill it in 4 hours; what fraction of the tank will the two pipes fill in an hour, running together?

51. If a field 200 yds. square provides grazing for 4 cows, how many cows can be grazed in a field 300 yds. square?

52. A man bought 126 pigs for £175 and sold them for £214. What profit, to the nearest shilling, did he make per dozen pigs?

148. EXAMPLE 1. **If 16 men plough 64 acres in 8 days, how many men must be employed to plough 27 acres in 9 days?**

Unitary Method in detail.

16 men plough 64 ac. in 8 days.

$\therefore$,, $\frac{64}{8}$ ac. (=8) in 1 day.

$\therefore \frac{16}{8}$,, 1 ac. in 1 day.

$\therefore \frac{16\times 27}{8}$,, 27 ,, 1 ,,

$\therefore \frac{16\times 27}{8\times 9}$,, 27 ,, 9 days.

$\therefore$ the required number of men $=\frac{16\times 27}{8\times 9}=6$.

Unitary Method shortened. *Preliminary Statement.*

64 acres are ploughed in 8 days by 16 men.

27 ,, ,, 9 ,, how many men?

The required number of men $=16\times\frac{27}{64}\times\frac{8}{9}$

$=6$.

The Reasoning.

In a fixed number of days, the *more* acres there are, the *more* men it will take to plough them.

27 is less than 64, *i.e. fewer* men are required.

$\therefore$ we multiply 16 (men) by the less (27) and divide by the greater (64).

With a fixed number of acres to be ploughed, the *more* days there are, the *fewer* men it will take to plough them.

9 is more than 8, *i.e. fewer* men are required.

$\therefore$ we multiply by the less (8) and divide by the greater (9).

EXAMPLE 2. **If 7 men earn £45. 12s. 6d. in 25 days of 10 hours each, how many hours a day must 12 men work to earn £54. 15s. in 21 days, at the same rate of pay?**

Unitary Method shortened. *Preliminary Statement.*

7 men earn £45. 12*s.* 6*d.* in 25 days of 10 hours each.

12 ,, £54. 15*s.* ,, 21 ,, how many hours each?

The required number of hours $=10\times\frac{7}{12}\times\frac{25}{21}\times\frac{£54.\ 15s.}{£45.\ 12s.\ 6d.}$.

The Reasoning.

Other things being constant, the *more* men there are, the *less* number of hours per day they need work.

i.e. 12 men take fewer hours per day than 7 men.

∴ we multiply by the less (7) and divide by the greater (12).

Again, other things being constant, the *more* days there are, the *less* number of hours per day they need work.

i.e. working for 21 days, they will work a greater number of hours per day than when working for 25 days.

∴ we multiply by the greater (25) and divide by the less (21).

Also, other things being constant, the *larger* the sum of money, the *larger* number of hours per day they must work to earn it.

i.e. they must work *more* hours per day to earn £54. 15*s.* than to earn £45. 12*s.* 6*d.*

∴ we multiply by the fraction $\frac{\text{£54. 15}s.}{\text{£45. 12}s.\ 6d.}$, for £54. 15*s.* is greater than £45. 12*s.* 6*d.*

The student should work a number of examples by the unitary methoa in full detail before resorting to the shortened method.

EXAMPLES XIX. b.

[*Show up all the working and avoid side sums.*]

1. If it costs £16. 10*s.* to keep 11 horses for 22 days, how many horses can be kept for 7 days at a cost of £10. 10*s.*?

2. If 21 men earn £56 in 15 days, how many men will earn £128 in 48 days?

3. If 14 people travel 150 miles for £10. 1*s.* 3*d.*, how much will it cost for 17 people to travel 320 miles?

4. If 16 men do three-sevenths of a piece of work in 21 days, how many men will be required to finish it in another 32 days?

5. If 35 men plough 70 acres in 3 days, how long will it take 42 men to plough 56 acres?

6. If 350 miners work 6 days a week of 8 hours each, how many must be employed to raise the same amount of coal working 5 days a week of 7 hours each?

7. If 10 men, on a tour of 11 months, spend £1283. 6*s.* 8*d.*, how much at the same rate would it cost a party of 7 men for 2 months?

8. If 4500 copies of a book of 11 sheets require 198 reams of paper, how much paper will be required for 5000 copies of a book of 25 sheets of the same size as the former?

9. If 5 men can do a piece of work in 48 days, how many men will perform another piece of work 7 times as great in one-fifth of the time?

10. A block of stone 5 ft. long, 3 ft. 9 in. broad and 2 ft. 6 in. deep weighs 3 tons 6 cwt. 108 lb.; what is the weight of a block of the same kind of stone 12 ft. 6 in. long, 6 ft. 6 in. broad and 8 ft. 3 in. deep?

11. If it costs £6 to dig a pit 24 ft. deep and 28 sq. ft. in horizontal section, what is the depth of a pit of horizontal section 14 ft. by 9 ft. which costs £9 to dig out?

12. If a family of 9 people can live comfortably in England for 1560 guineas a year, what will it cost per annum for a family of 8 to live in Belgium, prices being supposed to be three-fifths of what they would be in England?

13. Allowing each man 32 oz. a day, a certain amount of provisions suffice for 1000 men for 5 weeks; how long will they last for 1500 men, allowing each man $13\frac{1}{3}$ oz. a day?

14. If $42\frac{1}{2}$ yds. of cloth, 18 in. wide, cost £14. 18*s.* $6\frac{1}{2}$*d.*, what will be the cost of 119 yds. of the same kind of cloth, 36 in. wide?

15. A trench 110 yds. long, 3 ft. wide and 4 ft. deep is dug by 30 men in 5 days of 11 hours each; another trench is dug by half the number of men in 7 days of 9 hours each. How many cubic feet of water will it hold?

16. It takes 7 men 16 days to mow a field 1320 yds. long and 440 yds. wide. What will be the width of a field of the same length which 4 men can mow in 42 days?

17. If a cat and a half kill a rat and a half in a minute and a half, how long will 50 cats take to kill 50 rats?

18. A beam 16 ft. long, $2\frac{1}{4}$ ft. wide and 8 inches thick weighs 1280 lb.; find the length, to the nearest inch, of a beam of the same material $3\frac{1}{4}$ ft. wide and $7\frac{1}{2}$ in. thick which weighs 1014 lb.

19. 50 men do a piece of work, working for 12 days at 7 hours a day; how many hours a day must 15 men work in order to do the same amount in 35 days?

20. When wheat is at 4·75*s.* a bushel, a four-penny loaf weighs 3·45 lb.; what will be the cost of 95 lb. of bread when wheat is at 6·9*s.* a bushel?

21. 13 men can do a piece of work in 35 days. After having done one quarter of the work, 6 men strike; how much longer must the remaining men work in order to finish the task?

22. If 7 cwt. can be carried 124 miles for 35*s.*, what weight can be carried 104 miles for 65*s.*?

23. If 15 men build three-eighths of a wall in 21 days, how many men must be employed if the wall is to be finished within 16 days more?

24. At a public dinner 130 men drink £20 worth of wine at an average price of 6*s.* 8*d.* a bottle; what will be the wine bill, at the same rate of drinking, for 240 men if the price of the wine is 3*s.* 3*d.* a bottle?

25. If a 4 lb. loaf of bread costs 6*d.* when wheat is at 42*s.* a quarter, what should be the price of a 6 lb. loaf, to the nearest farthing, when wheat is 48*s.* a quarter?

26. If a man can plough an acre in a day, how many men must be employed to plough a rectangular field 330 yds. by 520 yds. in 4 days?

27. If a money-lender charges £6 for a loan of £50 for 7 months, what would he charge, to the nearest shilling, for a loan of £124 for a year?

28. If 630 flagstones 2 ft. by 18 in. will pave a courtyard, how many will be wanted for a courtyard one-third the size, when each flagstone is 14 in. by 9 in.?

29. If a tradesman with a capital of £500 gains £40 in 4 months, how much will he gain in 9 months with a capital of £3700?

30. If 80 men can dig a trench 3 miles long, 2 ft. deep and 3 ft. wide, in 4 days of 7 hours each, how many cubic feet does each man excavate per hour?

31. 12 men can build a wall in 14 days. After 4 days' work 4 men strike; how many days will the remaining men take to finish the wall?

32. It is stated in a text-book on arithmetic that: "252 men can dig a trench 210 yds. long, 3 yds. wide and 2 yds. deep in 5 days of 11 hours each." How many cubic feet does each man excavate per hour?

33. If 23 men can do a piece of work in 11 days, how many men will it take to do a piece of work 5 times as great in one-third of the time?

34. The cost of papering a room 16 ft. 6 in. long and 13 ft. 6 in. wide with paper 27 in. wide at 3*s.* per piece of 12 yds. is £1. 4*s.* Find the height of the room to the nearest inch.

35. A town which is defended by 3000 men is stocked with sufficient food for 10 weeks at the rate of 24 oz. per man per day. 1000 men are sent away and the rations are reduced to 20 oz. per day; how long will the food last for the remaining men?

36. If a form of 18 boys uses an average of 9 packets of paper a week at 1*s.* 3*d.* a packet, what will it cost to supply a school of 600 boys with paper for a school year of 40 weeks?

37. When meat is at 10*d.* a lb. it costs £13. 6*s.* 8*d.* to supply a family of 9 persons for 5 weeks; how much does it cost to supply a family of 15 persons for 3 weeks when meet is at 1*s.* a lb.?

38. It takes $13\frac{1}{2}$ days for 4 men to plough a square field; how many men can plough another square field whose side is half as long again as the other in $4\frac{1}{2}$ days?

39. If 7 boys do the same amount of work as 4 men, how long will 5 men and 7 boys take to do a piece of work which occupies 3 men and 6 boys for 7 days?

40. If the gas for 7 burners for 5 hrs. a night for 12 days costs 7*s.* 6*d.*, what will be the cost of 11 burners burning for 6 hrs. a night for 4 weeks?

41. In digging a trench 3 ft. wide and 2 ft. deep, 16 men dig 42 yards per day. How many men will it take to dig 63 yds. per day when the trench is 4 ft. wide and 2 ft. 6 in. deep?

XX. THE USE OF SQUARED PAPER. CO-ORDINATES.

149. TAKE two straight lines XOX′, YOY′, at right angles to one another.

For convenience sake we here use 'squared' paper. The thicker parallel lines are half an inch (or 1 cm.) apart, and the finer lines one-tenth of an inch (or 1 mm.) apart.

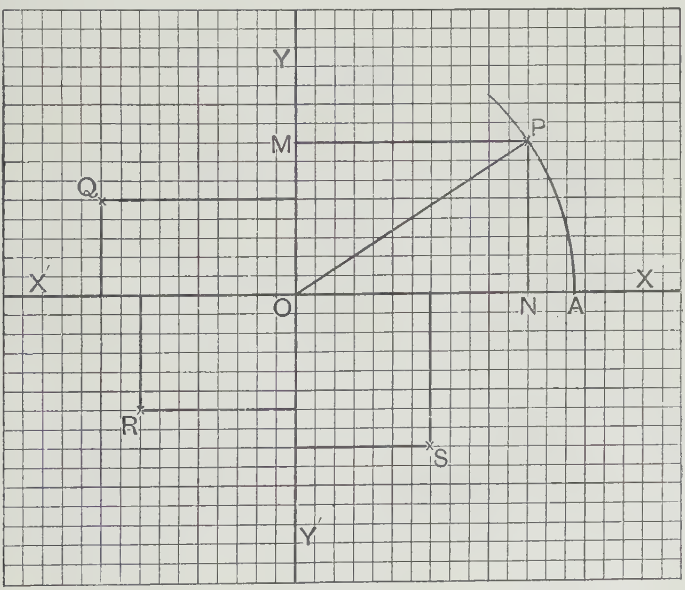


FIG. 27.

From P, any point in the plane of these lines, draw PN perpendicular to XOX′ and PM perpendicular to YOY′.

Let $PM = x$ and $PN = y$.

These values, x and y, determine the position of the point P; *i.e.* if we know the values of x and y, we can mark the point P.

For instance, if we know that $x = 12$ and $y = 8$, we measure $ON = 12$ along OX and $OM = 8$ along OY, and then draw MP and NP respectively parallel to OX and OY. P is the required point.

x and y are called the **co-ordinates** of the point P;

XOX′, YOY′, the **axes of co-ordinates**, or **lines of reference**; the point O is called the **origin**.

P is described as the point (x, y). In this case it is the same as the point (12, 8).

x is called the **abscissa** and y the **ordinate** of the point P.

150. ***If lines drawn in one direction are taken as positive, then lines drawn in the opposite direction must be taken as negative.***

Lines drawn in the directions OX, OY are usually considered positive, and therefore lines drawn in the directions OX′, OY′ are taken as negative.

In the diagram,

the abscissa of Q is negative and the ordinate positive.
" of R " " negative.
" of S is positive " "

In practice, the simplest way of obtaining the point (12, 8) is as follows:

Along OX take $ON = 12$, and at N draw NP perpendicular to ON in the direction OY, and make $NP = 8$.

To save space in printing, we have used one-tenth of an inch to represent unity. Any unit may be used, but it is important to choose a suitable one.

Take as large a unit as is convenient should be the general rule.

Example 1. **Plot the point (12, 8), P, and find the length of the line joining P to the origin.**

We have seen how to plot the point P (Fig. 27). With centre O and radius OP, describe an arc of a circle cutting OX at A.

We see that OA is between 14 and 15, a little nearer to the 14 line than to the 15.

We therefore estimate that OA = 14·4.

Hence OP = OA = 14·4.

[A larger unit than one-tenth of an inch might be used here with advantage.]

Verification by Pythagoras' Theorem. $OP^2 = ON^2 + NP^2 = 12^2 + 8^2 = 208$.

$\therefore$ $OP = \sqrt{208} = 14·4$ correct to one decimal place, by the method for finding a square root.

Note that we have, by the use of squared paper, determined an approximate value of $\sqrt{208}$.

EXAMPLE 2. **Plot the points (8, 12) and (−5, 7), and find the length of the line joining them.**

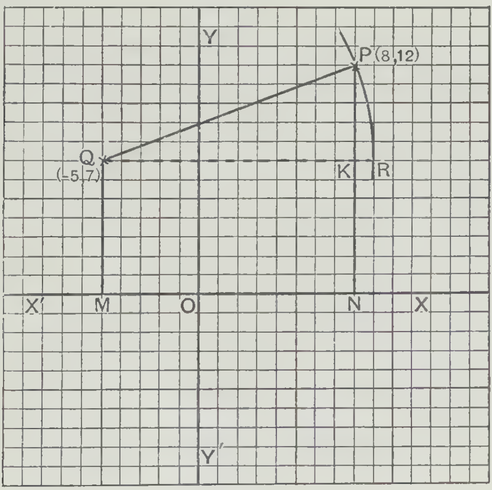


FIG. 28.

Along OX take ON = 8, and draw NP perpendicular to OX and in the direction of OY, taking NP = 12. P is the point (8, 12).

Along OX′ take OM = 5, and draw MQ perpendicular to OX′ in the direction of OY, taking MQ = 7. Q is the point (−5, 7).

To find the length of QP, with centre Q and radius QP, describe an arc of a circle cutting the line through Q parallel to XOX′ at R.

QP = QR = 14, by inspection.

Verification by Pythagoras' Theorem. Let PN cut QR at K.

By inspection, QK = 13 and PK = 5.

$$\therefore \quad QP^2 = QK^2 + KP^2 = 13^2 + 5^2 = 194;$$

$$\therefore \quad QP = \sqrt{194} = 14 \text{ nearly } (13·93).$$

EXAMPLE 3. **A man walks 12 miles to the East, then 9 miles to the North, then 20 miles to the West, and lastly 15 miles to the South. At what distance is he then from his starting point?**

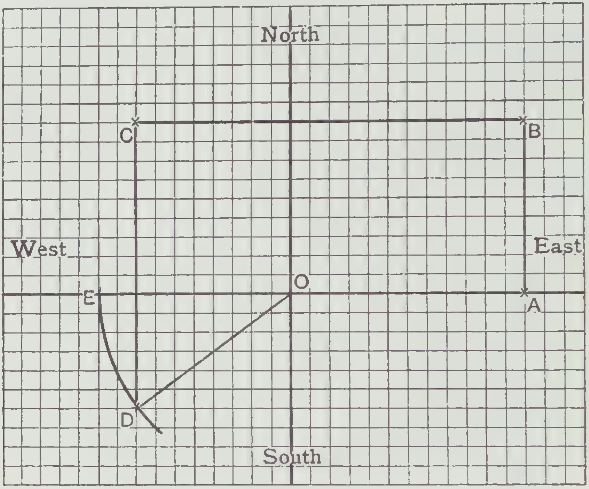


FIG. 29.

To the East	draw	$OA = 12$	tenths of an inch	to represent	12	miles.
,, North	,,	$AB = 9$	,,	,,	9	,,
,, West	,,	$BC = 20$	,,	,,	20	,,
,, South	,,	$CD = 15$	,,	,,	15	,,

By measurement, as in Example 2, $OD = 1$ inch.

$\therefore$ the required distance $= 10$ miles.

EXAMPLES XX. a.

Draw your diagrams as neatly as possible, with a well-pointed pencil.

Indicate the lines of reference, *i.e.* the axes of co-ordinates, clearly.

State definitely, on the diagram of each problem, the unit you employ.

Use as large a unit as you can without making the diagram inconveniently large.

A good plan is to use squared paper for the diagram only, writing the description and explanation on another sheet of paper.

A diagram without description and explanation is valueless.

In plotting points mark each with a small cross ($\times$).

1. Write down the co-ordinates (unit one-tenth of an inch) of the points P_1, P_2, P_3 ... in the diagram on next page (Fig. 30).

2. Using one-tenth of an inch as unit, plot the points (10, 18), (0, 9), (8, 0), (−9, 12), (−15, 0), (−9, −6), (0, −4), (0, −11), (7, −5), (7, −13).

Write the co-ordinates of each point against it.

3. Using an inch as unit, plot the points (1·2, 0·7), (1·6, 0), (2·9, −1·3), (0, −1·4), (−1·2, 2·7), (−2·5, 1·9), (0, −2), (−0·2, 2).

Write the co-ordinates of each point against it.

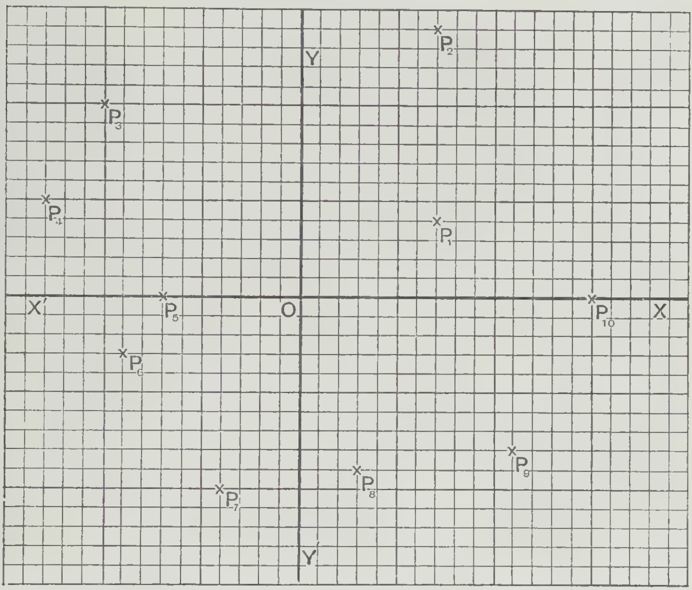


FIG. 30.

4. Plot the points given by the table below, using one inch as unit:

When $x=$	−2·5	−1	0	1·3	1·6	2	2·4
$y=$	−1·5	0	1	2·3	2·6	3	3·4

Join the points by straight lines. What do you see?

5. Using one inch as unit, plot the points (0·5, 1·5) and (−2, −1). Join them by a straight line, and produce it both ways. Write down in a similar form to the tabulation in question 4 above, the ordinates of the points on the line whose abscissae are: −3, −2·5, −0·5, 0·2, 0·8, 1·7, 2·1.

6. Copy the tabulation, etc., below; fill in the blank spaces; plot the points whose co-ordinates you thus obtain, and join them by straight lines. What do you notice?

$y=2x-3$. Let one-tenth of an inch represent unity.

When $x=$	-14	-13	-9	0	5	17	19	24
$2x=$	-28	-26						
$y=2x-3=$	-31	-29						

7. Using half an inch as unit, plot the points $(-1, 1)$, $(7, 5)$, and determine, from your figure, the co-ordinates of the middle point of the line joining them.

8. From a diagram find the length of the line joining the point $(6, 3)$ to the origin. Use Pythagoras' Theorem to show that this is a geometrical method of finding an approximate value of $\sqrt{45}$.

In each of the following examples, copy the table given, and fill in the blank spaces. Plot the points obtained and join them by straight lines.

9. $y=4x+3$.

When $x=$	0	2	4	6	8
$4x=$					
$y=4x+3=$					

10. $5y=6x-4$.

When $x=$	1	2	3	4	5	6
$6x=$						
$6x-4=$						
$y=$						

11. $4y=5x-20$.

$x=$	0	2	4	6	8
$5x=$					
$5x-20=$					
$y=$					

12. On squared paper take two lines AB, AC at right angles to one another, and each one inch long. Use Pythagoras' Theorem to show that $BC=\sqrt{2}$.

Hence find the value of $\sqrt{2}$ correct to one decimal place.

13. Take AB, AC as in the previous question, but each 5 inches long, instead of one inch. Use Pythagoras' Theorem to show that

$$BC=5\sqrt{2}.$$

Hence find the value of $\sqrt{2}$ correct to *two* decimal places.

14. On squared paper take a line AB one inch long, and letter the line BD at right angles to it. With centre A and radius 2 in. describe an arc of a circle cutting BD at C. Use Pythagoras' Theorem to prove that $BC=\sqrt{3}$, and from your diagram find the value of $\sqrt{3}$ correct to one decimal place.

15. Do question 14, taking $AB=2$ in. and drawing an arc of 4 in. radius. Show that $BC=2\sqrt{3}$, and from your diagram find the value of $\sqrt{3}$ correct to two decimal places.

16. A ladder whose foot is 14 feet from a vertical wall just reaches to a point on the wall 30 ft. from the ground. Use squared paper to find the length of the ladder to the nearest foot.

151. Before proceeding further, the student, who is not familiar with *similar triangles* and their properties, should work several of the following examples. If he works accurately, he will in each case obtain a straight line which passes through O, the origin. He should work neatly, and should mark each plotted point with a small cross (×).

EXAMPLES XX. b.

In each of the following examples plot the points given by the corresponding values of x and y in the tabulation, and then join them by straight lines.

Use one-tenth of an inch to represent unity in each example.

1.

When $x=$	0	5	8	10	14	20	21	40	42	50
$y=$	0	5	8	10	14	20	21	40	42	50

Note that at each point $y=x$, *i.e.* the ordinate = the abscissa.

2.

When $x=$	0	4	8	18	24	28	31	32	40
$y=$	0	8	16	36	48	56	62	64	80

At each point $y=2x$, *i.e.* the ordinate = twice the abscissa.

3.

When $x=$	0	5	8	12	18	21
$y=$	0	15	24	36	54	63

At each point $y=3x$, *i.e.* the ordinate = three times the abscissa.

See if the same holds at points on the straight line not plotted, *e.g.* at the points whose abscissae are 6, 9, 17, 19.

4.

When $x=$	0	10	14	20	26	30	40
$y=$	0	5	7	10	13	15	20

At each point $y=\frac{1}{2}x$ or $2y=x$.

See if the same holds at points on the straight line not plotted, *e.g.* at the points whose abscissae are 8, 12, 18, 24, 50; or at the points whose ordinates are 3, 9, 17, 18.

5.

When $x=$	0	3	6	9	15	30
$y=$	0	4	8	12	20	40

At each point $\frac{y}{x}=\frac{4}{3}$, or, multiplying these equal quantities by x,

$$y=\frac{4x}{3}.$$

See if the same holds at points on the straight line not plotted, *e.g.* at the points whose abscissae are 12, 18, 36.

6.

When $x=$	0	4	8	16	24	32
$y=$	0	5	10	20	30	40

At each of these points $\frac{y}{x}=\frac{5}{4}$ or $y=\frac{5x}{4}$.

See if the same holds at other points on the straight line not plotted, *e.g.* at the points where $x=12, 20, 28$.

7.

When $x=$	0	3	9	15	30
$y=$	0	5	15	25	50

At each of these points $\frac{y}{x}=\frac{5}{3}$ or $y=\frac{5x}{3}$.

Test for other points.

152. From the preceding examples we gather that if the co-ordinates of a series of points are connected by a relation in the form $\frac{y}{x} = k$ or $y = kx$, **where k is constant in value, all the points lie on a straight line passing through the origin.**

The straight line is called the **graph** of $y = kx$. This is one of the simplest forms of a graph. It will be seen later that a graph may be of any form.

153. We will now make use of this property in order to construct a **ready-reckoner** in the form of a graph. We shall still use *lines of reference*, but in problems of this kind we do not call them axes of co-ordinates. We give them names which suit the circumstances of each problem.

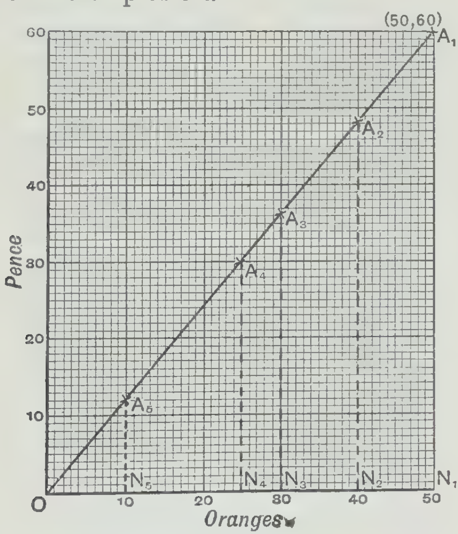


FIG. 31.

If 50 oranges cost		60 pence,		[Note that $\frac{60}{50} = \frac{6}{5}$.]		
5	,,	6	,,			
10	,,	12	,,	[	,,	$\frac{12}{10} = \frac{6}{5}$.]
20	,,	24	,,	[	,,	$\frac{24}{20} = \frac{6}{5}$.]
25	,,	30	,,	[	,,	$\frac{30}{25} = \frac{6}{5}$.]
30	,,	36	,,	[	,,	$\frac{36}{30} = \frac{6}{5}$.]

In each case,

$$\frac{\text{the \textbf{number} expressing the cost of the oranges in pence}}{\text{the \textbf{number} of oranges}} = \frac{6}{5}.$$

Examining the straight line OA_1 in the accompanying figure, we see that

at the point A_1	the abscissa ON_1	represents	50 oranges,
,,	,, ordinate A_1N_1	,,	60 pence, their cost.
,, A_2	the abscissa ON_2	,,	40 oranges,
,,	,, ordinate A_2N_2	,,	48 pence, their cost.
,, A_3	the abscissa ON_3	,,	30 oranges,
,,	,, ordinate A_3N_3	,,	36 pence, their cost.
,, A_4	the abscissa ON_4	,,	25 oranges,
,,	,, ordinate A_4N_4	,,	30 pence, their cost.

To sum the matter up:

The ordinate of any point on the line OA_1 represents the cost in pence of the number of oranges represented by the abscissa at that point.

The figure therefore is a ready-reckoner for oranges at the given price.

The printed figure is too small for practical purposes. The student should make a figure for himself *on a larger scale.*

One-tenth of an inch to represent 1 orange
and ,, ,, ,, 1 penny

will make a diagram from which the cost (to the nearest penny) of any number of oranges can be read off.

From the same figure can be determined the number of *whole* oranges which can be bought for any number of pence.

154. The interest on £100 for one year is £2. 10s. Construct a graph which will show the interest on any sum of money up to £100 for a year.

Across the page, let 1 mm. represent 1*s*.
Up ,, ,, 1 mm. ,, £1.

Plot the point (50, 100) A_1, and join A_1 to the origin **O**.
OA_1 is the graph required.

Interest on	£90	is the abscissa	(45*s.*)	of the point	A_2,	whose ordinate is	90.
,,	£72	,,	(36*s.*)	,,	A_3,	,, ,,	72.
,,	£54	,,	(27*s.*)	,,	A_4,	,, ,,	54.
,,	£35	,,	(17·5*s.*)	,,	A_5,	,, ,,	35 (*a*).
,,	£24	,,	(12*s.*)	,,	A_6,	,, ,,	24.

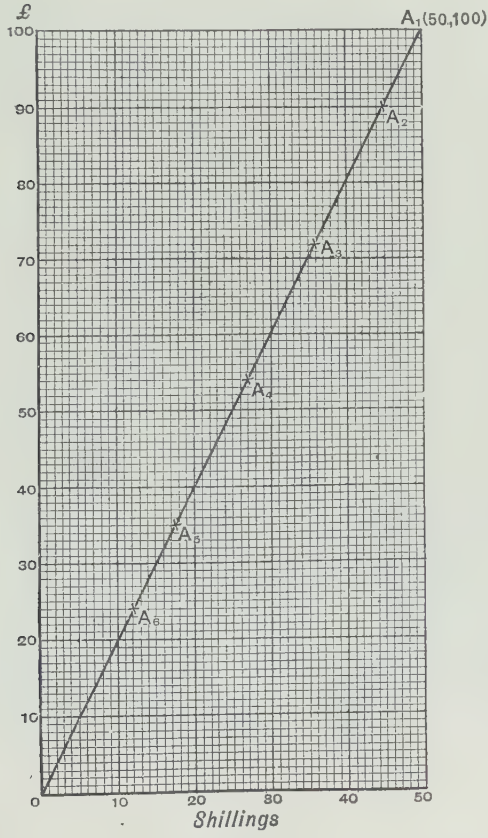


FIG. 32.

(*a*) At this point the abscissa lies half-way between 17 and 18.

This is much more easily seen if the figure is drawn on a larger scale; *e.g.* if one-tenth of an inch is used instead of 1 mm.

Conversely, from this graph we see that

if 17*s.* is the interest, £34 is the capital.
„ 40*s.* „ £80 „ and so on.

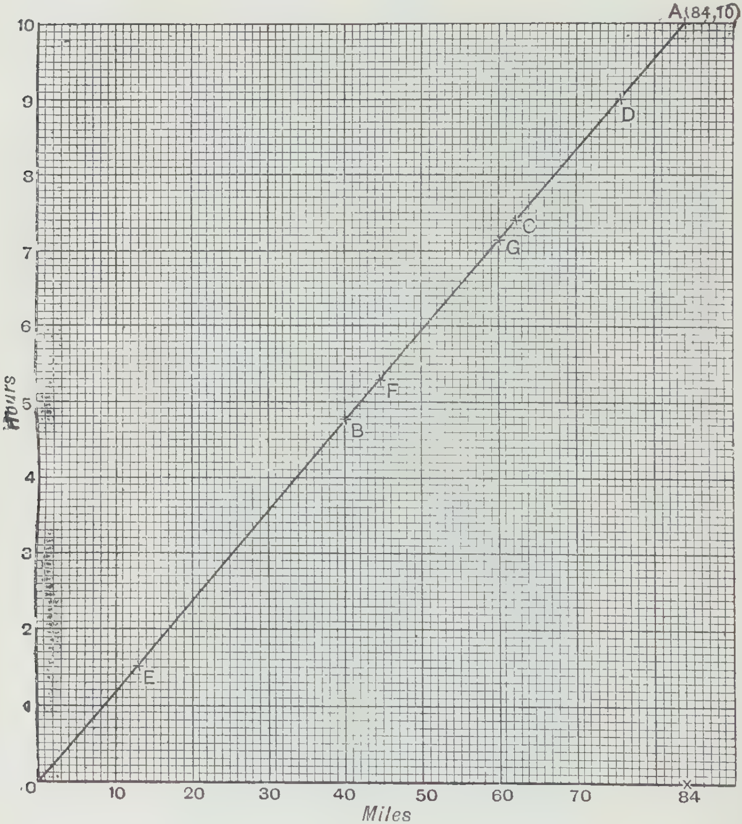


Fig. 33.

The accuracy of the graph can be tested by using the Unitary Method.

E.g. 50*s.* is the interest on £100.

∴ 1*s.* „ „ £2

∴ 17·5*s.* „ „ £(17·5 × 2), *i.e.* £35. [See (*a*).]

155. *A cyclist travels 84 miles in 10 hours at a uniform rate. Draw a graph to show his time over a given number of miles* (Fig. 33).

Across the page, take 1 mm. to represent 1 mile.
Up ,, ,, 1 cm. ,, 1 hour, *so that* 1 *mm. represents* 6 *minutes.*

Plot the point (84, 10) A, and join A to the origin O. OA is the required graph.

His time over 40 miles is the ordinate of the point B,
,, 62 ,, ,, ,, C,
,, $75\frac{1}{2}$,, ,, ,, D.

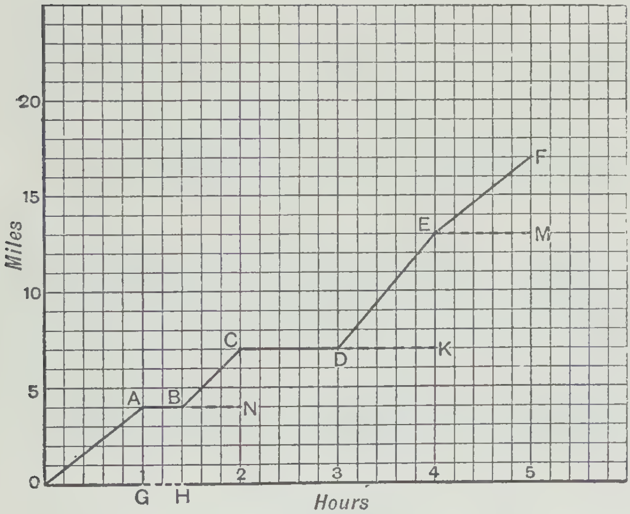


Fig. 34.

In $1\frac{1}{2}$ hours he travels $12\frac{1}{2}$ miles approx., the abscissa of point E.
,, 5 h. 18 m. ,, $44\frac{1}{2}$,, ,, ,, ,, F.
,, 7 h. 9 m. ,, 60 ,, ,, ,, ,, G.

A figure drawn on a larger scale will give the results more clearly.

Let us translate the graph in Fig. 34, which is that of a man walking or running along a road.

The position of the point A shows that he has travelled 4 miles in the first hour. Also OA is a straight line; therefore he has travelled this distance at a uniform rate.

The line AB shows that, in the time represented by GH, the distance travelled is nil; in other words, he rests for the time represented by GH, *i.e.* for two-fifths of an hour, 24 minutes.

Considering BC, we see that he travels a distance represented by NC in a time represented by BN, *i.e.* he travels 3 miles in the next 36 minutes.

CD (like AB) shows that he then rests for an hour.

In the next hour he travels a distance represented by KE, *i.e.* 6 miles.

In the last hour he travels a distance represented by MF, *i.e.* 4 miles.

We can gain further information from the graph.

In $3\frac{1}{2}$ hours he is 10 miles from the starting point.

The time taken to travel 15 miles is $4\frac{1}{2}$ hours.

Also the total distance travelled is 17 miles, and so on.

EXAMPLES XX. c.

Draw your diagrams as neatly as possible, with a well-pointed pencil.

Indicate the lines of reference, *i.e.* the axes of co-ordinates, clearly.

State definitely, on the diagram of each problem, the unit you employ.

Use as large a unit as you can without making the diagram inconveniently large.

A good plan is to use squared paper for the diagram only, writing the description and explanation on another sheet of paper.

A diagram without description and explanation is valueless.

In plotting points mark each with a small cross (×).

1. Given that 80 oranges cost 5*s.* 10*d.*, construct a graph which will show the cost of any number of oranges up to 80.

From the graph write down the cost of 60, 34 and 25 oranges respectively.

Determine, also from the graph, the number of *whole* oranges which can be bought for 2*s.* 6*d.*, 3*s.* 4*d.*, 4*s.* 3*d.* respectively.

Use one-tenth of an inch to represent one orange and one penny.

Check any *one* result by using the unitary method.

2. The price of 45 lb. of sugar is 10*s.* Construct a graph which will show the cost of any number of lb. of sugar up to 45.

Write down from the graph the cost (to the nearest penny) of 12 lb., 16 lb., 34 lb. respectively.

Also from the graph, write down how many lb. (to the nearest lb.) can be bought for 2*s.*, 3*s.* 6*d.*, 8*s.* 2*d.* respectively.

Use one-tenth of an inch across the page to represent 1 lb.
and half an inch up " " 10 pence.

Check any *one* result by using the unitary method.

3. A cyclist rides 55 miles in $4\frac{1}{2}$ hours. Draw a graph to show the distance covered in any given time. From the graph determine the distance covered, to the nearest mile, in 1 hour, $1\frac{1}{2}$ hours, 3 hrs. 36 min. respectively.

Also find, from the graph, as accurately as you can, the time he takes to ride 25 miles, 31 miles, 44 miles.

4. If £1 is worth 25 francs, construct a graph from which you can read off the value of any number of shillings up to £3 in francs. Write down from the figure the value of 32 shillings in francs and 42 francs in shillings.

5. Given that 30 inches = 76 cm., construct a graph which will give any number of inches up to 30 in centimetres. Read off the value of 24 in. in centimetres to the nearest centimetre; and the value of 50 cm. in inches to the nearest inch.

6. Given that 10 miles = 16 kilometres, make a graph from which you can read off any number of kilometres (up to 80) in miles.

From the graph read off the values of 35, 72, 27 km. in miles to the nearest mile.

7. Given that 22 lb. = 10 kilogrammes, draw a graph for converting lb. into kg. or *vice versa*, up to about 80 lb.

Read off, from the graph, the values of 35, 47, 80 lb. in kilogrammes.

8. Using one-tenth of an inch as unit, plot the points (10, 5), (30, 10), (35, 10), (40, 13), (45, 13), (50, 16). Join them by straight lines, point by point.

Now, supposing that one inch across the page (along OX) represents one hour, and that one-tenth of an inch up the page (along OY) represents one mile, translate the graph, having given that it represents a man's journey.

9. Using one-tenth of an inch as unit, plot the points (10, 15)A and (25, 0)B. Join OA, AB. With this figure take one inch across the page (along OX) to represent one hour, and one-tenth of an inch up the page (along OY) to represent one mile.

If OAB is the graph of a cyclist's journey, we see that in one hour he is 15 miles from his starting point. In $1\frac{1}{2}$ hours more, his distance from the starting point is nil. Hence AB denotes that he returns to his starting point, and represents the return journey.

How far from the starting point was he in 36 min., in $1\frac{1}{2}$ hours, in 2 hours?

10. A, starting at noon, bicycles at 10 miles an hour, and B, starting from the same place one hour later, rides at 7 miles an hour. Draw

graphs of their journeys in the same figure, and read off their distances apart (to the nearest half-mile) at 2.30 and at 3.36.

Also, from the graph, determine the times when they are 16 miles and $14\frac{1}{2}$ miles apart.

11. A, starting at noon, travels at 5 miles an hour, and B, starting from the same place 2 hours later, travels at 15 miles an hour. Draw a graph of their journeys, and from it determine when and where they are together.

Test your result by Arithmetic.

12. Take points P and Q, on squared paper, 4 in. apart up the page, and let PQ represent 40 miles. Draw lines across the page from P and Q, and along these let 1 in. represent one hour. A starts from P and travels to Q in 4 hours; B, starting at the same time from Q, travels to P in 3 hours. Draw their graphs, and find, as accurately as you can, when and where they meet.

13. A motorist does a journey of 10 miles in 25 minutes, and, without delay, does the return journey in 20 minutes. Draw his graph and read off his distance from home in 35 minutes from the start, and the times when he is 8 miles from home.

Graphs of Statistics.

156. Exhibition of Statistics by means of Graphs. The accompanying diagram gives a portion of a barometric chart, from which we can read off the height of the barometer at any hour of the dates given.

We determine the height of the barometer from the lines drawn up the page, and the date and hour from the lines drawn across the page.

Thus the height of the barometer at

4 a.m.	on the	14th	is given by	AL = 29·8 inches.
6 a.m.	„	15th	„	BM = 29·65 „
8 p.m.	„	15th	„	CN = 30·2 „
8 p.m.	„	16th	„	DR = 30·15 „

Also we see that the barometer was falling from midnight Thurs. 13th to 8 p.m. on Fri. 14th, and rising from 8 p.m. on the 14th to 8 a.m. on the 16th.

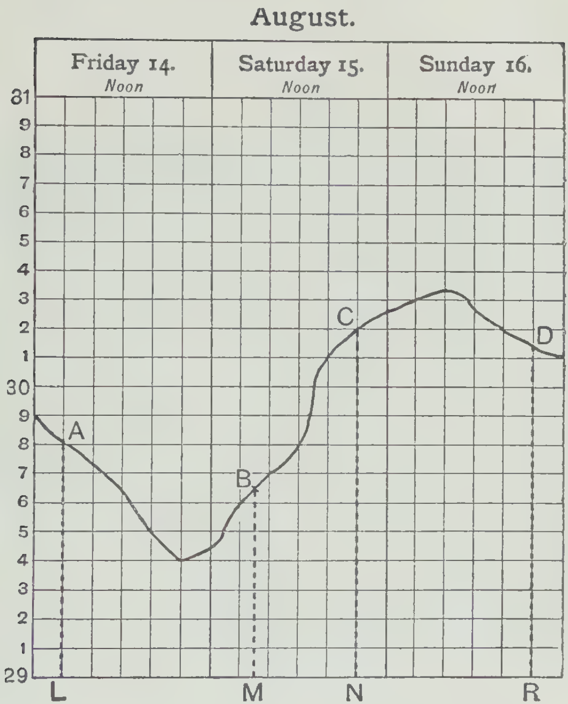


FIG. 35.

157. Construct a graph to exhibit the following:
Premiums of Life-insurance at various ages (for £100).

Age in years.	21	25	30	35	40	45	50	55	60
Premium.	£1. 16*s*.	£2	£2. 6*s*.	£2. 13*s*.	£3. 2*s*.	£3. 12*s*.	£4. 7*s*.	£5. 10*s*.	£7. 1*s*.

From the diagram estimate the premium at the ages of 32, 51 and 58.

Measuring the ages across the page, the premiums up the page, we plot the given points as shown in the diagram, the point O denoting age 20 and premium £1 (not premium £1 at age 20).

Joining the plotted points by an even curve, we have the required graph.

The dotted lines AB, CD, EF give the premiums at the ages 32, 51, 58 respectively.

They are £2. 9*s.*, £4. 11*s.*, £6. 8*s.*

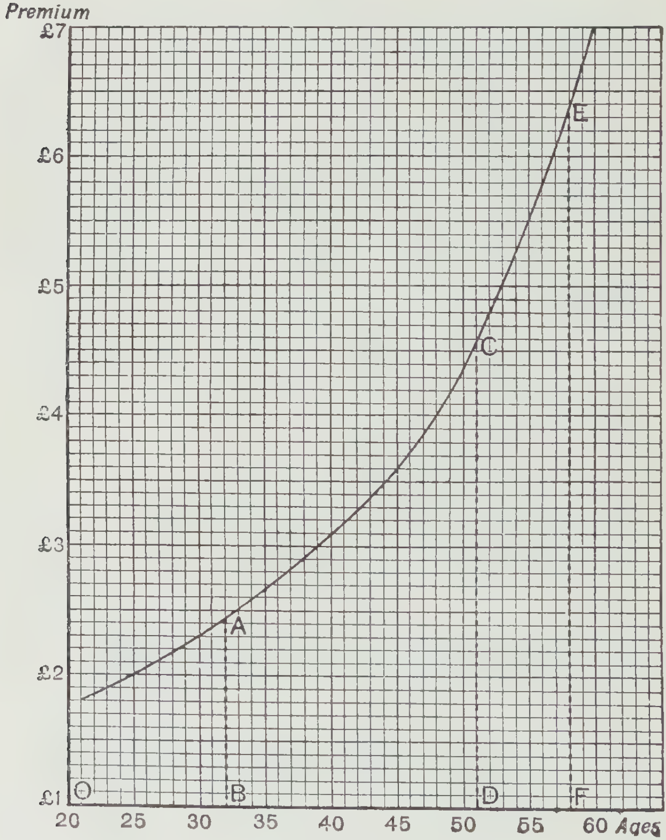


Fig. 36.

Note that in this case the point O is not the point (0, 0), the origin.

EXAMPLES XX. d.

1.

At the age of	25	30	35	40	45
A man paid in £	4·3	4·7	5·3	6	7

With suitable units, plot the points given by the above tabulation, and join them by an even curve.

Estimate what the man paid when he was 32 and 38 years old.

2. The temperature taken every two hours one day showed :

Midnight	36·2°	2 p.m.	35·2°
2 a.m.	34·6°	4 p.m.	37·5°
4 a.m.	32·8°	6 p.m.	37·6°
6 a.m.	32·6°	8 p.m.	37·2°
8 a.m.	33·3°	10 p.m.	35·9°
10 a.m.	33·9°	Midnight	34·3°
Noon	34·4°		

Draw a curve to show the variation of temperature throughout the day, and estimate the temperature at 3 a.m. and at 9 p.m.

3. The table below shows the number of cwt. of wool (in millions) used 'at home' in the years specified. Express these statistics in the form of a graph.

In the year	1897	98	99	1900	01	02	03	04	05	06
Cwt. of wool in millions.	4·4	5·1	4·7	4·5	4·8	4·4	4·0	3·9	4·2	4·6

4. The table below gives the boiling point of water in degrees Fahr. at different heights above sea-level :

Height above sea-level in feet.	0	1000	2000	3000	4000	5000	6000
Boiling point.	212°	210·1°	208·2°	206·3°	204·4°	202·5°	200·6°

Exhibit the above graphically, and read off the height above sea-level when the boiling point is 203°, and the boiling point at a height of 3700 feet.

5.

	1	2	3	4	5	6
Sq. root.	1	1·4	1·7	2	2·2	2·45

The above is a table of square roots (approximate).

Exhibit them in graphical form and estimate the square roots of 1·7, 2·5 and 3·6.

6. In experimenting with a lifting machine, the following results are obtained:

Effort in lb.	10	20	30	40
Wt. raised in lb.	60	140	250	410

Draw a graph to represent this, and estimate from it, as accurately as you can, the weights raised when the effort is (i) 14 lb., (ii) 32 lb.

7. The population of a village in 1851 was 2400, 2100 in 1861, 1900 in 1871, 1800 in 1881, 1500 in 1891, 1300 in 1901. Draw a graph to illustrate the above, and estimate the population in 1865 and in 1893.

8. The table below gives the values of British Exports (Domestic) in different years. Exhibit them as a graph.

Value of exports in millions of £.	223	213	263	226	291	280	283	290	300	329	375	426
Year. - -	1880	1885	1890	1895	1900	1901	1902	1903	1904	1905	1906	1907

Assuming the decrease between 1880 and 1885 to be steady, estimate the value of the exports in 1883, in millions of £.

9. The number of members of a certain Benefit Society on 1st Jan., at intervals of five years, was as follows:

1880— 8,973.	1895—17,665.
1885— 9,429.	1900—25,402.
1890—12,500.	1905—26,500.

Plot these points on a diagram drawn to a large scale; join each point to the next by a straight line, and from your drawing infer (i) the approximate number of members in 1898, and (ii) in what year the membership reached 16,500.

10. The table below gives the amount of the British National Debt, to the nearest million, for 12 years. Exhibit it in a graph.

Year. - -	1897	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908
Amount in millions of £. - -	645	636	635	639	706	765	798	795	797	789	774	760

XXI. RATIO.

158. If two quantities are of the same kind they have *ratio*; and the ratio of the first to the second is the quotient obtained

by dividing the first by the second, whether that quotient be integral or fractional. See Art. 78.

Thus the ratio of £6 to £5 is $\frac{6}{5}$.

If we want the ratio of 6 pence to 5 shillings, we must reduce both quantities to the same denomination.

$$5 \text{ shillings} = 60 \text{ pence}.$$

$$\therefore \text{ the ratio of 6 pence to 5 shillings} = \tfrac{6}{60} = \tfrac{1}{10}.$$

Ratio enables us to *compare* quantities. In the above case we see that 5 shillings is 10 times 6 pence.

We cannot find the ratio of things of different kinds. For instance, we cannot compare, or find the ratio of, 4 *geese* to 3 *sheep*.

On the other hand, if we know the *values* of 4 geese and 3 sheep, we can express the ratio of these *values*, for they are of the same kind, viz. money.

If a goose is worth 7*s.* 6*d.* and a sheep £2. 10*s.*,

$$\frac{\text{the value of 4 geese}}{\text{the value of 3 sheep}} = \frac{(7s.\ 6d.) \times 4}{(£2.\ 10s.) \times 3} = \frac{30s.}{150s.} = \frac{1}{5}.$$

This shows that 3 sheep are worth 5 times as much as 4 geese.

EXAMPLE. **What is the ratio of the values of 13s. and 14 francs, if 25 francs = 20s. ?**

$$1 \text{ franc} = \tfrac{20}{25} = \tfrac{4}{5}s. \quad \therefore \ 14 \text{ francs} = \tfrac{4 \times 14}{5} = \tfrac{56}{5}s.$$

$$\therefore \text{ the required ratio} = 13 \div \tfrac{56}{5} = 13 \times \tfrac{5}{56} = \tfrac{65}{56}.$$

159. In the ratio $\dfrac{a}{b}$, a is called the **antecedent** or **first term**, b the **consequent** or **second term**.

If a is greater than b, the ratio is called one of **greater inequality**.
„ less „ „ „ **less** „

If $a = b$, the ratio is said to be one of **equality**.

160. The **specific gravity** of any substance (liquid or solid) is the ratio of the weight of any volume of that substance to the weight of an equal volume of water at standard temperature.

The following may be used in working the examples below.

1 gallon of water weighs 10 lb.; 1 c. ft. of water weighs 62·3 lb.; 1 litre of water weighs 1 kilogramme.

EXAMPLE. **Find the specific gravity, correct to one decimal place, of a substance of which 1 c. in. weighs 1·4 oz.**

The specific gravity required

$$=\frac{\text{weight of 1 c. ft. of the substance}}{\text{weight of 1 c. ft. of water}}=\frac{1728\times 1{\cdot}4\text{ oz.}}{62{\cdot}3\times 16\text{ oz.}}$$

$$=\frac{1728\times 14}{623\times 16}=\frac{1728\times 2}{89\times 16}=\frac{108\times 2}{89}=\frac{216}{89}.$$

```
89 ) 216 ( 2·4
     178
     ---
      380
      356
      ---
       24
```

$\therefore$ the S.G. required $=2{\cdot}4$.

EXAMPLES XXI. a. (*Oral.*)

Express the following ratios in their simplest forms :

1. $\frac{3}{9}$. **2.** $\frac{6}{8}$. **3.** $\frac{15}{35}$. **4.** 7 shillings to 21 shillings.

5. 8*d.* to 1*s.* **6.** $1\frac{1}{2}:2\frac{1}{2}$. **7.** $\frac{1}{4}:1\frac{1}{4}$.

8. 9*d.* to 1*s.* 9*d.* **9.** 3 qr. to 1 cwt. **10.** 6*s.* 8*d.* to £1. 6*s.* 8*d.*

11. 3*s.* 4*d.* to £1. **12.** 2·5 : 3·5. **13.** ·4 : ·04.

14. 1 : 1·25. **15.** ·125 : 1. **16.** 3·125 : 1·125.

17. 73 days to a year. **18.** 3 yds. 1 ft. to 4 yds. **19.** $\frac{3}{4}:\frac{1}{4}$.

20. 7*s.* to £2. 10*s.* **21.** $\frac{1}{2}:\frac{1}{4}$. **22.** $\frac{1}{3}:\frac{3}{4}$.

23. $1\frac{1}{2}:1\frac{3}{4}$. **24.** $2\frac{1}{5}:2\frac{3}{4}$. **25.** $9\frac{1}{10}:1\frac{3}{10}$.

EXAMPLES XXI. b.

Express the following ratios in their simplest forms (give the result in fractional form) :

1. $2\frac{1}{2}:4$. **2.** $1\frac{1}{4}:5$. **3.** $3:4\frac{3}{4}$. **4.** $1\frac{1}{2}:1\frac{1}{4}$.

5. $\frac{5}{9}:\frac{7}{11}$. **6.** $8\frac{1}{3}:1\frac{2}{3}$. **7.** 3*s.* 4*d.* : 1*s.* 8*d.*

8. 1*d.* : £1. **9.** 1 ft. 3 in. : 2 ft. 6 in. **10.** $\frac{1}{2}$ cwt. : 104 lb.

11. 11 cm. : 1 m. **12.** 1 cm. 2 mm. : 3 cm. 6 mm.

13. Given that 1 shilling = 1·25 Swiss francs, find the ratio of the value of 10 shillings to 10 francs.

14. If a goose is worth 6*s.* 9*d.* and a turkey 15*s.* 4*d.*, find the ratio of the values of 4 geese and 3 turkeys.

15. If $a=2b$ and $b=5c$, find the ratio of a to c.

16. If $a=\frac{1}{3}b$ and $2b=c$, find the ratio of a to c.

17. Find the ratio of $\frac{2}{3}$ of £1 to $\frac{3}{4}$ of £2. 4*s*.

18. A runs a mile in 5 minutes and B runs 440 yds. in 62 secs.; find the ratio of A's speed to B's.

[*In Examples* 19–24 *express the result so that the consequent* (*the second term*) *of the ratio is unity.*]

19. The areas of three circles and their radii are increased, and the results are given in the table below. Find the ratio in each case of the area to the square of the radius, correct to 2 decimal places.

Radius of a circle in feet.	1	2	3
Area in sq. feet. - -	3·14	12·57	28·28

20. A man experiments with a weight-lifting machine, and his results are tabulated below. Find, correct to one decimal place, the ratio of the weight lifted to the effort used in each case.

Effort in lb. - -	7	9	11	13
Weight lifted in lb.	30	40	51	63

21. If a gallon of milk weighs 10·3 lb., what is its specific gravity?

22. A cubic inch of silver weighs 6·07 oz. (av.); what is its specific gravity correct to one decimal place?

23. If a cubic foot of mercury weighs 846·6 lb., what is its specific gravity to one decimal place?

24. A cubic foot of honey weighs 90·335 lb. Find its specific gravity.

25. What is the weight to the nearest lb. of a cubic foot of chalk whose specific gravity is 2·45?

26. What is the weight to the nearest tenth of an oz. of a cubic inch of granite whose specific gravity is 2·5?

Two equal Ratios form a Proportion.

161. Thus $\frac{3}{4}=\frac{6}{8}$ forms a proportion, and this is expressed by saying 3 is to 4 as 6 is to 8.

It is often written thus: $3:4::6:8$,

or $3:4=6:8$.

For working purposes it is best to use the fractional shape, $\frac{3}{4}=\frac{6}{8}$.

In general terms, if a, b, c, d are in proportion,

$$\frac{a}{b}=\frac{c}{d}.$$

a and d are called the **extremes**, b and c the **means**.

162. The following are very important, and the student should be thoroughly familiar with them.

If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$; or, the product of the extremes = the product of the means.

This is at once seen by multiplying each of the equal ratios by bd.

If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$.

This is proved by dividing unity by the equal fractions $\frac{a}{b}, \frac{c}{d}$.

If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$.

Proof. Multiply each of the equal ratios by $\frac{b}{c}$, and we have $\frac{a}{c}=\frac{b}{d}$.

163. The **inverse** or **reciprocal** of 3 is $\frac{1}{3}$.

The inverse or reciprocal of x is $\frac{1}{x}$.

" " " $\frac{1}{5}$ is 5, *i.e.* $1 \div \frac{1}{5}$.

" " " $\frac{3}{4}$ is $\frac{4}{3}$, *i.e.* $1 \div \frac{3}{4}$, and so on.

Definition. If a, b, c, d are **inversely or reciprocally proportional,**

$$\frac{a}{b}=\frac{\frac{1}{c}}{\frac{1}{d}} \text{ or } \frac{a}{b}=\frac{d}{c}.$$

In this case, a and b are **directly** proportional to **d and c** and **inversely** proportional to **c and d.**

We often have to deal with inverse proportion.

Let us take the case of a number of men doing a certain piece of work.

If 1 man does a piece of work in 9 days,

2 men do the work in $\frac{9}{2}$ days.

3 " " " $\frac{9}{3}$ "

6 " " " $\frac{9}{6}$ "

i.e. the number of men required to do a piece of work is **inversely proportional** to the number of days they take to do it.

164. *Definition.* If $\frac{a}{b}=\frac{b}{c}$ or $a:b=b:c$,

b is said to be a *mean proportional* between a and c.

EXAMPLE 1. **Find the mean proportional between 3 and 27.**

Let x be the mean proportional.

Then, by definition, $$\frac{3}{x}=\frac{x}{27}.$$

Multiply these equal quantities by $27x$.

$$\frac{3\times 27x}{x}=\frac{x\times 27x}{27},\quad i.e.\ 81=x^2;$$

$$\therefore\ x=\sqrt{81}=9.$$

Definition. If $\frac{a}{b}=\frac{b}{c}$ or $a:b=b:c$,

c is said to be a **third proportional** to a and b.

EXAMPLE 2. **Find the third proportional to 108 and 12.**

Let x be the third proportional.

By definition, $$\frac{108}{12}=\frac{12}{x}.$$

Multiply these equal quantities by $12x$.

$$\frac{108\times 12x}{12}=\frac{12\times 12x}{x},\quad i.e.\ 108x=144;$$

$$\therefore\ x=\frac{144}{108}=\frac{4}{3}.$$

165. The idea of proportion is illustrated in the accompanying figure.

Draw any number of ordinates A_1N_1, A_2N_2, etc., from points on the straight line OA_1.

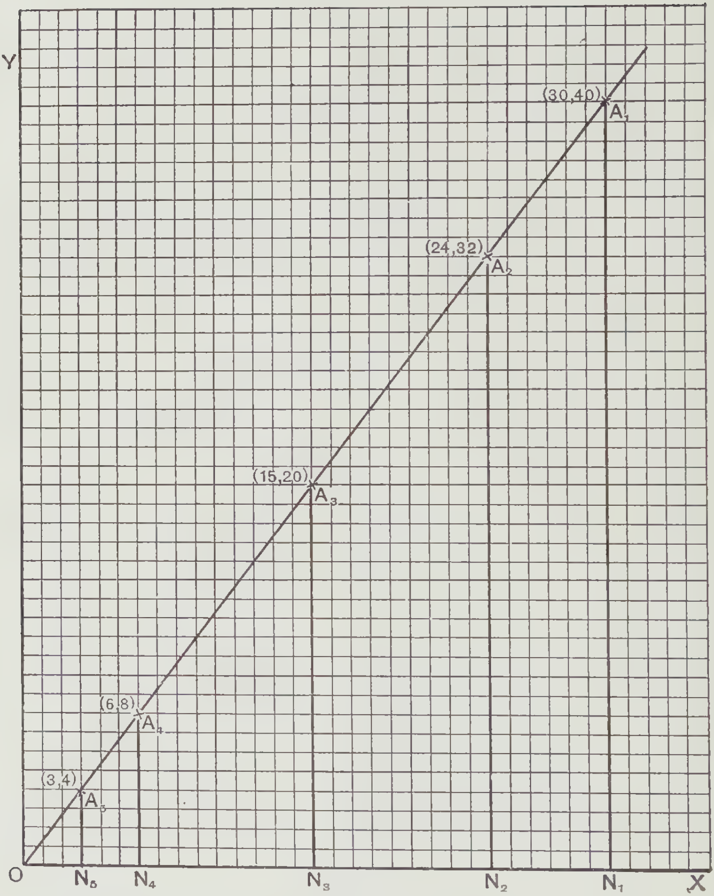


FIG. 37.

By inspection or measurement, we see that in every case

$$\frac{\text{the ordinate}}{\text{the abscissa}} = \frac{4}{3}.$$

$$\frac{A_1N_1}{ON_1} = \frac{40}{30} = \frac{4}{3}. \qquad \frac{A_2N_2}{ON_2} = \frac{32}{24} = \frac{4}{3}.$$

$\therefore \dfrac{A_1N_1}{ON_1} = \dfrac{A_2N_2}{ON_2}$ or $\dfrac{A_1N_1}{A_2N_2} = \dfrac{ON_1}{ON_2}$, and so on for other points.

This is expressed by saying that ***the ordinate is proportional to the abscissa for any point on a straight line which passes through the origin.***

166. Suppose we know that 12 articles cost 16 pence.

If we *double* the number of articles, we *double* the cost,

„ *halve* „ „ *halve* „

„ *treble* „ „ *treble* „

and so on.

$$\therefore \frac{12 \text{ articles}}{24 \text{ articles}}=\frac{16 \text{ pence}}{32 \text{ pence}}, \text{ or } \frac{12 \text{ articles}}{24 \text{ articles}}=\frac{\text{the cost of 12 articles}}{\text{the cost of 24 articles}}.$$

In the same way, $\dfrac{12 \text{ articles}}{36 \text{ articles}}=\dfrac{\text{the cost of 12 articles}}{\text{the cost of 36 articles}}$.

Or, in words, the number of articles is proportional to their cost.

167. *In an examination paper, maximum* 80, *the marks of six candidates are respectively:* 31, 36, 48, 59, 70, 76. *Draw a graph to determine their marks when the maximum is raised to* 100.

(The figure is left for the student to draw. Paper ruled in millimetres gives a diagram of convenient size.)

In each case the number of marks is proportional to the maximum.

Along Ox mark off 100 units and along Oy 80 units, and plot the point (100, 80). Join OA.

From the graph we see that

When $y=31$	36	48	59	70	76
$x=39$	45	60	74	88	95

to the nearest integer; and these abscissae are the required marks.

EXAMPLES XXI. c.

[*Check your result in each case.*]

Find the value of x in each of the following cases:

1. $\frac{x}{3}=\frac{4}{9}$. **2.** $\frac{5}{x}=\frac{10}{3}$. **3.** $\frac{7}{8}=\frac{x}{16}$. **4.** $\frac{1}{5}=\frac{2}{x}$.

5. $\frac{x}{5}=\frac{1\frac{1}{3}}{2\frac{2}{3}}$. **6.** $\frac{x}{0\cdot5}=\frac{1}{2}$. **7.** $\frac{x}{6}=\frac{3\frac{3}{4}}{1\frac{1}{5}}$.

Find the value of x in each of the following cases :

8. $\dfrac{x}{a}=\dfrac{\frac{1}{b}}{\frac{1}{c}}$. **9.** $\dfrac{\frac{1}{x}}{\frac{1}{a}}=\dfrac{\frac{1}{b}}{\frac{1}{c}}$. **10.** $\dfrac{3x}{5}=\dfrac{6}{25}$.

Find a fourth proportional to

11. 6, 5 and 12. **12.** 9, 5 and 3. **13.** $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{2}$.

Find a third proportional to

14. 6 and 9. **15.** 3 and $\sqrt{21}$. **16.** 5 and 13.

Find a mean proportional to

17. 3 and 27. **18.** 125 and 5. **19.** $\frac{1}{7}$ and $\frac{7}{9}$. **20.** $\frac{7}{11}$ and $\frac{9}{308}$.

Find the value of x in each of the following cases (21–30) :

21. $x : 5 :: 6 : 10$. **22.** $8 : x = 1 : 4$. **23.** $x : \frac{1}{2} :: 1 : 2$.

24. $\frac{3}{4} : x = 5 : 8$. **25.** $8 : 9 = x : 3$. **26.** $11 : 7 = 33 : x$.

27. $\dfrac{1}{x} : \dfrac{1}{2} = 3 : 4$. **28.** $3 : \dfrac{1}{x} = 6 : \dfrac{1}{4}$. **29.** $\dfrac{1}{x} : \dfrac{1}{3} = \dfrac{1}{2} : \dfrac{1}{5}$. **30.** $\dfrac{1}{7} : x = \dfrac{1}{14} : 9$.

31. If $7x=4y$, what is the ratio of x to y?

32. If 7 apples cost the same as 4 oranges, what is the ratio of the cost of an apple to the cost of an orange?

33. If $3x+y=5y$, find the ratio of x to y.

34. If $3x-y=5y$, ,, ,, ,,

35. If $2y+x=8x$, ,, ,, ,,

36. If $3x+y=5y+x$, ,, ,, ,,

37. If $5x-y=7y-x$, ,, ,, ,,

38. If $3x-4y=2x+4y$, ,, ,,

39. If $7x-4y=9x-8y$, ,, ,,

40. If $8x-3y=x+11y$, ,, ,,

If x and y are reciprocally proportional to a and b, find the value of x :

41. When $y=4$, $a=2$ and $b=3$. **42.** When $y=3a$ and $b=1$.

43. ,, $y=6$ and $a=4b$. **44.** ,, $y=\dfrac{a}{5}$ and $b=10$.

THERMOMETERS.

	Freezing Point.	Boiling Point.
Centigrade.	0°	100°
Réaumur.	0°	80°
Fahrenheit.	32°	212°

45. Draw a graph to convert temperatures in Centigra equivalents in the Réaumur scale and *vice versa.*

Read off, to the nearest degree, 44° and 75° Centig Réaumur scale and 15°, 35° Réaumur in the Centigrade sc...

46. Draw a graph to convert temperatures in Centigrade to their equivalents in the Fahrenheit scale and *vice versa.*

Read off, to the nearest degree, 42°, 79° Centigrade in the Fahrenheit scale and 64°, 102° Fahrenheit in the Centigrade scale.

47. Draw a graph to convert temperatures in Réaumur to their equivalents in the Fahrenheit scale and *vice versa.*

Read off, to the nearest degree, 36°, 47° Réaumur in the Fahrenheit scale and 36°, 190° Fahrenheit in the Réaumur scale.

XXII. PROPORTIONAL PARTS AND PARTNERSHIPS.

168. We deal here with quantities which have to be divided into a number of parts in a particular manner, generally in a given proportion.

Example 1. **Divide the number 1001 into two parts which are in the ratio of 3 to 8.**

The ratio $3x : 8x$ is the same as the ratio $3 : 8$.

Hence we may suppose $3x$ and $8x$ to be the required parts.

Then
$$3x+8x=1001,$$
$$i.e.\ 11x=1001.$$
$$\therefore\ x=91.$$
$$\therefore \text{ the required parts are } 3\times 91 \text{ and } 8\times 91,$$
$$i.e.\ 273 \text{ and } 728.$$
[*Check.* $273+728=1001$.]

Example 2. **Divide £973 into four parts which are proportional to $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$.**

Multiply each of the fractions by 120, the L.C.M. of 2, 3, 5, 8. The products are respectively 60, 40, 24, 15; and we have to divide the sum of money proportionally to these numbers.

Hence we may take $60x$, $40x$, $24x$, $15x$ to be the required parts.
$$60x+40x+24x+15x=973,$$
$$i.e.\ 139x=973.\ \therefore\ x=7.$$
$$\therefore\ £60\times 7,\ £40\times 7,\ £24\times 7,\ £15\times 7,$$
or £420, £280, £168, £105 are the required parts.

[*Check.* $420+280+168+105=973$.]

EXAMPLE 3. **Divide £4193 among four people A, B, C and D, so that A's share : B's share = 3 : 5 ; B's share : C's share = 2 : 7 ; and C's share : D's share = 9 : 4.**

Let a, b, c, d denote the respective shares in £.

Then $\frac{b}{5}=\frac{a}{3}$. $\therefore\ b=\frac{5a}{3}$ ....(1)

$\frac{c}{7}=\frac{b}{2}$. $\therefore\ c=\frac{7b}{2}=\frac{35a}{6}$ from (1) ....(2)

$\frac{d}{4}=\frac{c}{9}$. $\therefore\ d=\frac{4c}{9}=\frac{140a}{54}=\frac{70a}{27}$ from (2).

But the sum of the shares = £4193.

$$\therefore\ a+\frac{5a}{3}+\frac{35a}{6}+\frac{70a}{27}=4193,$$

$$i.e.\ \frac{54a+90a+315a+140a}{54}=4193.$$

$$\frac{599a}{54}=4193.$$

$$\therefore\ a=4193\times\frac{54}{599}=7\times54=378.$$

And
$$b=\tfrac{5}{3}\times378=5\times126=630,$$
$$c=\tfrac{35}{6}\times378=35\times63=315\times7=2205,$$
$$d=\tfrac{70}{27}\times378=\tfrac{70}{3}\times42=70\times14=980.$$

$\therefore$ the required shares are £378, £630, £2205, £980.

[*Check.* $378+630+2205+980=4193$.]

EXAMPLE 4. **Two men invest £133 and £327 in a business, and make a profit of £92. How should this profit be divided between them?**

We have to divide £92 into two parts which are in the ratio of 133 to 327. $\therefore$ we may take $133x$ and $327x$ to be the parts.

$$133x+327x=92,\quad i.e.\ 460x=92,$$

and
$$x=\tfrac{92}{460}=\tfrac{1}{5}.$$

$\therefore$ the required parts are £$\frac{133}{5}$ and £$\frac{327}{5}$;

i.e. £26. 12*s.* and £65. 8*s.*

[*Check.* £26. 12*s.* + £65. 8*s.* = £92]

EXAMPLE 5. **Two partners in a speculation contribute respectively £3000 for 9 months and £4800 for 6 months, and make a profit of £1920. How should the profit be divided between them?**

£3000 for 9 months is equivalent to £27000 for 1 month.
£4800 ,, 6 ,, ,, £28800 ,,

Hence we must divide the profit in the ratio of 27000 to 28800.

The working is left to the student.

N.B.—$\frac{27000}{28800}=\frac{270}{288}=\frac{30}{32}=\frac{15}{16}$.

EXAMPLES XXII. a.

[*Show up a check in each case.*]

Divide the following quantities into parts as instructed below:

1. 651 into two parts in the ratio of 3 to 4.
2. 1136 " " " 11 " 5.
3. 1356 " three parts proportional to 3, 4 and 5.
4. 925 " two parts in the ratio of $\frac{1}{2}$ to $\frac{1}{3}$.
5. £12 " four parts proportional to 1, 2, 3 and 4.
6. £65 " two parts in the ratio of $\frac{1}{4}$ to $\frac{1}{9}$.
7. £35. 3*s.* 6*d.* into two parts in the ratio of 5 to 16.
8. £58. 10*s.* into three parts proportional to $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{7}$.

9. Divide £3. 13*s.* 4*d.* between three people in the proportion of 1, 3 and 7.

10. A worked for 5 days, B for 7 and C for $3\frac{1}{2}$, and their wages amounted to £3. 2*s.* Find how much each man received.

11. Divide £4. 13*s.* 6*d.* between A, B and C so that A receives twice as much as B and three times as much as C.

12. Divide £9. 2*s.* 1*d.* between A, B and C so that A's share is $\frac{2}{3}$ of B's share and B's share is $\frac{2}{3}$ of C's share.

13. Divide £4422 into three shares in the proportion of $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{8}$.

14. Three men contribute £3400, £7200, £1500 respectively to a business, and make a profit of £786. 10*s.* Find how this profit should be divided amongst them.

15. Gunpowder is made of saltpetre, charcoal, and sulphur in the proportion of 15 lb. of saltpetre to 3 lb. of charcoal and 2 lb. of sulphur. How much sulphur is wanted for one cwt. of gunpowder?

16. A sum of money was divided between A, B and C in the proportion of 7, 5 and 9, and £21. 4*s.* 8*d.* was the amount of A's share. What was the sum of money?

17. A and B receive legacies in the ratio of $\frac{1}{2}$ to $\frac{1}{5}$. If A received £312. 1*s.* 3*d.*, how much did B receive?

18. The profits of a business were divided between three partners A, B and C in the proportion of 6, 1 and 9. If A's and B's shares together amounted to £431. 10*s.* 5*d.*, what was C's share?

19. A man paid £21. 3*s.* 6*d.* to three workmen, A, B and C, for a job. If A worked all the time, B worked $\frac{1}{2}$ of the time and C $\frac{1}{3}$ of the time, how much did each receive?

20. A man divided his property, valued at £16080, into two parts in the ratio of 7 to 5. He left the first part in equal shares to 3 daughters and the other part in equal shares to 4 sons. How much did each daughter and son receive?

A man leaves a sum of £2575 to 2 sons and 3 daughters, but daughter is to receive twice as much as each son. Find the e of a daughter's share.

2. Divide £13. 5*s.* 10*d.* between A and B so that $\frac{2}{3}$ of A's share may be equal to $\frac{4}{5}$ of B's share.

23. A sum of £8. 9*s.* is divided between 4 men, 7 women and 9 children so that each man receives twice as much as each woman, and each woman twice as much as a child. How much does each child receive?

24. Divide £723. 10*s.* 6*d.* among three people, A, B and C, so that A's share : B's share = 4 : 5, and B's share : C's share = 2 : 3.

25. A sum of money is divided between three men, A, B and C, so that A has three times as much as B and B has one half of what C has. If C's share is £72. 4*s.* 6*d.*, find the sum of money divided amongst them.

26. Three boys, A, B and C, gain 36, 81 and 92 marks respectively in an examination, and the examiner alters the marks proportionally so that C gets 100. Find, to the nearest integer, the altered marks of A and B.

27. Two men, A and B, enter into partnership, A contributing £3000 for 9 months and B £4000 for 8 months. How should a profit of £767 be divided between them?

28. Three brothers each invest £500, and two sisters each invest £400 in a company which brings them in a total of £103. 10*s.* per annum. Find how much each brother and each sister gets per annum.

29. A man makes a will by which he leaves a sum of £16500 to his three sons A, B, C, in the proportion of 4, 5, 6; but A dies, and then the man increases the legacies of B and C by $\frac{3}{5}$ of their former amounts. Find by how much the sum to be disposed of has been increased in the interval.

30. A and B invest money in a business in the proportion of $\frac{1}{2}$ and $\frac{1}{3}$. At the end of 4 months they withdraw $\frac{1}{2}$ and $\frac{1}{3}$ of their respective capitals. At the end of a year, the profits being £275. 10*s.* 8*d.*, what should each receive?

Mixtures.

169. EXAMPLE 1. **If a man mixes 12 lb. of tea at 1s. 4d. a lb. with 24 lb. at 1s. 9d., at what price must he sell the mixture so as to gain 5s. profit on the whole?**

12 lb. of tea at 1*s.* 4*d.* a lb. are worth 16*s.*
24 ,, 1*s.* 9*d.* ,, ,, 42*s.*
∴ 36 ,, cost the man 58*s.*
∴ he sells 36 lb. for 63*s.*
∴ ,, 1 lb. ,, $\frac{63}{36}=\frac{7}{4}=1\frac{3}{4}s.=1s.\ 9d.$

Alternative Method. If x shillings per lb. is the selling price,

$36x$ shillings = cost + 5*s*.

$$\therefore\ 36x = 12 \times 1\tfrac{1}{3} + 24 \times 1\tfrac{3}{4} + 5$$
$$= 16 + 42 + 5$$
$$= 63.$$
$$\therefore\ x = \tfrac{63}{36} = \tfrac{7}{4} = 1\tfrac{3}{4}.$$

The selling price = 1*s*. 9*d*.

EXAMPLE 2. **Coffee at 1s. 3d. a lb. is mixed with coffee at 1s. 10d. a lb. In what proportion are the two kinds mixed, if 1 lb. of the mixture is worth 1s. 6d.?**

Suppose x lb. of the first kind are mixed with y lb. of the second.

The value of x lb. of the first and y lb. of the second kind

$$= (15x + 22y) \text{ pence.}$$

But the value of x lb. of the first and y lb. of the second kind also

$$= 18(x + y) \text{ pence,}$$

for the $(x + y)$ lb. are worth 1*s*. 6*d*. a lb.

$$\therefore\ 18(x + y) = 15x + 22y,$$
$$i.e.\ 18x + 18y = 15x + 22y.$$

Subtracting

$15x$ from each of these equal quantities, we have $3x + 18y = 22y$.

$18y$,, ,, ,, ,, $3x = 4y$.

Dividing ,, ,, ,, by $3y$,, $\dfrac{x}{y} = \dfrac{4}{3}$.

$\therefore$ the mixture is in the proportion of 4 lb. of the first to 3 lb. of the second.

Check.	4 lb.	of the first kind	are worth	60	pence.
	3	,, second	,,	66	,,
	$\therefore$ 7	,, mixture	,,	126	,,
	$\therefore$ 1	,, ,,	is worth	18	,,

Alternative Method.

On every lb. of the first kind in the mixture there is a *gain* of 3 pence.

,, ,, second ,, ,, *loss* of 4 ,,

$\therefore$ to make gains balance losses, 4 lb. of the first must be mixed with 3 lb. of the second kind.

EXAMPLE 3. **In what proportion must a man mix beer at 1s. 1d. per gallon with beer at 1s. 4d. a gallon so as to gain $\frac{1}{8}$ of his outlay by selling at 1s. 5d. per gallon?**

Suppose he mixes x gallons of the first kind with y gallons of the second.

$(x + y)$ gallons cost him $(13x + 16y)$ pence.

$\therefore$ he sells these for $(13x + 16y) \times \frac{9}{8}$ pence.

$$\therefore\ (x + y) \times 17 = (13x + 16y) \times \tfrac{9}{8}.$$

Multiplying these equal quantities by 8, we have

$$(x + y) \times 136 = (13x + 16y) \times 9,$$
$$i.e.\ 136x + 136y = 117x + 144y.$$

Subtracting

$117x$ from each of these equal quantities, we have $19x+136y=144y$.

$136y$ " " " " $19x=8y$.

Dividing " " " by $19y$ " $\frac{x}{y}=\frac{8}{19}$.

∴ he mixes 8 gallons of the first with 19 gallons of the second kind.

EXAMPLES XXII. b.

1. A man mixed 16 lb. of sugar at $2\frac{1}{4}d.$ per lb. with 14 lb. at $3\frac{1}{2}d.$ per lb. If he made a total profit of $2s.$ $11d.$ on selling, what price per lb. did he sell at?

2. A wine-merchant bought 3 doz. of wine at $21s.$ a dozen, 7 doz. at $23s.$ and 6 doz. at $27s.$ If he sold them at an average price of $28s.$ a dozen, what profit did he make?

3. A tea-dealer mixes 16 lb. of tea at $2s.$ $3d.$ a lb. with 18 lb. at $3s.$ $2d.$ At what price must he sell the mixture in order to make a total profit of $9s.$?

4. A man bought 15 gallons of milk at $10d.$ a gallon, and after adding 4 gallons of water sold the mixture at $1s.$ a gallon. Find his total profit.

5. A tea-dealer mixed equal quantities of tea at $1s.$ $9d.$, $2s.$ $1d.$ and $2s.$ $5d.$ per lb., and sold the mixture at $2s.$ $6d.$ per lb. What profit per lb. did he make on the average?

6. A man bought 7 doz. eggs at $10d.$, 8 doz. at $9d.$ and 5 doz. at $13d.$ a dozen. If he sold the lot for $15s.$ $2d.$, find his average loss per dozen.

7. A man bought 24 gallons of milk at $10d.$ a gallon, and, after adding water to it, sold the mixture at the same price per gallon and made a profit of $\frac{1}{4}$ of his outlay. How much water did he add?

8. In what ratio must tea at $1s.$ $9d.$ a lb. be mixed with tea at $2s.$ $4d.$ a lb. in order that the mixture may be worth $2s.$ $2d.$ a lb.?

9. Assuming that a florin would be worth $2s.$ if made of pure silver, what proportion of valueless alloy does it contain if the coin is only worth $1s.$ $4d.$?

10. How many lb. of sugar at $3\frac{1}{2}d.$ per lb. and how many lb. at $5\frac{1}{2}d.$ per lb. will together form a mixture of 24 lb. worth $4d.$ a lb.?

11. 15 lb. of tobacco at $5s.$ $4d.$ per lb. are mixed with 25 lb. of a more expensive kind, and the mixture is worth $5s.$ $9d.$ per lb. What is the value of the more expensive kind?

12. In what proportion must spirits at $14s.$ a gallon and $18s.$ a gallon be mixed if the mixture is sold so as to make a profit of $1s.$ a gallon when 100 gallons are sold for £90?

13. One cask contains 15 gallons of water, two others contain 60 and 85 gallons of milk respectively, all the casks being full. If the contents of the casks are mixed and the casks refilled, find how much water each cask will contain, expressing the answer in pints.

XXIII. REVISION PAPERS.

(1) *Show up all the working, including the check.*
(2) *Avoid side sums.*
(3) *Give explanations of the steps.*
(4) *Use factors if possible.*
(5) *Revise your work before proceeding to the next example.*

XXIII. a.

1. A square room is 3 m. 5 cm. high and contains 47 c.m. 1700 c. cm. of air. Find the length of its side to the nearest centimetre.

Give a rough check by finding the cubic content of a room of height 3 m. and side the nearest whole number of metres to your answer.

2. If the wages of 20 men working 9 hours a day for 8 days be £39, how much must be paid to 12 men working 8 hours a day for 16 days?

3. Find the value of £3·3125 + 14·832*s.* + 2·266*d.*

4. How long will it take to walk round a square field of $5\frac{5}{8}$ acres, at the rate of 3 miles in 32 minutes?

5. In an examination paper, maximum 50, the marks of four candidates are respectively 15, 24, 37, 41. From a graph determine their marks when the maximum is raised to 80. Check one result by Arithmetic.

6. How must two kinds of tea be mixed, one at 2*s.* 4*d.* a lb. and the other at 2*s.* 11*d.* a lb., to make the mixture worth 2*s.* 8*d.* a lb.?

7. A and B are two stations. One train leaving A at noon reaches B at half-past two, and another leaving B at noon reaches A at a quarter to three. At what time, to the nearest minute, do they meet?

XXIII. b.

1. A pole is divided into two parts, one of which is five-eighths of the other. What fraction is each part of the whole pole, and what fraction is the difference of the parts of the whole pole? If the difference in the lengths of the parts be 3 ft. $4\frac{1}{2}$ in., what is the length of the whole pole?

2. The external dimensions of a closed wooden box are 7, 6 and 5 centimetres; how many sq. cm. of wood 3 mm. thick will be required for its construction?

3. A contractor has to excavate 3150 c. yds. of earth in 50 days, and employs 60 men; but at the end of 35 days they have only done 1800 c. yds. How many extra men must be put on to complete his contract in time?

4. Using no more figures than are necessary, find the value of (correct to three decimal places):

(1) $3·04625 \times 0·0468$; (2) $0·6712537 \div 0·03412$.

5. What sum of money bears the same ratio to £5 that 13*s.* $11\frac{1}{4}$*d.* does to £3. 14*s.* 4*d.*?

6. Three boys in an examination paper, maximum 60, gain respectively 35, 47 and 52 marks. Find graphically their marks when the maximum is raised to 80.

7. Three men, A, B, C, join in business. A puts in £500 for 3 months, B £650 for 8 months and C £300 for 11 months. They gain £420. Find the share of each.

XXIII. c.

1. The area of a circle is 3·14159 times the square on its radius. Find to the nearest sq. dm. the area of a circle of radius 3·89 metres. Give a rough check of the work.

2. Find the depth of water in a tank which is 2·25 m. long, 1·34 m. wide and contains 1809 litres of water.

3. If a number is divided by 42 by means of successive factors 3, 7 and 2, the remainders are respectively 1, 5 and 1. Find the complete remainder. Give reasons for your result.

4. If 6 men can do a piece of work in 30 days of 9 hours each, how many men can do ten times as much in 25 days of 8 hours each?

5. Find the rent of 57 ac. 2 roods 8 poles at £10. 12*s.* 6*d.* per acre.

6. A clock which is right at 9 a.m. indicates 4 minutes past 10 when it should indicate 10 a.m. Assuming that it gains uniformly, find the true time when the clock indicates 5 p.m. on the same day.

7. A man's income is diminished by £150, but the income-tax being raised from 1*s.* to 1*s.* 2*d.* in the £, he pays the same amount as before. What was his income? [See note on page 110].

XXIII. d.

1. Find, in tons and decimal of a ton, the weight of water in a cistern 18 ft. 8 in. long, 18 ft. 4 in. broad and 6 ft. 9 in. deep, supposing a cubic foot of water to weigh 1000 oz.

2. Find the cost of 30 cwt. 2 qrs. 14 lb. at £1. 7*s.* $8\frac{1}{4}$*d.* per cwt. Give the answer to the nearest penny.

3. A watch set right at noon on March 1st loses $31\frac{1}{2}$ seconds a day; what will be the time by it at noon on March 31st?

4. Two sets of telegraph wires are carried on opposite sides of a railway on posts whose distances apart are 275 ft. in one case, and 135 ft. in the other. An engine starts from a point where two posts are exactly opposite one another, runs an exact number of quarter miles and stops at a point where two posts are again exactly opposite one another. Find the least distance which the engine can have travelled.

5. Find a man's gross rental if, after paying an income-tax of 8*d.* in the £ and a rate of 2*s.* 6*d.* in the £ on two-thirds of his rental, he has a net income of £398. 16*s.* 6*d.*

6. A rectangular grass lawn-tennis court, whose sides are in the ratio 17 : 8, has an area of 1224 sq. yds. Find the cost, to the nearest shilling, of surrounding it with a gravel path 5 ft. wide at 4*d.* a sq. yard.

7. A invests £500, B £400 in a business, and at the end of three months A withdraws £200 and B £50. If the profits for a year are £285, what will each receive?

XXIII. e.

1. Two lines are 41·06328 and ·0438 inches long respectively. How many lines as long as the latter can be cut from the former? What will be the length of the remaining line?

2. A man, after paying 7*d.* in the £ income-tax, has £349. 10*s.* left; what is his gross income?

3. The soil removed from a trench 47 metres long, 1·2 metres broad and ·85 metre deep is spread uniformly over a plot of ground whose area is 1500 square metres. Find the thickness of the layer to the nearest millimetre.

4. A garrison of 420 have food enough to last them 35 days. After 5 days they are reinforced by 210 men, bringing no food with them. How much longer will the food last?

5. Find the cost of papering the walls of a room 32 ft. long, 20 ft. broad and 16 ft. high with paper 3 ft. wide at $4\frac{1}{2}d.$ a yard; allowing for a door 8 ft. by 4 ft. and 4 windows each 7 ft. by 5 ft.

6. The ratio of a kilogramme to a pound is 2·2046 : 1. Express 1 cwt. in kilogrammes correct to 3 decimal places.

7. Three partners put £480, £720 and £240 respectively into a business; how should they share a profit of £2880?

XXIII. f.

1. Reduce $3\frac{3}{4}d.$ to the decimal of 10*s.*, and divide the result by 12·5.

2. Find the cost of papering a room 25 ft. long, 18 ft. 6 in. wide and 10 ft. high with paper 2 ft. wide at 3*s.* 6*d.* a piece of 12 yards.

3. Divide £500 between 3 people so that the first has £20 more than the second, and the second has £20 more than the third.

4. How much water must be added to 36 gallons of spirit to reduce the price from 17*s.* 6*d.* to 16*s.* a gallon?

5. Express $\sqrt{\frac{2}{3}}$ as a decimal fraction correct to 4 significant figures.

6. The speeds of two locomotive engines are in the ratio 15 : 17. If the slower travels at 40 miles an hour, find the speed of the faster.

7. A man travelling at a uniform speed and starting at noon does a journey of 30 miles in 2 hours, then rests for half an hour, and then returns home, arriving there at 4 p.m. Draw a graph to represent his journey, and from your diagram determine, as accurately as you can:

(1) His distance from home at 3 o'clock.

(2) The times when he was 12 miles from home.

State clearly the units employed.

XXIII. g.

1. If 1 gallon equals 277·274 cubic inches, and a cubic foot of water weighs 1000 oz., what is the weight of 39·25 gallons of water to the nearest ounce?

2. Reduce £5. 13*s.* $7\frac{1}{2}$*d.* to the decimal of £6.

3. If 3 men plough 8 ac. 3 roods in 3 days of 7 hours each, how many hours a day must 2 men work to plough 20 acres in 9 days?

4. Find the cost of papering a room 27·7 ft. long, 19·55 ft. broad and 12·4 ft. high with paper 2·7 ft. broad at 1*s.* 3*d.* per yard.

5. Find the square root of 5,197,689,025.

6. A sum of £103. 16*s.* 8*d.* is divided between two boys proportionally to their ages. If they are 11 and 17 years old, find their respective shares.

7. Each of the children in a school received 3 apples and 2 pears. The apples were bought at 3*d.* a score and the pears at $2\frac{1}{2}$*d.* a dozen, and the former cost $4\frac{1}{2}$*d.* more than the latter. How many children were there in the school?

XXIII. h.

1. One litre contains ·220096 gallon, and a dozen bottles contain 2 gallons: express the contents of one bottle in litres correct to two decimal places.

2. A man undertakes a journey; he walks $\frac{1}{5}$ of the distance the first day, ·25 of what remained on the second day, $\frac{1}{3}$ of what still remained on the third day, thus leaving 12 miles for the fourth day. Find the length of the journey.

3. A square field is bounded by a path 3 yds. wide, the field and path together occupying $2\frac{1}{2}$ acres; find the cost of covering the path with gravel at 1*s.* 6*d.* per square yard.

4. If 24 Englishmen in 72 days of 10 hours each can dig a trench 16 ft. deep and 28800 sq. yds. in horizontal section, how many Chinamen in 24 days of 15 hours each can dig a trench 15 ft. deep, 90 yds. wide and 400 yds. long, the work of an Englishman being to that of a Chinaman as 8 : 5?

5. Find the value of $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}-\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ correct to three places of decimals.

6. On a journey of 43 miles a lorry travels 14 miles at 24 miles per hour, 26 miles at 15 miles per hour and the remaining 3 miles through towns at 6 miles per hour. Find the average speed for the whole journey to the nearest mile per hour.

7. What is the cubic capacity of a box which on the outside measures 2 ft. $9\frac{1}{2}$ in. long, 2 ft. $6\frac{3}{4}$ in. broad and 2 ft. $3\frac{1}{4}$ in. high, the sides and top being $\frac{3}{4}$ inch thick and the bottom 1 inch thick?

How many complete pounds of water will 19 such boxes hold, a gallon of water weighing 10 lb. and containing 277·274 cubic inches.

XXIII. k.

1. If 8 guineas be spent in buying carpet $\frac{3}{4}$ yd. wide at 3*s.* 6*d.* a yard for a room 27 ft. long and 16 ft. 9 in. wide, how much of the floor will remain uncovered?

2. If 28 men mow 672 acres in 24 days of $8\frac{1}{2}$ hours each, how many hours a day would 120 men take to mow 1080 acres in 9 days?

3. A metre contains 39·3708 in.: express an inch as a decimal fraction of a decimetre correct to 4 decimal places.

4. Find the cost of 14 cwt. 2 qrs. 8 lb. at £2. 12*s.* 6*d.* per ton.

5. A cubic foot of water is poured into a rectangular tank whose length is 18 in. and breadth 15 in.; find the depth of water in the tank.

6. A rate of £4125 is to be raised from property of which the rental is £133,875; find, correct to the nearest farthing, how much must be charged in the pound so as to obtain the full sum required.

7. A bicycles at 11 miles an hour, and B, starting from the same place in the same direction but half an hour later, drives a motor car at 20 miles an hour. Find (to the nearest mile) where B overtakes A.

XXIII. l.

1. If the population of Great Britain be 40,595,700 and the area be 121,115 sq. miles, find, correct to two decimal places, the number of inhabitants to each square mile.

2. A bankrupt's debts amount to £48725, and his assets to £7370; if the legal expenses of liquidation are £615, find, correct to the nearest farthing, what dividend he will pay in the £.

3. An oblong grass plot measures 15 yds. by 20 yds., and is surrounded by a walk 4 ft. broad. What will be the price of enough gravel, at 6*s.* 9*d.* per cubic yard, to cover the walk to the depth of 4 inches?

4. A garrison of 40 white men and 70 black men require a ton of flour a fortnight. Assuming that the daily ration of a white man is half as much again as that of a black man, find the weight of flour required by a garrison of 25 white men and 84 black men in 26 days.

5. How many pieces of paper, each 12 yds. long and 1 ft. 8 in. wide, will be required for a room 17 ft. long, 14 ft. 6 in. wide and 10 ft. high?

6. Find the square root of 172·305 correct to two decimal places.

7. Given that 5 cubic inches = 82 cubic centimetres, make a graph to convert c. cm. into c. in. (up to 100 c. cm.), and read off the values of 30 and 47 c. cm. in c. in. and of 3·2 c. in. in c. cm., as accurately as you can.

XXIII. m.

1. Find the weight of water in a dock 24 feet deep and $\frac{1}{10}$ of an acre in extent, if a cubic fathom of water weighs 6 tons.

2. Add together ·07890625 of a ton and $47\frac{1}{4}$ lb.

3. The floor of a room, 17 ft. 6 in. long by 16 ft. 3 in. wide, is to be covered with tiles at £2. 5*s.* per thousand, the size of each being 6 inches by 3 inches. What is the cost of the necessary tiles?

4. A grocer buys tea at £8 the cwt. He packs it in canisters holding 14 lb. each and sells the canisters for 24*s.* 6*d.* each. The canisters cost him 7*d.* each and the average expense of their delivery is 1*s.* $4\frac{1}{2}$*d.* How much profit does he make on each cwt.?

5. A train 71 yds. long and running at 21 miles an hour is overtaken by another train 105 yds. long running at 36 miles an hour on parallel lines. How long does the faster take to pass the slower train?

How long would they take to pass one another if they were travelling in opposite directions?

6. A man mixed 16 gallons of spirits at 17*s.* 6*d.* a gallon with 18 gallons at £1 a gallon. How much water did he add if he sold the mixture at 16*s.* a gallon without loss?

7. The speeds of two trains are in the ratio 12 : 17. If the faster travels 119 miles in $3\frac{1}{2}$ hours, how long will the slower take to travel 240 miles?

XXIII. n.

1. A piece of land is divided into 3 portions. The first is ·42 of the whole and the second is ·75 of the first. What decimal fraction of the whole is the third?

2. If light takes 8 min. 34 secs. to reach the Earth from the Sun, a distance of 92,520,000 miles, and takes 200 years (of $365\frac{1}{4}$ days) to come from Arcturus, travelling at the same rate, find the distance of Arcturus in millions of miles.

3. Two cyclists ride from two places A and B, 60 miles apart, towards each other, starting at the same instant and travelling at 11 and 13 miles per hour respectively. Find the distance from A to the point where they meet.

4. How far can 59 people travel for half the amount paid for the railway fares of 17 people for a journey of 295 miles?

5. If £11. 10*s.* is divided into equal numbers of half crowns, florins and sixpences, how many coins are there of each kind?

6. Multiply 3·01678 by ·0123709 correct to six decimal places, using no more figures than are necessary.

7. Find the value of $\frac{1}{3{\cdot}1416}$ correct to four decimal places, and use the result to find, correct to the tenth of an inch, the radius of a circle whose area is 4 sq. inches. (Area of a circle $= 3{\cdot}1416 \times a^2$, where a is the radius.)

XXIII. o.

1. A person calculates that his necessary expenses absorb $\frac{4}{7}$ of his income, and that $\frac{2}{5}$ of the remainder is sufficient for his amusements. If he has still £72 to dispose of, find his income.

2. In a school of 240 pupils the number of absentees on the six days of a week were 27, 13, 17, 14, 21, 25. Find the average number absent daily, and express it as a decimal fraction of the whole number of pupils.

3. Find the cost of 104 acres, 2 roods, 35 poles at £27. 14*s.* 8*d.* per acre.

4. On the liquidation of an estate the creditors only receive 6*s.* 4*d.* in the £. Find the loss on a claim of £725. 2*s.* 6*d.*, neglecting halfpence?

5. If 9 men and 5 boys can do in a day as much work as 5 men and 11 boys, find the ratio of the value of a man's work to a boy's.

6. A sum of money is divided between three men in the proportion of 2 : 13 : 5. If the third man receives £14. 3*s.* 4*d.*, find the shares of the other men.

7. A train travelling 30 miles per hour passes a signal box at one o'clock and 25 minutes later another train passes it, travelling at 60 miles an hour. Find graphically the time at which the second train catches up the first.

XXIII. p.

1. Simplify (1) $\dfrac{\frac{1}{3}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{12}} \div \dfrac{(\frac{1}{3}+\frac{1}{4}) \text{ of } \frac{1}{12}}{\frac{1}{3}\times(\frac{1}{4}-\frac{1}{12})}$.

(2) ·021875 of £3 + ·375 of a guinea − ·275 of 7*s.* 6*d.*

2. One ounce Troy of standard gold is worth £3. 17*s.* $10\frac{1}{2}$*d.* Find the sum of the weights of 934 sovereigns and one half-sovereign.

3. If it costs £2. 10*s.* to paper the walls of a room 16 ft. long, 14 ft wide and 10 ft. 6 in. high, what will be the cost of papering a room 26 ft. long, 16 ft. wide and 11 ft. high with the same kind of paper?

4. In consequence of the fall in value of the rupee* from 1*s* $11\frac{1}{2}$*d.* to 1*s.* 10*d.* an Indian official, who was paid in rupees, lost £30. 14*s.* 9*a.* per annum. Find his salary in rupees.

* The value of the rupee has now been stabilized at 1*s.* 6*d.*

5. Coffee at £5. 10*s.* per cwt. is mixed with chicory at £2. 10*s.* a cwt. in the proportion of 8 cwt. of coffee to 3 cwt. of chicory. Find, to the nearest penny, the value of a pound of the mixture.

6. A mounted troop went into battle, each man riding one horse and leading another. They were defeated and fled on horseback, leaving 17 men and 30 horses dead, while 80 horses were captured by the enemy. Assuming that each man in his flight had only the horse he rode, determine how many men were originally in the troop.

7. Given that the area of a circle $= \pi a^2$, where a is the radius and $\frac{1}{\pi} = \cdot 31831$, find the diameter of a circle, to the nearest tenth of an inch, whose area is 463 sq. inches.

XXIII. q.

1. A half-crown weighs $218\frac{2}{11}$ grains, while 1 lb. Avoirdupois contains 7000 grains. How often would a grocer use a half-crown instead of a half-ounce weight in weighing small packets of tea before the deficiency thus caused amounted to one ounce?

2. A cubical tank, open at the top, costs £16. 6*s.* 8*d.* to line with lead at 1*s.* 4*d.* per sq. ft. Find its volume.

3. If $\frac{9}{10}$ of a litre of wine is mixed with $\frac{1}{4}$ of a litre of water, find the weight of the mixture if the wine is $\cdot 9$ times as heavy as water.

4. How many cubes each $3\frac{1}{2}$ inches to the edge can be cut from a block of wood 16 inches long, 12 inches wide and 9 inches thick?

How many such cubes could be cut from a similar block of metal, which could be melted down and re-cast?

5. From observation of the eclipses of Jupiter's satellites, it is found that the time taken by light to travel from the Sun to the Earth is 499 seconds (with a possible error of 2 seconds either way). If the velocity of light is 186,330 miles per second, find the distance from the Sun to the Earth, and the possible error in your result. [It will be sufficient to give your result and the possible error to the nearest hundred thousand miles.]

6. In a school there are five classes, and the average number in each class is exactly 29. If the average for the 1st, 3rd and 5th classes is 30, find the total number of students in the 2nd and 4th classes.

What modification will your answer need if the above average of 29 is true only to the nearest integer?

7. When a man's income is £700 he pays income-tax on the whole amount, but if his income is between £600 and £700 he is allowed to deduct £70 and pay income-tax on the remainder. A's gross income is £697 and B's is £700. If the income-tax is 1*s.* 2*d.* in the pound, show that A's *net* income is greater than B's.

C's gross income is less than A's and his net income equal to B's Find C's gross income to the nearest penny. [See note on page 110].

XXIII. r.

1. Forty pounds Troy of gold are coined into 1869 sovereigns. Find the smallest number of sovereigns which weigh an exact number of pounds Avoirdupois.

2. Find, to the nearest penny, the cost of 517 tons 13 cwt. 3 qrs. 21 lb. of coal at £1. 3*s*. 9*d*. a ton.

3. The breadth of a room is 14 feet; the cost of papering the walls at 1*s*. a square yard is £4, and that of carpeting the floor at 4*s*. 6*d*. a square yard is £5. 12*s*. Determine the height and length of the room.

4. The area of the surface of a sphere is $4\pi a^2$, where a is its radius. Taking $\pi = 3{\cdot}1416$, find, to the nearest hundredth of a sq. in., the surface of a sphere whose radius $= 3{\cdot}68$ inches.

5. A man walks $3\frac{3}{4}$ miles an hour. How long can he rest, if he must get round a square field containing 13 ac. 81 sq. yds. in 10 minutes?

6. A green-grocer mixes 100 apples at 2 a penny with 200 at 3 a penny; at what price per 100 must he sell them so as to gain $\frac{1}{8}$ of the cost price?

7. A cubic foot of water weighs 1000 oz.; and a pipe whose bore is $3\frac{1}{2}$ sq. inches discharges 252 lb. per minute. Find the velocity in feet per minute (to the nearest foot) of the issuing water.

XXIII. s.

1. Express 34512876 sq. ft. as a decimal of an acre correct to five places.

2. In 9 hours a pump fills a reservoir 84 ft. long, 27 ft. wide and 15 feet deep. It also takes 48 hours to fill another which is 90 ft. long by 42 ft. wide. What is the depth of the latter?

3. If 3 men, 5 women, or 8 children could do a piece of work in $26\frac{1}{2}$ hours, in what time would 2 men, 3 women and 4 children all working together do the work?

4. By selling a horse for £65 a man loses one-quarter of what it cost him. What did it cost him?

5. The Scottish Education Department requires for each pupil in a school-room 8 square feet of floor space and 80 cubic feet of air space. How many pupils may use a room 40 feet long, 30 feet wide and 12 feet high? Explain how you obtain your result.

6. The volume of a wedge which has a rectangular base is $(2l+e)\dfrac{bh}{6}$ cubic inches, where e is the length in inches of the edge, h the height in inches of the wedge, l and b the length and breadth in inches of the base. Find, to the nearest cubic inch, the volume of a wedge whose height is 13 inches and edge 35 inches, and the base of which is 15 inches by 8 inches.

7. How many gallons of water (to the nearest thousand) will be required to fill a water main 5 miles long and 2 ft. in diameter? At what speed would the water flow in this main if the water flowing in 6 hours gave a day's supply to a town of 20,000 inhabitants, allowing 30 gallons per head per day? Give your result to the nearest half foot per second.

[A cubic ft. of water is 6·2 gallons. Area of a circle of radius $a = \pi a^2$, where $\pi = 3·14$.]

XXIII. t.

1. A person, paid at the rate of 8*s.* 4*d.* in the £, received £140. 16*s.* 8*d.*; what was the whole debt due to him?

2. Taking the value of gold as 3 francs 10 centimes per gramme, find its value in English money per ounce Troy (to the nearest shilling), assuming that one gramme is 15·5 grains, and that £1 is worth 25·25 francs.

3. By selling a horse for £72 a man loses one quarter of what it cost him; how much should he sell it for in order to gain one quarter of what it cost him?

4. From a rectangular field part is to be taken for a building and another part for a road. That for the building is a triangle in one corner of the field measuring 150 metres along each of two sides of the field. The road is to be 11·5 metres wide and have the hypotenuse of the triangle for one side. Find, to the nearest pound, the cost of the ground taken at 10*s.* a square metre.

5. A certain liquid is 1·8 times as heavy as an equal volume of water. How many cubic centimetres of this liquid must be added to 50 c. cm. of water that the weight of 1 c. cm. of the mixture may be 1·5 grams?

(1 c. cm. of water weighs 1 gram.)

6. Two tons can be carried 150 miles for £18, the rate per ton per mile for the second 50 miles being 0·9 of the initial rate, and for the third 50 miles 0·8 of the initial rate. How far can 5·6 tons be carried for £8·4?

7. Two workmen, A and B, are paid the same rate of wages per hour. They begin work on the same day, and each on that day works for the same number of hours. On each succeeding day A works for a quarter of an hour less, and B works for 20 minutes less, than on the day before. At the end of 5 days, the ratio of A's earnings to B's is 31 : 30. How long did they work on the first day?

XXIII. u.

1. For a payment of 2*s.* $3\frac{1}{2}$*d.* the Post Office will insure a parcel sent abroad for £120; that is, will pay £120 if the parcel is lost. How many such parcels must be insured in order that the Post Office shall not lose if *one* is lost?

2. Find, to the nearest ounce, the weight of a cylindrical bar of metal 5 feet long and 1 inch in diameter. The volume of a cylinder is $\pi a^2 h$, where $\pi = 3{\cdot}142$, and a is the radius of a circular section of the cylinder and h is the length of the cylinder. One cubic foot of the same metal weighs 7900 ounces.

3. In a four-sided field ABCD, AB and AD are each 96 yards; BC = 64 yards; CD = 48 yards; and BCD is a right angle. Find its area to the nearest square yard.

4. Express as a decimal

$$1 - 6 \times (0{\cdot}9) + 15 \times (0{\cdot}9)^2 - 20 \times (0{\cdot}9)^3 + 15 \times (0{\cdot}9)^4 - 6 \times (0{\cdot}9)^5 + (0{\cdot}9)^6.$$

5. If 63·725 pints of wine worth 2·44 shillings a pint are mixed with 49·475 pints worth 3·64 shillings a pint, find in shillings and pence, correct to the nearest farthing, the value of a pint of the mixture.

6. Find, correct to the nearest penny, the cost of fencing in a square field whose area is 3 acres at the rate of 2*s.* 11*d.* per yard.

7. A's income is one-fourth as much again as B's and one-eighth as much again as C's. Together they pay £116. 19*s.* 4*d.* for income-tax at the rate of 8*d.* in the £. What are their respective incomes?

XXIII. v.

1. What is the least number that can be divided by 8, 9, 10 and 12, with remainder 5 in each case?

2. After paying an income-tax of 5*d.* in the £, a man has an income of £468. 4*s.* 9*d.* left. What was his gross income?

3. If a man buys eggs at 20 for a shilling, how many must he sell for a guinea in order to gain 3*d.* on every 100 eggs he buys?

4. A sovereign weighs $123\frac{1}{3}$ grains and $\frac{11}{12}$ of this weight is gold; whilst a shilling weighs $87\frac{1}{4}$ grains and $\frac{37}{40}$ of this weight is silver. If gold is worth £4. 5*s.* per oz. of 480 grains and silver is worth 2*s.* 1*d.*, find the value of the gold in a sovereign and the value of the silver in a shilling to the nearest farthing in each case.

5. A motor car was timed over a measured quarter mile and the time given as 34 seconds. The error made in taking the time may have been a second. Within what limits does the speed (in miles per hour) lie?

6. Find, to the nearest gram, what weight of a substance worth 12·8 francs per kilogram should be given in exchange for $14\frac{1}{4}$ lb. of a substance worth 5*s.* $10\frac{1}{2}$*d.* per pound. (A franc = $9\frac{1}{2}$*d.*)

7. A box is made of wood $\frac{3}{4}$ of an inch thick. When closed, its external measurements are 24, 18 and 15 inches. Find its weight if a piece of the same wood 2 feet long, 6 inches broad and $\frac{3}{4}$ of an inch thick weighs 4 lb.

XXIV. PERCENTAGE.

170. In comparing vulgar fractions with reference to magnitude, the regular way is to reduce them to a common denominator. It would be convenient, therefore, if all fractions were **expressed with a standard denominator** so as to require no reduction. The method of percentage accomplishes this object. In every case the numerator alone is mentioned, the omitted denominator being always 100.

Thus 5 per cent. of anything means five-hundredths of it; and $\frac{5}{100}=\frac{1}{20}$, *i.e.* 5 per cent. $=\frac{1}{20}$.

Similarly, 25 per cent. $=\frac{25}{100}=\frac{1}{4}$; and 12 per cent. $=\frac{12}{100}=\frac{3}{25}$. 'Per cent.' is an abbreviation for 'per centum,' and is further abbreviated by a symbol; for, according to the usual notation, the words 'per cent.' are indicated by %.

Thus seven per cent. is written 7 %.

It may be noticed that 100 % of anything = the whole of it.

After some oral practice it becomes easy to change immediately a vulgar fraction into the percentage-form, and *vice versa* percentage into a fraction. Any fraction, when multiplied by 100, gives the number necessary to express it as a percentage.

Thus $\frac{3}{4}=\frac{3}{4}$ of 100 per cent. $=75$ per cent.

$\frac{1}{5}=\frac{1}{5}$ of 100 per cent. $=20$ per cent.

$\frac{1}{8}=12\frac{1}{2}$ per cent.

Commission.

171. In business transactions, when an agent is employed, it is usual to make the amount of reward for his services depend upon the magnitude of the transaction. This is conveniently done by agreeing that he shall have a certain percentage of the amount. Thus, if a house-agent sold a house for £3000 on the understanding that his fee should be 2 %, he would receive for his services 2 % of £3000, *i.e.* £60.

Example. **Find the commission at 3 % on £475. 8s. 4d.**

Here we have to take three-hundredths, *i.e.* we must multiply by 3 and divide by 100.

The division by 100 is conveniently done by marking off the two final figures. These figures will be the remainder at each step.

	£.	s.	d.
	475	8	4
			3
(The remainder £26 must be reduced to shillings.)	14,26	5	0
	20		
(Reduce the remainder 25*s.* to pence.)	5,25		
	12		
	3,00		

Commission = £14. 5*s.* 3*d.*

This method of dividing money by 100 is important, and should be carefully learnt.

Conversion of Percentage into a Decimal or other Fraction.

172. This follows immediately from the meaning of percentage:

E.g. $13\% = \frac{13}{100} = \cdot 13$; $3\% = \frac{3}{100} = \cdot 03$; $5\% = \cdot 05$.

EXAMPLE 1. **Find 11 % of 325.**

$$\tfrac{11}{100} \text{ of } 325 = \tfrac{3575}{100} = 35\cdot 75.$$

EXAMPLE 2. **What per cent. of 32 is 13?**

Let it be x per cent.

x must be the same fraction of 100 as 13 is of 32.

i.e.
$$\frac{x}{100} = \frac{13}{32};$$

$$\therefore\ x = \tfrac{13}{32} \times 100 = 40\tfrac{5}{8};$$

$$\therefore\ 13 \text{ is } 40\tfrac{5}{8} \text{ per cent. of } 32.$$

173. The rapid calculation of 5 per cent. of a sum of money is simplified by the fact that pounds are turned into shillings, since one shilling = 5 % of £1.

Thus 5 % of £43 = 43 shillings = £2. 3*s.*

5 % of £13. 5*s.* = 5 % of £$13\frac{1}{4}$ = $13\frac{1}{4}$ shillings = 13*s.* 3*d.*

5 % of £76. 2*s.* 6*d.* = 5 % of £$76\frac{1}{8}$ = $76\frac{1}{8}$ shillings = £3. 16*s.* $1\frac{1}{2}$*d.*

In fact for every £ we put down 1 shilling,

for 10 shillings „ „ 6*d.*,

for 5 shillings „ „ 3*d.*,

for 2*s.* 6*d.* „ „ $1\frac{1}{2}$*d.*;

and the odd pence, if any, must be divided by 20.

In actual business, where a rough approximation is sufficient, the result may be easily worked out mentally.

$7\frac{1}{2}$ per cent. may be found by taking 5 % and increasing this by half of itself.

EXAMPLE 1. **Find 5 % of £23. 14s. 2d.**

5 % of £23. 10*s.* + 5 % of 4*s.* 2*d.* = 23*s.* 6*d.* + $2\frac{1}{2}$*d.* = £1. 3*s.* $8\frac{1}{2}$*d.*

EXAMPLE 2. **Find 5 % of £17. 8s. 7d.**

5 % of £17 + 5 % of 5*s.* + 5 % of 43*d.*
= 17*s.* + 3*d.* + 2*d.*
= 17*s.* 5*d.*, the $\frac{3}{20}$ of a penny being neglected.

EXAMPLES XXIV. a. (*Oral.*)

1. When $\frac{2}{5}$ is expressed as a fraction with denominator 100, what is the numerator?

2. Express $\frac{2}{5}$ as a percentage.

3. When $\frac{3}{20}$ is expressed with denominator 100, what is the numerator?

4. Express $\frac{3}{20}$ as a percentage.

5. If the birth rate in a town is 26 in every 1000, what is the percentage?

Express the following percentages as fractions in their lowest terms:

6. 4 %. **7.** 10 %. **8.** 25 %. **9.** 60 %. **10.** $12\frac{1}{2}$ %.
11. 40 %. **12.** $33\frac{1}{3}$ %. **13.** 75 %. **14.** $6\frac{1}{4}$ %.
15. $37\frac{1}{2}$ %. **16.** 35 %. **17.** $7\frac{1}{2}$ %. **18.** $66\frac{2}{3}$ %.

Express as percentages

19. $\frac{1}{20}$. **20.** $\frac{1}{4}$. **21.** $\frac{1}{5}$. **22.** $\frac{1}{8}$.
23. $\frac{1}{16}$. **24.** $\frac{1}{12}$. **25.** $\frac{3}{5}$.

Express as decimals

26. 24 %. **27.** 13 %. **28.** 3 %. **29.** 7·5 %. **30.** $38\frac{3}{4}$ %.

31. To increase anything by 30 %, by what fraction do you multiply it?
32. " " 15 % " " "
33. 13 is 25 % of what number?
34. 18 " 5 % " "
35. 27 " 3 % " "
36. 5 " $12\frac{1}{2}$ % " "
37. What percentage is equivalent to 1*s.* in the £?
38. " " " 4*s.* "
39. " " " 2*s.* 6*d.* "
40. " " " 1*s.* 6*d.* "

41. What is 10 % of £37?
42. " " £403?
43. " " £612. 10*s.*?
44. What is the sum of which £17. 12*s.* is 10 per cent.?
45. Find 3 % of 1700.
46. Find 4 % of 75.
47. Find 8 % of 112·5.
48. Find 7 % of 31.
49. Find $6\frac{1}{4}$ % of 32.

From the fact that 5 % is 1*s.* in the £ and 6*d.* in 10*s.*, and so on, find 5 % of

50. £257.
51. £74.
52. £28. 10*s.*
53. £32. 5*s.*
54. £2. 2*s.* 6*d.*
55. £13. 15*s.*
56. £67. 13*s.* 4*d.*
57. £43. 2*s.* 6*d.*
58. £18. 5*s.*

In a village of 300 persons

59. the men were 20 %; how many were there?
60. " women " 25 %; " "
61. " children " 55 %; " "

62. 68 % of a debt was paid. What per cent. remained unpaid?

63. Of a regiment numbering 800 men, 11 % were killed. How many was that?

64. Of a regiment numbering 850 men, 12 % were killed. How many was that?

65. The population of a town numbering 2400 was increased by 25 %. What was the population after the increase?

66. If the population of a place decreases by 7 %, what is the ratio of the new number to the original?

67. What multiplier will cause a number to increase 17 %?

68. A population of 800 increases every year by 25 %. What is the number at the end of 2 years?

69. What is the sum of money of which £4. 9*s.* is 25 %?

70. A tank which contained 250 gallons is found to contain only $237\frac{1}{2}$ gallons. What percentage has gone away?

71. 7 % of x is 42. What is x?

72. x % of 1100 is 132. What is x?

73. 25 % of x is 17. What is x?

74. $12\frac{1}{2}$ % of x is 15. What is x?

75. A man loses 23 % of his capital. What percentage remains?

76. A railway company had an income of £1,800,000 in 1907 and £1,980,000 in 1908. What was the increase per cent.?

77. The average number of passengers per day on a railway increased from 35,000 to 42,000. What was the increase per cent.?

78. A broker, paid at the rate of 3 % on the purchase money, gets £2. 17*s*. as his commission. What was the purchase-money?

79. A charitable institution receives subscriptions amounting to £64 through a collector. The actual amount handed over by him is £60. 16*s*. What percentage does the collector keep?

80. A man receiving pay at the rate of £300 a year gets a 10 % increase and subsequently a 10 % decrease of pay. To what salary does this reduce him?

174. Percentage is largely used in denoting rate of increase or decrease in such matters as wealth, expenditure, population and so on. Its usefulness is shown by instances of the following sort.

EXAMPLE 1. **A town of 2500 inhabitants had an increase of 73 in one year, and another town, with a population of 12800, had an increase of 160 in the same time. Compare their rates of increase.**

Let the increases be respectively x and y per cent.

Then $$\frac{x}{100}=\frac{73}{2500}; \qquad \therefore\ x=\frac{73}{25}=\frac{292}{100}=2{\cdot}92.$$

$$\frac{y}{100}=\frac{160}{12800}; \qquad \therefore\ y=\frac{160}{128}=\frac{5}{4}=1{\cdot}25.$$

Thus, although in the second town the increase was larger than that in the first, it was not *proportionately* as large. This is clearly shown by the percentage 1·25 compared with 2·92.

If the second town had increased at as great a rate as the first, the increase would have been 2·92 % of 12800, *i.e.* about 374 instead of 160.

EXAMPLE 2. **In an examination, in which there were 2275 candidates, 156 failed. Find the percentage of failures.**

Let the failures be x per cent. of the candidates.

Then $$\frac{x}{100}=\frac{156}{2275}.$$

$$\therefore\ x=\frac{15600}{2275}=\frac{624\cancel{00}}{91\cancel{00}}=\frac{48}{7}.$$

∴ the percentage was 6·86 approximately.

175. One great point in considering percentage is to recognise what is the original quantity on which you are calculating the percentage. When you say 15 per cent., for instance, you mean 15 per cent. of *what*?

In reckoning profit per cent. in a business transaction, you mean that you take the percentage on your *outlay*, not on your *selling price*.

In reckoning increase of wealth you take the percentage on the wealth as it stood *before the increase.*

The confusion which may possibly arise on such points is exemplified by the following instance.

Example 1. **A workman receiving £5 a week obtained an increase of 10% during a time of good trade, and in a subsequent time of depression was subject to a decrease of 10%. Were his wages the same as originally?**

His increase was 10% of £5, *i.e.* 10 shillings.

His wages then were 110 shillings.

∴ his decrease was 10% of 110 shillings, *i.e.* 11 shillings.

He therefore came down to £4. 19*s*. a week.

The increase was calculated on the *wages at the time.*

The decrease was also calculated on the current wages, *i.e.* on the wages he was receiving *at the time.*

Example 2. **A cask, which contained 75 gallons, lost $12\frac{1}{2}$% of its contents. How many gallons remained in it?**

$$\text{The amount lost} = \frac{12\frac{1}{2}}{100} \text{ of } 75 = \frac{1}{8} \text{ of } 75 = 9{\cdot}375 \text{ gallons.}$$

$$\text{The remainder} = 75 - 9{\cdot}375 = 65{\cdot}625 \text{ gallons.}$$

Insurance.

176. If a man wishes to recover the value of his property in case of its destruction, *e.g.* by fire, he can do so if he has previously made an arrangement with an Insurance Company. This arrangement is to the effect that he shall be repaid a certain sum, at which he has valued the property, in consideration of an annual payment of a certain percentage on that sum.

If a house worth £2000 is insured against fire at the rate of $\frac{1}{8}$ per cent., the owner has to pay annually to the Insurance Company $\frac{1}{8}$ of a £ on every hundred pounds of the value. Therefore his yearly payment is 20 times $\frac{1}{8}$ of a £, *i.e.* £2. 10*s.*

The Insurance Company on their part are liable to pay him £2000 in the event of the house being totally destroyed by fire.

The written agreement on this subject is called the **Policy**, and the yearly payment is called the **Premium.**

By Life-insurance is meant obtaining the promise of a sum of money to be paid at the death of the person insured. The promise is made by an Insurance Company in return for a pay-

ment (called the premium) which is made annually to the Company.

This premium has, of course, to be calculated according to the number of years that the person is likely to live. Consequently the younger a man is when he begins to pay a premium, the smaller will that annual premium be.

The various Insurance Companies do not all charge quite alike, but, on an average, a man who is 26 when he insures his life (*i.e.* when he insures a sum of money to be paid at his death) pays annually £1. 17*s.* 6*d.* on every £100 insured; whereas, if he puts off insurance till he is 46, he must pay annually £3. 9*s.* on every £100, or £3. 9*s.* per cent.

If a ship and its cargo are insured for a certain voyage at a certain percentage, it is natural for the owner to insure such a sum as will in case of loss repay him the premium as well as the value of ship and cargo.

EXAMPLE. **A ship and cargo, worth £7200 together, are insured at 4 %. What must be the sum insured so that in case of loss the owner shall recover the amount of the premium as well as the value of the ship and cargo?**

If £x be the sum to be insured, the premium $=\frac{4x}{100}$.

$$\therefore\ x \text{ must cover not only the 7200 but the } \frac{4x}{100} \text{ also.}$$

$$\therefore\ x=7200+\frac{4x}{100}.$$

$$\therefore\ \frac{96x}{100}=7200.$$

$$\therefore\ x=\frac{100}{96} \text{ of } 7200=\frac{25}{24} \text{ of } 7200=7500.$$

The sum insured must be £7500.

177. It is important, with a view to saving labour, to notice that a number or a quantity may be increased by a fraction of itself by means of multiplication.

For instance, if it has to be increased by half, this may be done by multiplying by $1\frac{1}{2}$, *i.e.* by $\frac{3}{2}$. If it has to be increased by 3 %, the result is got by multiplying by $1\frac{3}{100}$, *i.e.* by $\frac{103}{100}$.

In the same way a number may be decreased 19 % by multiplying it by $1-\frac{19}{100}$, *i.e.* by $\frac{81}{100}$.

EXAMPLE. **The population of a country increased from 750,000 by 12 % in a certain year, by 15 % the next year and by 10 % in the next, and in the next year decreased by 30 %. What was the population then?**

The result may be found in one step as follows:

$$\begin{aligned}\text{The population} &= 750000 \times \tfrac{112}{100} \times \tfrac{115}{100} \times \tfrac{110}{100} \times \tfrac{70}{100}\\ &= 750000 \times \tfrac{112}{100} \times \tfrac{23}{20} \times \tfrac{11}{10} \times \tfrac{7}{10}\\ &= 75 \times \tfrac{112}{10} \times \tfrac{23}{2} \times \tfrac{11}{1} \times \tfrac{7}{1}\\ &= 15 \times 28 \times 23 \times 11 \times 7\\ &= 743820.\end{aligned}$$

Inverse Questions on Percentage.

178. Such questions and the method of working them are illustrated by the following example:

EXAMPLE. **The population of a town after increasing in one year by 15 % was 4738. What was it at the beginning of the year?**

Let x = the population at the beginning of the year.

100 in a year becomes 115.

x „ „ 4738.

$$\therefore \frac{x}{100} = \frac{4738}{115}.$$

$$\therefore x = \frac{473800}{115} = \frac{94760}{23} = 4120.$$

To test this we must increase 4120 by 15 % of 4120, *i.e.* by 618.

$$4120 + 618 = 4738.$$

EXAMPLES XXIV. b.

(*It will suffice if percentages are correct to 2 decimal places.*)

1. What per cent. of 105 is $3\frac{1}{2}$? **2.** What per cent. of 68 is $8\frac{1}{2}$?

3. „ „ 528 „ $49\frac{1}{2}$? **4.** „ „ 426 „ 71?

5. „ „ 632 „ 57? **6.** „ „ 3125 „ 111?

7. „ „ 1185 „ 18?

8. Find the commission on £923. 6*s*. 3*d*. at 2 %.

9. „ „ „ £37. 15*s*. 7*d*. at 5 %.

10. „ „ „ £84. 4*s*. 2*d*. at $7\frac{1}{2}$ %.

11. „ „ „ £1374. 3*s*. 8*d*. at $\frac{1}{2}$ %.

12. A man, whose age is 21, finds that, if he wishes to insure his life, he must pay 2 % annual premium on the sum insured. What does he pay annually to insure his life for £4380?

13. A man of 36 has to pay an annual premium of £2. 9*s*. 3*d*. per cent. for life-insurance. What premium does he pay to insure a sum of £2800?

14. What is the annual cost of insuring a house for £2625 at $\frac{1}{8}$ per cent.?

15. A cistern containing 775 gallons lost 32 % of its contents. How many gallons remained?

16. In a population of 14800 there were 793 deaths. What was the death-rate per cent.?

17. An agent, collecting rents to the amount of £825, receives $2\frac{1}{2}$ % commission. How much is that?

18. A broker sells a property for £2350. He hands over £2314. 15*s.* What is his rate of brokerage?

19. The total receipts of a railway company were £4,442,375. 18*s.*; and 5 % of this was for parcels. What was the sum received for parcels?

20. A South American railway, carrying 8640 tons of hides in one year, carried 8856 tons the next year. What was the increase per cent.?

21. A canal company, whose traffic receipts for one year were £69700, found the year's working expenses to be £24139. What percentage of the receipts were the working expenses?

22. The traffic receipts of a railway company were £2,054,000, and its working expenses £1,174,111. What per cent. was this?

23. In 1891 the population of England and Wales was 29 millions. The increase in the next 10 years was 3,525,318. What was the increase per cent.?

24. During the same period the population of Scotland, beginning with 4026 thousand, increased by 446,456. What was the increase per cent.?

25. For Ireland the figures were: population 4705 thousand, decrease 245,975. Find the decrease per cent.?

26. A country with a population of 2·09 millions has in a certain time an increase of 272,865. What percentage is this?

27. The population of a town has risen from 34,292 to 37,125. What is the increase per cent.?

28. A bankrupt's estate realises £2357. 5*s.* before expenses are deducted; and the expenses amount to 10 per cent. If his debts are £3143, what does he pay in the £ to his creditors?

29. Of a regiment, after 1·5 per cent. have been killed, 8·75 per cent. wounded and 12 per cent. invalided, there still remain 933 men available. What was its original strength?

30. In a cricket match between two schools, A and B, A made 85 more runs than B in the first innings; but in the second innings B made 64 % more runs than A, and won the match by 43 runs. What were the scores in the second innings?

31. A ship and cargo worth £22800 are insured at 5 %. For what sum must they be insured so that the owner may in case of loss recover both the value and the premium?

32. A ship worth £20240 is insured at 8 %. For what sum must she be insured so as to return the value and the premium to the owner in case of total wreck?

33. The inhabitants of a Swiss town speak either French or German or both; 73 % speak French, and 87 % speak German. What percentage speak both?

34. In a forest 5 per cent. of the trees are blown over in a gale; and, after 3 per cent. of those remaining have been cut down, there still stand 55290 trees. How many were there before the gale?

35. The Navy Estimates for the year 1903 were £35,837,000, and for 1904 they were £38,328,000. Find, to the nearest tenth, the percentage increase.

36. In 1903 our exports to foreign countries were valued at £240,889,483, and to British possessions at £119,484,189. Find what percentage of the whole exports (correct to 1 decimal place) went to foreign countries.

37. In 1907 the figures were 370,523,086 and 147,454,081. Calculate the percentage as before.

38. The population of a town decreased in 10 years from 47,438 to 46,012. What was the decrease per cent. correct to 2 decimal places?

39. Three garrisons number respectively 26,529 with 9·3 per cent. sick, 12,743 with 13·9 per cent. sick and 2339 with 18·4 per cent. sick. What percentage (to one decimal place) of the whole are the sick?

40. An alloy of silver is mixed with an alloy of gold in the ratio of 57 to 13; the percentage of lead in the silver alloy is 13·75 and that in the gold alloy 16·25. What is the percentage of lead in the mixture?

XXV. INTEREST.

179. Interest is the rent charged for the use of borrowed money. Just as the rent of a house *might* be calculated by the number of rooms in it, so the interest on a sum of money lent is actually reckoned by the number of hundreds of pounds lent.

The sum of money lent is called the **Principal**.

The principal increased by the interest is called the **Amount**.

When the interest is calculated each year on the original principal, it is called **Simple Interest**.

If the interest were not paid to the lender each year, but were allowed to remain as a debt, the principal on which interest would be calculated would be increased every year.

Thus if £1000 were lent at 5 % per annum, the debt at the end of the 1st year would be £1000 + 5 % of £1000, *i.e.* £1050.

The interest for the 2nd year would not be £50 as in the 1st year, but 5 % of £1050, *i.e.* £52. 10*s.*; and the principal would become £1102. 10*s.* For the 3rd year the interest would be 5 % of £1102. 10*s.*; and so on.

Interest calculated in this way is called **Compound Interest.**

Simple Interest.

180. To find the Simple Interest on a principal of £P at 3 % per annum for 4 years, we need only observe that the interest for 1 year is $\frac{3}{100}$ of P, and it follows that we must multiply by 4 to find the interest for 4 years.

Similarly, if the time had been $5\frac{1}{2}$ years, it would have been necessary to multiply by $5\frac{1}{2}$ instead of 4.

If the question were "*What is the Simple Interest on £P at r per cent. for n years?*" the working would be as follows:

$$\text{The interest for 1 year } = \frac{r}{100} \text{ of } \mathsf{P} = \frac{\mathsf{P}r}{100}.$$

$$\therefore \quad \text{,,} \quad \text{,,} \quad n \text{ years} = \frac{\mathsf{P}rn}{100};$$

i.e. to find Simple Interest multiply the Principal by the rate per cent. and by the number of years (whole or fractional) and divide by 100.

Division by 100 is effected by marking off the last two digits and treating them as a remainder.

EXAMPLE 1. **Find the Simple Interest on £876. 14s. 2d. at 4 % for $3\frac{1}{3}$ years.**

The interest on £100 for $3\frac{1}{2}$ yrs. $= 4 \times 3\frac{1}{2} = 14$.

,, £1 ,, $= \frac{14}{100}$.

∴ ,, £876. 14*s.* 2*d.* for $3\frac{1}{2}$ yrs. = £876. 14*s.* 2*d.* $\times \frac{14}{100}$.

$$\begin{array}{r@{\;.\;}r@{\;.\;}r}
\text{£} & s & d \\
876 & 14 & 2 \\
\multicolumn{3}{r}{2} \\ \hline
1753 & 8 & 4 \\
\multicolumn{3}{r}{7} \\ \hline
122{,}73 & 18 & 4 \\
20 & & \\ \hline
14{,}78 & & \\
12 & & \\ \hline
9{,}40 & &
\end{array}$$

Interest = £122. 14*s.* 9·4*d*

EXAMPLE 2. **Find the S.I. on £775 at 4 % from April 16th to June 23rd. Find also the Amount.**

Here it is a matter of common sense to reckon only one of the given days, April 16th and June 23rd, in finding the length of time.

In April (after the 16th)	14 days
,, May	31 ,,
,, June	23 ,,
	68 days $=\frac{68}{365}$ of a year.

The interest on £1 for 68 days $=\frac{4}{100}\times\frac{68}{365}$.

$\therefore$ the interest on £775 $=775\times\frac{4}{100}\times\frac{68}{365}$

$$=775\times\frac{1}{25}\times\frac{68}{365}=\frac{31\times 68}{365}=\frac{2108}{365}$$

$$=£5\tfrac{283}{365}=£5.\ 15s.\ 6d. \text{ approximately.}$$

Bankers do not recognise in their accounts any halfpence or farthings. Therefore, in questions of this sort it is generally sufficient to give the answer to the nearest penny.

Here the answer is true to the nearest farthing.

The Amount = £775 + £5. 15*s.* 6*d.* = £780. 15*s.* 6*d.*

EXAMPLE 3. **Find the Simple Interest on £347. 18s. 9d. for 219 days at 5 % per annum.**

£347. 18*s.* 9*d.* = £347·9375.

$$219 \text{ days} = \frac{219}{365}=\frac{3}{5} \text{ of a year.}$$

The interest on £1 for 219 days $=\frac{5}{100}\times\frac{3}{5}=\frac{3}{100}$;

$\therefore$ the interest on £347·9375

$=347{\cdot}9375\times\frac{3}{100}$

$=3{\cdot}479375\times 3$

$=10{\cdot}438125$

$=£10.\ 8{\cdot}7625s. = £10.\ 8s.\ 9d.$ to the nearest penny.

181. In some cases the fraction $\frac{nr}{100}$, by which the Principal has to be multiplied, may be reduced to much lower terms.

For instance, if we have to find the interest for 4 years at 5 % on £635. 12*s.* 6*d.*, the multiplier $\frac{4\times 5}{100}$ becomes $\frac{1}{5}$.

$\therefore$ the interest $=\frac{1}{5}$ of £635. 12*s.* 6*d.* = £127. 2*s.* 6*d.*

Approximation by means of the 'Third, Tenth and Tenth Rule.'

182. $365=5\times 73$.

Therefore, in finding interest to a certain number of days, it is generally necessary to divide by 73.

Approximately $\frac{1}{73} = \cdot 0137 = \frac{1\cdot 37}{100} = \frac{1}{100}\left\{1 + \frac{37}{100}\right\}$

$= \frac{1}{100}\left\{1 + \frac{111}{300}\right\}$

$= \frac{1}{100}\left\{1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300}\right\} = \frac{1}{100}\left\{1 + \frac{1}{3} + \frac{1}{10} \text{ of } \frac{1}{3} + \frac{1}{10} \text{ of } \frac{1}{10} \text{ of } \frac{1}{3}\right\}$.

Example 1. **To divide £2718. 15s. by 73, the process is as follows:**

$$2718\cdot 75 \div 73 = 27\cdot 1875\left\{1 + \tfrac{1}{3} + \tfrac{1}{10} \text{ of } \tfrac{1}{3} + \tfrac{1}{10} \text{ of } \tfrac{1}{10} \text{ of } \tfrac{1}{3}\right\}$$

$$= \begin{cases} \ 27\cdot 1875 \\ +\ \ 9\cdot 0625 \\ +\ \ \ \ \cdot 9063 \\ +\ \ \ \ \cdot 0906 = 37\cdot 2469 = £37.\ 4s.\ 11d. \text{ approximately.} \end{cases}$$

The error is about $1d$. This process gives a result about a farthing too great for every £10 of the answer.

Example 2. **Find the interest on £451. 5s. from Sept. 29th to Dec. 25th at $3\frac{1}{2}$%.**

$$\text{The interest} = 451\cdot 25 \times \frac{7}{2} \times \frac{87}{365} \times \frac{1}{100} = \cdot 45125 \times \frac{7 \times 87}{73}$$

$$= \frac{274\cdot 811}{73} = \begin{cases} \ 2\cdot 74811 \\ +\ \ \cdot 91604 \\ +\ \ \cdot 09160 \\ +\ \ \cdot 00916 = 3\cdot 7649 = £3.\ 15s.\ 4d. \end{cases}$$

EXAMPLES XXV. a. (*Oral.*)

What is the Simple Interest on

1.	£200	for	1	year at	1	per cent.?
2.	£300	,,	1	,,	3	,,
3.	£300	,,	2	,,	1	,,
4.	£400	,,	6 months		2	,,
5.	£50	,,	1 year		4	,,
6.	£50	,,	2	,,	3	,,
7.	£20	,,	1	,,	5	,,
8.	£11	,,	1	,,	5	,,
9.	£1	,,	2	,,	5	,,
10.	£25	,,	4	,,	3	,,
11.	£700	,,	2	,,	4	,,
12.	£300	,,	5	,,	3	,,
13	£600	,,	4	,,	2	,,
14.	£130	,,	4	,,	$2\frac{1}{2}$	,,
15.	£750	,,	2	,,	6	,,
16.	£50	,,	3	,,	3	,,
17.	£815	,,	1	,,	5	,,
18.	£195. 10s.	,,	2	,,	$2\frac{1}{2}$	,,

19.	£235. 15*s.*	for	2	years at	5	per cent. ?
20.	£78	,,	5	,,	4	,,
21.	£156	,,	1	,,	$7\frac{1}{2}$	,,
22.	£234. 8*s.*	,,	2	,,	5	,,

EXAMPLES XXV. b.

[*In questions on Interest the words 'per annum' are understood after 'per cent.' Show up a rough check in each case.*]

Find the Simple Interest on

1.	£1175	for	2	years at	5	per cent.
2.	£410	,,	2	,,	4	,,
3.	£1161. 10*s.*	,,	6	,,	5	,,
4.	£1080	,,	$4\frac{1}{2}$	,,	2	,,
5.	£1575	,,	4	,,	7	,,
6.	£312. 3*s.* 9*d.*	,,	3	,,	4	,,
7.	£278. 6*s.* 8*d.*	,,	5	,,	3	,,
8.	£1125. 8*s.* 4*d.*	,,	3	,,	$3\frac{1}{2}$	,,
9.	£547. 3*s.* 9*d.*	,,	$3\frac{1}{2}$	,,	4	,,

[*In Examples* 10–19 *give the result to the nearest penny where farthings would occur.*]

10.	£1386. 6*s.* 7*d.*	for	4	years at	5	per cent.
11.	£397. 11*s.* 9*d.*	,,	3	,,	4	,,
12.	£198. 14*s.* 5*d.*	,,	3	,,	$2\frac{1}{2}$	,,
13.	£300. 8*s.* 4*d.*	,,	6	,,	7	,,
14.	£1465. 3*s.* 10*d.*	,,	10	,,	3	,,
15.	£192. 7*s.* 5*d.*	,,	3	,,	$6\frac{1}{4}$	,,
16.	£301. 5*s.*	,,	4	,,	$2\frac{1}{3}$	,,
17.	£2365	,,	3	,,	3	,,
18.	£182. 6*s.* 10*d.*	,,	$2\frac{1}{2}$	,,	6	,,
19.	£2365. 10*s.* 8*d.*	,,	7	,,	$3\frac{1}{3}$	,,

Find to the nearest penny the Amount (at Simple Interest) of

20.	£1365	for	$3\frac{1}{2}$	years at	$2\frac{1}{2}$	per cent.
21.	£3286. 14*s.* 11*d.*	,,	2	,,	$3\frac{1}{2}$	,,
22.	£793. 16*s.*	,,	2	,,	$2\frac{1}{2}$	,,
23.	£653. 2*s.* 6*d.*	,,	3	,,	$4\frac{1}{2}$	,,
24.	£315. 10*s.* 8*d.*	,,	5	,,	$3\frac{1}{2}$	,,
25.	£1675	,,	$4\frac{1}{2}$	,,	$4\frac{1}{4}$	,,
26.	£186. 13*s.* 4*d.*	,,	$2\frac{1}{4}$	,,	$4\frac{1}{2}$	,,
27.	£237. 18*s.* 4*d.*	,,	3	,,	$4\frac{1}{2}$	,,

Find the Simple Interest on

28.	£3704. 12*s*. 8*d*.	for 10	years	at $2\frac{1}{2}$	per cent.
29.	£751. 13*s*. 4*d*.	„ 3	„	$3\frac{1}{3}$	„
30.	£705. 12*s*. 6*d*.	„ 30	„	$4\frac{1}{3}$	„
31.	£50. 3*s*. 4*d*.	„ 100	„	$4\frac{1}{2}$	„
32.	£323. 6*s*. 8*d*.	„ 3	months	at 4	„
33.	£141. 6*s*. 8*d*.	„ 9	„	$3\frac{1}{3}$	„
34.	£66. 1*s*. 8*d*.	„ 15	„	4	„
35.	£707. 18*s*. 4*d*.	„ 8	„	3	„
36.	£32. 10*s*.	„ 7	„	4	„
37.	£230	„ 11	„	$3\frac{1}{2}$	„

Find the Simple Interest to the nearest penny on

38.	£4651. 17*s*. 6*d*.	for 7 months	at $3\frac{1}{2}$	per cent.
39.	£24720	„ $4\frac{1}{2}$ months	„ $4\frac{1}{2}$	„
40.	£91. 13*s*. 4*d*.	„ 3 yrs. 6 mos.	„ $4\frac{1}{4}$	„
41.	£637. 10*s*.	„ 5 yrs. 8 mos.	„ $3\frac{4}{5}$	„
42.	£850	„ 4 months	„ $4\frac{1}{4}$	„
43.	£834. 15*s*. 6*d*.	„ 11 months	„ $4\frac{3}{8}$	„
44.	£5750	„ 36 days	„ $4\frac{1}{2}$	„
45.	£695	„ 146 days	„ 5	„

46. £291. 13*s*. 4*d*. from Jan. 29th to June 24th at $3\frac{1}{2}$ per cent.

47. £379. 10*s*. from May 12th to Sept. 24th at $3\frac{1}{2}$ „

48. £737. 6*s*. from March 8th to April 2nd at $7\frac{1}{2}$ „

49. A man borrowed £10 from a money lender for a year at the rate of 9*d*. per £ per month. How much interest did the man pay?

Inverse Simple Interest.

183. Under this head come questions of the following types, in all of which we denote by P the Principal, **I** the Interest, A the Amount, n the number of years, r the rate per cent.

(*a*) Given **I** (or A) and n and r, find P.

(*b*) Given P and **I** (or A) and r, find n.

(*c*) Given P and **I** (or A) and n, find r.

The methods to be pursued are fundamentally the same in all cases.

(*a*) Find the Interest (or Amount) of £1 under the same data, and hence get P by division.

(*b*) Find the Interest for 1 year under the same data, and hence get n by division.

(*c*) Find the Interest at 1 % under the same data, and hence get r by division.

[In (*b*) and (*c*), if A is given, the first thing to do is to find **I** by subtraction.]

(*a*) Example 1. **What principal will amount to £345 in 3 years at 5 per cent.?**

£100 amounts to £115 under the given conditions.

What ,, £345 ,, ,,

The required sum $= \frac{100 \times 345}{115} =$ £300.

The following solution is fundamentally wrong. It is left to the student to discover why.

£100 amounts to £105 in one year.

What ,, £345 ,, three years?

The sum required $= 100 \times \frac{345}{105} \times \frac{1}{3}$

$=$ etc.

Example 2. **What principal will give £26. 13s. 4d. interest in 2 years at $2\frac{1}{2}$ per cent.?**

£$2\frac{1}{2} \times 2$ is the interest on £100.

£$26\frac{2}{3}$,, ,, how much?

$$\text{The required sum} = \frac{100 \times 26\frac{2}{3}}{2\frac{1}{2} \times 2} = \frac{100}{5} \times \frac{80}{3}$$

$$= \frac{1600}{3} = £533.\ 6s.\ 8d.$$

(*b*) Example 3. **In what time will £3775 amount to £4105. 6s. 3d. at $3\frac{1}{2}$ per cent.?**

Here the interest $=$ £4105. 6*s*. 3*d*. $-$ £3775 $=$ £330. 6*s*. 3*d*.

£$3\frac{1}{2}$ is the interest on £100 for 1 year.

£330. 6*s*. 3*d*. ,, £3775 ,, how many years?

$$\therefore \text{ the required number of years} = 1 \times \frac{330\frac{5}{16}}{3\frac{1}{2}} \times \frac{100}{3775}$$

$$= \frac{5285}{16} \times \frac{2}{7} \times \frac{100}{3775}$$

$$= \frac{7 \times 755 \times 2 \times 100}{8 \times 2 \times 7 \times 5 \times 755} = \frac{100}{40} = 2\tfrac{1}{2}.$$

(*c*) EXAMPLE 4. **At what rate will the interest on £213. 6s. 8d. become £25. 12s. in 3 years?**

£$25\frac{3}{5}$ is the interest on £$213\frac{1}{3}$ for 3 years.
What ,, £100 ,, 1 year?

$$\text{The rate per cent.} = 25\tfrac{3}{5} \times \frac{100}{213\frac{1}{3}} \times \frac{1}{3}$$
$$= \frac{128}{5} \times \frac{100}{640}$$
$$= \frac{100}{5 \times 5} = 4.$$

Alternative Methods for (b) and (c).

(*b*) EXAMPLE 3. **In what time will £3775 amount to £4105. 6s. 3d. at $3\frac{1}{2}$ per cent.?**

Here the interest = £4105. 6*s*. 3*d*. − £3775 = £330. 6*s*. 3*d*.

The interest for 1 year $= 3775 \times 3\frac{1}{2} \times \frac{1}{100} = \frac{3775 \times 7}{200} = £\frac{1057}{8}$.

The interest for x years $= £330\frac{5}{16}$;

$\therefore\ x = 330\frac{5}{16} \div \frac{1057}{8} = \frac{5285}{16} \times \frac{8}{1057} = \frac{5}{2} = 2\frac{1}{2}$.

The required time is $2\frac{1}{2}$ years.

(*c*) EXAMPLE 4. **At what rate will the interest on £213. 6s. 8d. become £25. 12s. in 3 years?**

The interest at 1 % $= 213\frac{1}{3} \times \frac{3}{100} = \frac{64}{10} = £\frac{32}{5}$.

The interest at x % = £25. 12*s*.

$\therefore\ x = 25\frac{3}{5} \div \frac{32}{5} = \frac{128}{5} \times \frac{5}{32} = 4$.

The required rate is 4 per cent.

Inverse Questions in Interest reduced to direct.

184. By representing the unknown quantity by a letter of the alphabet, we can proceed just as if the question were a *direct* one.

EXAMPLE 1. **What principal will amount to £345 in 3 years at 5 per cent.?**

The amount of £P for the given time and at the given rate

$$= P + \frac{P \times 3 \times 5}{100} = P + \frac{3P}{20} = \frac{23P}{20}.$$

$$\therefore\ \frac{23P}{20} = \text{the amount} = 345.$$

Multiplying both sides by $\frac{20}{23}$, we get

$$P = 345 \times \frac{20}{23} = 15 \times 20 = 300.$$

The required principal is £300.

EXAMPLE 2. **In what time will £3775 amount to £4105. 6s. 3d. at $3\frac{1}{2}$ per cent.?**

The interest here in n years $=\dfrac{3775\times n\times 3\frac{1}{2}}{100}=\dfrac{151\times 25\times n\times 7}{25\times 4\times 2}=\dfrac{1057n}{8}$.

But, by subtraction, the interest $=£330\frac{5}{16}$;

$$\therefore\ \frac{1057n}{8}=330\tfrac{5}{16}=\frac{5285}{16}.$$

Multiply both sides by $\dfrac{8}{1057}$;

$$\text{then}\quad n=\frac{5285}{16}\times\frac{8}{1057}=\frac{5\times 8}{16}=\frac{5}{2}=2\tfrac{1}{2}.$$

EXAMPLE 3. **At what rate will the interest on £213. 6s. 8d. become £25. 12s. in 3 years?**

The interest on $£213\frac{1}{3}$ for 3 years at r per cent.

$$=\frac{640}{3}\times\frac{3r}{100}=\frac{32r}{5};$$

$$\therefore\ \frac{32r}{5}=\text{the given interest}=25\tfrac{3}{5}=\frac{128}{5};$$

$$\therefore\ r=\frac{128}{5}\times\frac{5}{32}=4.$$

The rate is 4 per cent.

EXAMPLES XXV. c. (*Oral.*)

What is the length of time in which the

1.	Interest on	£200 at 1	per cent. becomes	£10?
2.	„	£300 „ 2	„	£24?
3.	„	£50 „ 6	„	£12?
4.	„	£150 „ 2	„	£18?
5.	„	£50 „ 1	„	£3. 10s.?
6.	„	£25 „ 8	„	£6?
7.	„	£400 „ $2\frac{1}{2}$	„	£70?

In what time does

8.	£100	amount to	£106 at 2	per cent.	per annum?
9.	£100	„	£112 „ 3	„	„
10.	£100	„	£120 „ 4	„	„
11.	£200	„	£212 „ 3	„	„
12.	£200	„	£210 „ 5	„	„
13.	£300	„	£324 „ 4	„	„
14.	£300	„	£342 „ 7	„	„

At what rate per cent. does

15. £100 become	£107	in 2 years?
16. £100	„	£106	„ 6 months?
17. £100	„	£125	„ 5 years?
18. £100	„	£103. 10*s.*	„ 6 months?
19. £50	„	£53	„ 1 year?

20. If £100 amounts to £109 in 2 years, what will be the amount in 3 years?

EXAMPLES XXV. d.

1. £150 amounts to £157. 10*s.* at $2\frac{1}{2}$ %. Find the length of time.

2. The interest on £575 for 8 yrs. is £195. 10*s.* Find the rate per cent.

3. In how many years does a sum of money double itself at $12\frac{1}{2}$ per cent.?

4. Find the principal whose interest at $2\frac{1}{2}$ % for 1 yr. 9 months is £24. 10*s.*

5. At what rate does £175 become £211. 15*s.* in $3\frac{1}{2}$ yrs.?

6. What principal amounts to £534. 12*s.* 6*d.* in $3\frac{1}{2}$ yrs. at 5 %?

7. In what time does £450 amount to £497. 5*s.* at 3 %?

8. £350 amounts to £358. 8*s.* in 146 days. What is the rate?

9. What principal amounts to £472. 10*s.* in 2 yrs. at $2\frac{1}{2}$ per cent.?

10. In what time does a sum of money double itself at 8 %?

11. The interest on £182. 10*s.* is £1. 1*s.* 6*d.* for 43 days. What is the rate?

12. What amounts to £191. 17*s.* 6*d.* in 8 months at $3\frac{1}{2}$ per cent.?

13. £3166. 13*s.* 4*d.* gives as interest £253. 6*s.* 8*d.* in 2 yrs. What is the rate?

14. In how many years does the interest on £166. 13*s.* 4*d.* become £8. 6*s.* 8*d.* at $2\frac{1}{2}$ per cent.?

15. What principal amounts to £766. 13*s.* 4*d.* in 2 yrs. 6 months at 6 per cent.?

16. In what time does £840 become £903. 14*s.* at $3\frac{1}{4}$ %?

17. £182. 10*s.* amounts to £184 4*s.* in 85 days. What is the rate?

18. What principal amounts to £429 in 160 days at $9\frac{1}{8}$ per cent.?

19. £160 amounts to £178. 4*s.* at $3\frac{1}{2}$ per cent. What is the length of time?

20. Find the principal which will become £294. 8*s.* in 75 days at 4%.

21. Find the rate at which £315. 10*s*. 8*d*. amounts to £591. 12*s*. 6*d*. in 7 yrs.

22. The interest on £5750 for $2\frac{1}{2}$ yrs. is £646. 17*s*. 6*d*. What is the rate?

23. In what time will £425 amount to £516. 10*s*. 4*d*. at 7·6 per cent.?

24. What is the principal, if the amount is £1618. 3*s*. 9*d*., the time 5 yrs. 8 months and the rate $4\frac{3}{4}$ per cent.?

25. In 7 yrs. 3 months £7630 amounts to £10672. 9*s*. 3*d*. What is the rate?

26. What sum of money lent at 4 % for 4 months amounts to £3466. 11*s*.?

27. At what rate per cent. will £3235 yield £420. 11*s*. interest in 4 years?

28. How much money does a man borrow for 3 years at $3\frac{1}{2}$ per cent. if he pays £77 interest?

29. A man borrows £5 from a money-lender and pays 2*s*. 6*d*. per month as interest. How much per cent. per annum does he pay?

30. What sum of money amounts to £486. 17*s*. in 2 years at $3\frac{1}{2}$ per cent.?

31. At what rate per cent. will £1625 amount to £1844. 7*s*. 6*d*. in 3 years?

32. What sum of money will amount to £350. 19*s*. $4\frac{1}{2}$*d*. in $6\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.?

33. If a sum of money trebles itself in 24 years at simple interest, what is the rate per cent.?

34. At what rate per cent. will £142. 10*s*. amount to £163. 13*s*. $11\frac{1}{4}$*d*. in $4\frac{1}{4}$ years?

35. In how many years will a sum of money double itself at 4 per cent. per annum, simple interest?

36. In how many days will the simple interest on £608. 6*s*. 8*d*. at 5 per cent. per annum amount to £12. 10*s*.?

37. How much does a man borrow at $3\frac{1}{2}$ per cent. if he pays £5. 15*s*. 6*d*. per annum as interest?

38. On what sum will the interest for 175 days at $2\frac{1}{2}$ per cent. amount to £7. 17*s*. 6*d*.?

39. £265. 10*s*. is the interest on a sum of money at 4 per cent.; what is the interest at 5 per cent.?

40. At what rate per cent. will 100 guineas amount to £107. 12*s*. 6*d*. in 8 months?

41. A man buys a house for £1725 and lets it for £100 per annum. If he pays 1*s*. in the £ income-tax on the rent, and an average of

£17. 7*s.* 6*d.* per annum for repairs, what rate per cent. does he get on his outlay?

42. A man has a house worth £2325. What rent must he charge in order to get 4 per cent. per annum for his money, if he has to pay £17. 10*s.* per annum for income-tax and repairs?

43. In $3\frac{1}{3}$ years £356. 6*s.* 8*d.* amounts to £409. 15*s.* 8*d.* at simple interest. What is the rate per cent.?

44. If the interest on a certain sum of money for two-fifths of a year at $4\frac{1}{2}$ per cent. is 11 guineas, what would be the interest on the same sum for three-quarters of a year at $3\frac{1}{2}$ per cent.?

45. In how many years will £355. 16*s.* 8*d.* amount to £403. 17*s.* 5*d.* at $4\frac{1}{2}$ per cent.?

46. At what rate per cent. simple interest will £868. 0*s.* 3*d.* produce £144. 13*s.* $4\frac{1}{2}$*d.* in $5\frac{1}{3}$ years?

47. If the rate of interest is £3. 6*s.* 8*d.* per cent. per annum, what is the principal upon which the interest will amount to £67. 1*s.* 2*d.* in 13 months?

Compound Interest.

185. It has been explained (Art. 179) that the amount of a sum of money at Compound Interest is found by increasing the Principal at the end of each year by adding the Interest for that year.

In questions on Compound Interest it is advisable to work throughout in decimals.

EXAMPLE 1. **Find the Compound Interest on £1750. 12s. 6d. for 3 years at 4 %.**

The amount at the end of each year is the Principal for the next year.

Each year the Principal for that year has to be multiplied by $\frac{4}{100}$ to obtain the Interest.

The multiplication by $\frac{4}{100}$ is done by multiplying by 4 and moving the resulting line of digits 2 places to the right before adding it to the corresponding Principal.

$\frac{4}{100}$	1750·625	Principal for 1st year.
	70·02500	Interest ,,
$\frac{4}{100}$	1820·65	Principal for 2nd year.
	72·8260	Interest ,,
$\frac{4}{100}$	1893·476	Principal for 3rd year.
	75·73904	Interest ,,
	1969·21504	Amount.

The total Interest may be found either by adding up the three sums mentioned as Interest for the 1st, 2nd and 3rd years, or better by deducting the original Principal from the final Amount.

$$\begin{array}{r}
1969{\cdot}21504 \\
1750{\cdot}625 \\
\hline
218{\cdot}59004 \\
20 \\
\hline
11{\cdot}8008 \\
12 \\
\hline
\underline{9{\cdot}6}
\end{array}$$

Interest = £218. 11*s.* 10*d.* to the nearest penny.

In these questions, in order to get the result to the nearest penny, we want it correct to the 3rd decimal place, and so we work to 5 places.

EXAMPLE 2. **Find the Amount of £258. 15s. in 3 years at $4\frac{1}{2}$ %.**

$$\begin{array}{r|r}
 & 258{\cdot}75 \\
\frac{4}{100} & 10{\cdot}3500 \\
\frac{1}{200} & 1{\cdot}29375 \\
\hline
 & 270{\cdot}39375 \\
\frac{4}{100} & 10{\cdot}81575 \\
\frac{1}{200} & 1{\cdot}35197 \\
\hline
 & 282{\cdot}56147 \\
\frac{4}{100} & 11{\cdot}30246 \\
\frac{1}{200} & 1{\cdot}41281 \\
\hline
 & 295{\cdot}27674 \\
 & 20 \\
\hline
 & 5{\cdot}5348 \\
 & 12 \\
\hline
 & \underline{6{\cdot}42}
\end{array}$$

Here multiply 282·561 by $\frac{4}{100}$, but allow for the figure carried from the neglected part.

The Amount = £295. 5*s.* 6*d.* to the nearest penny.

From the 295·27674 we might have written down the result, at a glance, as approximately

£295 + $2\frac{1}{2}$ florins + rather less than 26 farthings,

i.e. £295. 5*s.* 6*d.* to the nearest penny.

EXAMPLE 3. **Find the Amount of £416. 13s. 4d. in 2 years at 3 %.**

We can write the Principal as £416·66667 or we can multiply it by 3, making it £1250, and divide the result by 3.

$$\begin{array}{r|r}
 & 416{\cdot}66667 \\
\frac{3}{100} & 12{\cdot}50000 \\
\hline
 & 429{\cdot}16667 \\
\frac{3}{100} & 12{\cdot}87500 \\
\hline
 & \underline{442{\cdot}04167}
\end{array}
\qquad
\begin{array}{r|r}
 & 1250 \\
\frac{3}{100} & 37{\cdot}50 \\
\hline
 & 1287{\cdot}5 \\
\frac{3}{100} & 38{\cdot}625 \\
\hline
3\,) & 1326{\cdot}125 \\
 & \underline{442{\cdot}04167}
\end{array}$$

By either method the Amount = £442. 0*s.* 10*d.*

EXAMPLE 4. **Find the Amount of £866. 5s. in 2 years at $2\frac{1}{2}$% C.I.**

Here we have to multiply by $2\frac{1}{2} \div 100$ to find each year's interest, *i.e.* we have to multiply by $\frac{1}{40}$.

The division by 40 is easily done in one line.

$$\begin{array}{r|l} \frac{1}{40} & 866{\cdot}25 \\ & 21{\cdot}65625 \\ \hline \frac{1}{40} & 887{\cdot}90625 \\ & 22{\cdot}19766 \\ \hline & 910{\cdot}10391 \end{array} \qquad \text{Amount} = £910.\ 2s.\ 1d.$$

186. If the length of time were $3\frac{1}{2}$ years, the process would be the same as for 4 years, but only one half of the 4th year's interest would be taken.

187. In some cases the Interest is payable more frequently than once a year; *e.g.* it may be required to *find the amount of* £1000 *in* 2 *years at* 6 *per cent. per annum, the interest being paid half-yearly.*

Here the Interest for the 1st half-year is $\frac{3}{100}$ of £1000.

This is added in, and the Interest for the 2nd half-year is calculated in the same way.

Thus the process is equivalent to calculating the amount for twice as many years at half the given rate.

188. In finding the Amount of £P in 1 yr. at 3%, we add $\frac{3}{100}$ of P to the £P. The result is $\frac{103}{100}$ of P.

This Amount, forming the Principal for the 2nd year, gives, when multiplied by $\frac{103}{100}$, the Amount at the end of 2 years; and so on.

Thus the Amount of £P in 7 years at 3% is $P \times (\frac{103}{100})^7$.

EXAMPLE 1. **What Principal amounts to £1432. 4s. 3·6d. in 2 years at 3% compound interest?**

$$\begin{array}{r|l} 12 & 3{\cdot}6 \\ \hline 20 & 4{\cdot}3 \\ \hline & {\cdot}215 \end{array}$$

$$P \times \left(\frac{103}{100}\right)^2 = 1432{\cdot}215.$$

Multiply both sides by $\left(\frac{100}{103}\right)^2$.

$$\therefore\ P = 1432{\cdot}215 \times \left(\frac{100}{103}\right)^2$$

$$= \frac{14322150}{103 \times 103} = \frac{139050}{103} = 1350.$$

The required Principal is £1350.

EXAMPLE 2. **Assuming that £1 will in 20 years at 5 per cent. compound interest amount to £2·6533, show that the amount of £1 in 40 years will be about £7·04 and in 50 years about £11·47.**

In one year P amounts to $P + \frac{5P}{100}$, *i.e.* $P\left(1 + \frac{5}{100}\right)$, *i.e.* $P \times 1{\cdot}05$.

This sum in the 2nd year is multiplied again by 1·05.

$\therefore$ the amount in 2 years is $P \times (1{\cdot}05)^2$; and so on.

$(1{\cdot}05)^{20}$ = the amount of £1 in 20 years = £2·6533 (given).

The amount in 40 years $= (1{\cdot}05)^{40}$ = the square of $(1{\cdot}05)^{20}$

$= (2{\cdot}6533)^2 = 7{\cdot}04$ approximately.

$$\begin{array}{r} 2{\cdot}6533 \\ \underline{2{\cdot}6533} \\ 5{\cdot}3066 \\ 1{\cdot}5920 \\ {\cdot}1327 \\ {\cdot}0080 \\ \underline{{\cdot}0008} \\ \underline{\underline{7{\cdot}040}} \end{array}$$

$$(1{\cdot}05)^{10} = \sqrt{(1{\cdot}05)^{20}} = \sqrt{2{\cdot}6533} = 1{\cdot}629.$$

The amount in 50 years $= (1{\cdot}05)^{50} = (1{\cdot}05)^{40} \times (1{\cdot}05)^{10}$

$= 7{\cdot}04 \times 1{\cdot}629$

$= 11{\cdot}47$ approximately.

Compound and Simple Interest graphically compared.

189. Compound Interest on

	5	10	15	20	25 years.
£10 at 4 %,	£2·17	£4·80	£8·01	£11·91	£16·66.
Simple Interest	£2	£4	£6	£8	£10.

Across the page (Fig. 38) let each inch represent 5 years.
Up " " " " £2.

For Compound Interest mark the points (0, 0), (5, 2·17), (10, 4·80), etc., and connect these 6 points by a curve.

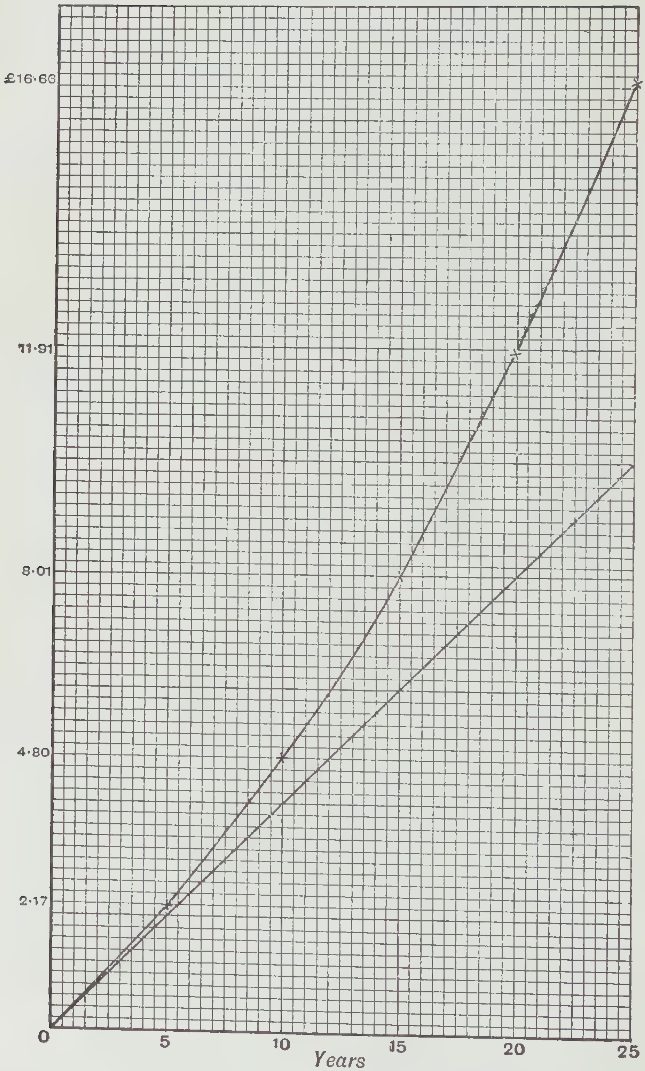


Fig. 38.

For Simple Interest the points are (0, 0), (5, 2), (10, 4), etc., and these all lie on a straight line.

Hints. $$567 \times 3\tfrac{1}{3} \times \frac{1}{100} = 567 \times \frac{1}{30} = \frac{56 \cdot 7}{3}.$$

$$682 \times 2\tfrac{1}{2} \times \frac{1}{100} = 682 \times \frac{1}{40} = \frac{68 \cdot 2}{4}.$$

EXAMPLES XXV. e.

[*Results required to the nearest penny. Check each line of multiplication before proceeding to the next.*]

Find the amount at compound interest of

1.	£300	in 2	years at	5	per cent.
2.	£750	„ 2	„	4	„
3.	£825	„ 2	„	6	„
4.	£6000	„ 3	„	5	„
5.	£400	„ 2	„	3	„
6.	£1450	„ 2	„	4	„
7.	£2000	„ 3	„	4	„
8.	£984	„ 2	„	$2\frac{1}{2}$	„
9.	£2290	„ 3	„	4	„

Find the compound interest on

10.	£3000	for 2	years at	5	per cent.
11.	£3125	„ 3	„	4	„
12.	£1500	„ 3	„	3	„
13.	£5000	„ 4	„	5	„

14. £450. 16*s*. 9*d*. for 2 years at 4 per cent.

15.	£5000	for 5	years at	4	per cent.
16.	£10000	„ 5	„	5	„

17.	£156. 6*s*. 8*d*.	for 3	years at	3	per cent.
18.	£168. 15*s*. 6*d*.	„ 3	„	4	„

19. £1000000 for 5 years at 4 per cent. (to the nearest £).

20.	£5432. 12*s*.	„ 2	„	$4\frac{1}{2}$	„
21.	£6480	„ 2	„	$3\frac{1}{3}$	„
22.	£378. 15*s*.	„ 2	„	$3\frac{1}{3}$	„
23.	£40	„ 3	„	$2\frac{1}{2}$	„
24.	£900	„ 3	„	$3\frac{1}{2}$	„
25.	£1840	„ 3	„	$4\frac{1}{2}$	„

Find the amount at compound interest of

26.	£154. 2*s.*	in $2\frac{1}{2}$ years	at 5	per cent.
27.	£166. 13*s.* 4*d.*	,, $2\frac{1}{2}$,,	6	,,
28.	£463. 2*s.* 6*d.*	,, $2\frac{1}{2}$,,	4	,,
29.	£400	,, 2 ,,	$3\frac{3}{4}$	,,
30.	£1200	,, 3 ,,	$3\frac{3}{4}$	,,
31.	£640. 3*s.* 9*d.*	,, $2\frac{1}{2}$,,	4	,,

If the interest be payable half-yearly, find the amount at compound interest of

32.	£729	in $1\frac{1}{2}$ years	at 6	per cent. per annum.
33.	£390. 12*s.* 6*d.*	,, 2 ,,	8	,, ,,
34.	£550	,, $1\frac{1}{2}$,,	5	,, ,,
35.	£1072	,, 2 ,,	5	,, ,,

Find the sum of money which at compound interest amounts to

36.	£848. 14*s.* 5*d.*	in 2 years at	3 %.
37.	£6749. 3*s.* 8*d.*	,, 3 ,,	4 %.
38.	£183. 17*s.* 2*d.*	,, 2 ,,	$2\frac{1}{2}$ %.
39.	£179. 19*s.* 7*d.*	,, 3 ,,	4 %.
40.	£582. 15*s.* 9*d.*	,, 3 ,,	3 %.

41. If the compound interest for 3 years at 3 % is £46. 7*s.* $3\frac{1}{4}$*d.*, what is the principal?

42. To what would £2500 amount at compound interest in 3 years if the rate for the 1st year were 3 %, for the 2nd 4 % and for the 3rd 5 %?

43. Find the difference in the compound interest on £100 at 4 % for 2 years according as the interest is paid yearly or half-yearly.

44. What sum at compound interest will amount to £650 at the end of the 1st year and £676 at the end of the 2nd year?

45. Assuming that at 4 % compound interest £1 becomes £1·4802 in 10 years, find the amount in 20 years, and in 25 years.

46. Money is invested at such a rate as to double itself at compound interest in 24 years. Find, to the nearest pound, how much £1000 will amount to in 12 years.

47. How many complete years must elapse before a sum of money, accumulating at 3 % compound interest, is increased by more than $12\frac{1}{2}$ per cent.?

48. Find the compound interest on £130 in 3 years at 4 per cent. per annum, interest being paid half-yearly.

49. Find the compound interest on £500 for $2\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum, interest being paid half-yearly.

50. Find the difference between the simple and compound interest on £1750 for 3 years at 4 per cent.

51. Given that £1 amounts in 5 years to £1·276 at 5 per cent. compound interest, find the amount in 15 years.

XXVI. DISCOUNT.

190. If I owe a man £255, the payment of which, according to agreement, has to be made 6 months hence, what sum of money ought I to pay him if I wish to discharge the debt at once?

I must evidently pay him such a sum as he can make into £255 by the end of the 6 months. The question is "How much can he add in 6 months to the sum which I pay him?" This depends upon the rate of interest. Suppose we consider it to be 4 per cent. The question then becomes: "*What sum of money at 4 per cent. will in 6 months amount to £255?*"

£100 amounts to £102.

What ,, ,, £255?

$$\text{The required sum} = \frac{100 \times 255}{102} = £250.$$

This sum of £250 paid now is as valuable to him as £255 paid at the end of the 6 months (assuming the rate to be 4 per cent.).

This £250 is called the **Present Worth** of the £255, and the deducted £5 is called the **Discount.**

It must be noticed that this £5, which I deduct, is what he can make by using the £250 at 4 per cent.

In fact **Discount is the Interest on the Present Worth.**

Also Discount is the sum due minus its Present Worth.

Present Worth may be denoted by P.W.

191. As we have shown above, the Present Worth of £255 is found by solving the question, "What will amount to £255 in 6 months at 4 per cent.?"

The Discount is found by subtracting from the £255 its Present Worth.

But is it not possible to find the Discount without going through the process of finding Present Worth? The following paragraph will show that it is.

At the given rate £100 becomes £102;

$\therefore$ £100 is the P.W. of £102;

$\therefore$ £2 is the Discount on £102.

What ,, ,, ,, £255?

Discount on £255 $= 2 \times \frac{255}{102} =$ £5.

Commercial Discount.

192. If we were finding the Simple Interest on £255 under the same conditions, we should multiply by 2 and divide by 100. The result would be £5. 2*s*.

There is not a great difference between the Interest and the Discount, and the Interest is more easily found: for the divisor is 100 instead of something rather greater than 100.

Commercially it is the custom to allow *Interest* to be deducted from an account, and to call it *Discount.*

Commercial Discount is therefore the same as **Simple Interest**; and what we have defined as Discount is generally called **True Discount.**

The difference between the Interest and the True Discount is sometimes known as Banker's Gain.

EXAMPLE 1. **Find the Present Worth of £621. 17s. 3d. due 6 months hence at 7 per cent.**

In such cases the sum due may be expressed either decimally or fractionally in terms of a £.

Here the interest on £100 is £$\frac{7}{2}$.

$\therefore$ £100 amounts to £$103\frac{1}{2}$,

i.e. £100 is the P.W. of £$103\frac{1}{2}$.

What ,, ,, £621. 17*s*. 3*d*.?

$$\therefore \text{ the P.W. required} = 100 \times \frac{621\frac{207}{240}}{103\frac{1}{2}} = \frac{200}{207} \times 621\frac{207}{240}$$

$$= 200 \times 3\tfrac{1}{240} = 200 \times \tfrac{721}{240}$$

$$= \tfrac{5 \times 721}{6} = \tfrac{3605}{6} = £600.\ 16s.\ 8d.$$

The True Discount = Sum due − P.W. = £21. 0*s*. 7*d*.

Without first finding the P.W. we might get the Discount as follows:

£$3\frac{1}{2}$ is the Discount on £$103\frac{1}{2}$.

What ,, ,, £621. 17*s*. 3*d*.?

$$\text{The required Discount} = 3\tfrac{1}{2} \times \frac{621\frac{207}{240}}{103\frac{1}{2}} = \frac{7}{207} \times 621\tfrac{207}{240}$$

$$= 7 \times 3\tfrac{1}{240} = 21\tfrac{7}{240}$$

$$= £21.\ 0s.\ 7d.$$

EXAMPLE 2. **Find the True Discount on £156. 19s. 6d. due 6 months hence at 8 per cent.**

£4 is the Discount on £104.

What " " £156. 19*s*. 6*d*.?

$$\text{The Discount} = 4 \times \frac{156{\cdot}975}{104} = \frac{156{\cdot}975}{26} = \frac{12{\cdot}075}{2}$$

$$= 6{\cdot}0375 = £6.\ 0s.\ 9d.$$

EXAMPLES XXVI. a.

[*Answers required to the nearest penny. Unless otherwise notified, Simple Interest is used in calculating Present Worth and Discount.*]

Find the Present Worth of

1.	£1219	due 2 years hence	at 3	per cent.
2.	£6838	" 1 year	" 4	"
3.	£555. 18*s*.	" 18 months	" 6	"
4.	£3965. 10*s*. 2*d*.	" 5 months	" 4	"
5.	£427. 7*s*.	" 3 months	" 7	"
6.	£6165	" 6 months	" 5½	"
7.	£1380. 7*s*. 6*d*.	" 9 months	" 3	"
8.	£3995. 4*s*.	" 3 years	" 4½	"
9.	£247. 19*s*. 9*d*.	" 6 months	" 4	"
10.	£1087. 14*s*. 6*d*.	" 2½ years	" 12	"
11.	£540. 1*s*. 5*d*.	" 3 years	" 4½	"

Find the True Discount on

12.	£622. 15*s*.	due in 2 years	at 3	per cent.
13.	£1297. 16*s*.	" 8 months	" 4½	"
14.	£1224. 1*s*. 4*d*.	" 2 months	" 10	"
15.	£226. 2*s*. 8*d*.	" 18 months	" 4	"
16.	£181. 18*s*. 3*d*.	" 2 years	" 5	"
17.	£457. 10*s*.	" 25 days	" 4	"
18.	£295. 15*s*.	" 73 days	" 7	"
19.	£1466	" 30 days	" 5	"
20.	£2038. 6*s*.	" 56 days	" 10	"

Find the Present Worth of

21. £1616. 2*s*. 8*d*. due in 2 years at $7\frac{1}{2}$ per cent.
22. £2700. 7*s*. 1*d*. „ 3 years „ $4\frac{1}{2}$ „
23. £4913. 8*s*. 11*d*. „ 4 months „ $4\frac{1}{2}$ „

Find the True Discount on

24. £930. 0*s*. 10*d*. due in 6 months at 8 per cent.
25. £187. 0*s*. 11*d*. „ 60 days „ 10 „
26. £796. 19*s*. 9*d*. „ 3 months „ 6 „

27. Find, to the nearest penny, the difference between the commercial and true discount on £765. 10*s*. due in 9 months at 4 per cent.

28. How much, to the nearest penny, does a banker gain by charging commercial instead of true discount on £750 due in 6 months at 4 per cent.?

193. It is important to bear in mind what has already been stated, viz., that **True Discount is the Interest on the Present Worth.**

Moreover, the Sum due − Present Worth = Discount.

∴ Interest on Sum due − Interest on P.W. = Interest on Discount.

i.e. **Interest − Discount = Interest on Discount.**

This is sometimes useful.

EXAMPLE 1. **What is the sum of money on which the difference between Interest and Discount for 4 months at 5 per cent. is 3s. 4d.?**

$$3s.\ 4d. = \text{Interest on the Discount}$$

$$= \text{Discount} \times \frac{1\frac{2}{3}}{100} = \text{Discount} \times \frac{1}{60};$$

$$\therefore\ \text{Discount} = £\tfrac{1}{6} \times 60 = £10.$$

$$\text{The Interest} = \text{Discount} + 3s.\ 4d. = £10\tfrac{1}{6};$$

$$\therefore\ £10\tfrac{1}{6} = \text{Sum due} \times \frac{1\frac{2}{3}}{100} = \text{Sum due} \times \frac{1}{60};$$

$$\therefore\ \text{Sum due} = £10\tfrac{1}{6} \times 60 = £610.$$

EXAMPLE 2. **The difference between Simple Interest and True Discount on a sum of money for $2\frac{1}{2}$ years at 4 per cent. is £4. 1s. 2d. What is the sum of money?**

If we denote the sum by £x, we have

$$\text{the Interest} = \frac{x \times 2\frac{1}{2} \times 4}{100} = \frac{x}{10}.$$

£10 is the True Discount on £110.
What „ „ £x?

$$\text{The True Discount} = \frac{10x}{110} = \frac{x}{11}.$$

But the difference between the Interest and True Discount $=£4.\ 1s.\ 2d.$

$$=£4\tfrac{7}{120};$$

$$\therefore\ \frac{x}{10}-\frac{x}{11}=4\tfrac{7}{120}=\frac{487}{120};$$

$$\therefore\ \frac{x}{110}=\frac{487}{120}.$$

Multiplying both sides by 110, we get

$$x=\frac{487}{120}\times 110=\frac{487\times 11}{12}=\frac{5357}{12}=446\tfrac{5}{12};$$

$\therefore$ the required sum $=£446.\ 8s.\ 4d.$

194. It may be sometimes required to find Present Worth on the assumption that Compound Interest is to be reckoned.

Example. **Find the P.W. of £2298. 8s. due 2 years hence at 4 per cent. Compound Interest.**

$$\begin{array}{r} 100 \\ 4 \\ \hline 104 \\ 4{\cdot}16 \\ \hline 108{\cdot}16 \end{array}$$

£100 is the P.W. of £108·16.

What ,, £2298·4?

$$\text{The required P.W.}=\frac{100\times 2298{\cdot}4}{108{\cdot}16}=\frac{2298{\cdot}4}{1{\cdot}0816}$$

$$=£2125.$$

EXAMPLES XXVI. b.

1. If £250 is the present worth of £255 due in 6 months, what is the rate of interest?

2. If £8. 8*s*. 9*d*. is the true discount on £383. 8*s*. 9*d*. due in 9 months, at what rate is the interest calculated?

3. £495 is the present worth of £504. 18*s*., interest being calculated at the rate of 4 per cent. When is the sum due?

4. If £13. 5*s*. is the true discount on a sum of money due in 8 months at 3 per cent., what is the sum due?

5. What is the difference (correct to a penny) between the commercial and true discount on a bill of £1775 due in 4 months at 3 per cent.?

6. What is the amount of a sum of money due in 4 years if its present worth is £450 and interest is calculated at the rate of $3\frac{1}{2}$ per cent (simple interest)?

7. The interest on £473. 2*s*. 6*d*. at 6 per cent. is equal to the true discount on £492. 1*s*. for the same time at the same rate. When is the latter sum due?

8. The interest on a sum of money for a year is £47, and the true discount on the same sum due in a year is £45. What is the sum of money?

9. The difference between the interest and true discount on a sum of money due in 6 months at $3\frac{1}{3}$ per cent. is £2. What is the sum?

10. If the interest on £4275 for 9 months is equal to the true discount on £4403. 5*s.* due in the same time, what is the rate per cent.?

11. If £7. 10*s.* 6*d.* is the true discount on £383. 15*s.* 6*d.* due in 8 months, what is the rate per cent.?

12. If £352. 13*s.* 4*d.* is the present worth of £365. 17*s.* 10*d.* due in 9 months, what is the rate per cent.?

13. The difference between the interest and the true discount on a sum of money for 7 months at 4 per cent. is 3*s.* 6*d.* Find the sum of money to the nearest penny.

14. Find the present worth of £385. 17*s.* 6*d.* due in 2 years at 5 per cent. (compound interest).

Bills of Exchange.

195. These are either Inland or Foreign; but the principle is the same in both sorts.

As an introduction to the idea, it is as well to consider what happens when a man makes at a shop a purchase for which he does not pay at once. The shop-keeper sends him an account of what he owes, and expects him to pay the sum mentioned. In the same way, if R. Jones consigns goods of the value of £1000 to T. Evans (on the understanding that payment is to be made after a certain time), it is R. Jones who draws the Bill requesting T. Evans to pay £1000 after the appointed interval. T. Evans, the debtor, 'accepts' the Bill by writing his name across it.

This is an Inland Bill.

£1000. 0s. 0d. MANCHESTER,
Jany. 15th, 1909.

Three months after date pay to our order
One thousand pounds for value received.

R. JONES.

Accepted Jany. 16th, 1909.
Payable at N.P. Bank, London.
T. EVANS.

The acceptance may be written anywhere on the face of the Bill. The document, which was a *Draft* before acceptance, becomes a *Bill* when the acceptance has been written on the face of it; and it is now negotiable, *i.e.* it can be passed from hand to hand as a means of exchange; but each successive holder of it must endorse it (*i.e.* sign his name on the back) before parting with it. An endorsement is a promise to pay on the part of the endorser in case the acceptor fails to pay.

In addition to its convenience as a means of transferring value from place to place, one advantage of such a document is that R. Jones, without pressing T. Evans for immediate payment, can get cash as soon as he likes by selling the Bill to a Banker or Bill-broker. The amount paid is the sum named *minus the discount.* The deduction which the Banker or Broker makes is *commercial discount*, or in other words *simple interest.*

If in any of the following questions the *theoretical discount* is required, it will be called *true discount.*

(A foreign Bill generally differs from the above form in that it contains the name of a third person, an agent to whom the drawer requests the acceptor to make the payment. Moreover, instead of "Three months after date," it may say "Three months after sight"; in which case the three months are reckoned from the date of the *acceptance*; the acceptor being supposed to have signed and dated the Bill as soon as it was presented to him.)

Days of Grace.

196. By a curious survival from a time when communication was much less rapid, it is still customary to allow '3 days' grace' before presenting a Bill for payment. So that one nominally due on October 7th is really due on October 10th.

Where a Bill is drawn at so many months it is always understood that they are calendar months. A Bill drawn on Feb. 5th at 5 months would be *nominally* due on July 5th, and *actually* (because of the 3 days' grace) on July 8th.

Suppose that the holder of such a Bill found, say on March 16th, that he required ready money. He could take the Bill to a Broker, who would reckon up the days which it had still to run

before 'maturing.' He would say: "A Bill drawn Feb. 5th at 5 months matures on July 8th: from now, March 16th, to July 8th there are 114 days, and I can discount it for you at ... per cent." The sum named on the Bill minus the commercial discount for the 114 days is handed to the holder who is parting with the Bill.

EXAMPLES XXVI. c.

[*Answers should be calculated to the nearest penny.*]

Find the commercial discount on a bill of

		Drawn.	Discounted.
1.	£560	May 1st at 6 months,	Aug. 23rd at $7\frac{1}{2}$ %.
2.	£472. 12*s.* 6*d.*	Jan. 13th at 8 months,	Feb. 9th at 4 %.
3.	£297. 3*s.* 5*d.*	March 27th at 3 months,	May 11th at 6 %.
4.	£389. 10*s.*	Aug. 9th at 4 months,	Sept. 28th at 10 %.
5.	£1277. 10*s.*	Feb. 20th at 2 months,	March 9th at 8 %.
6.	£87. 9*s.*	April 11th at 6 months,	May 21st at 5 %.
7.	£4250. 2*s.* 5*d.*	April 27th at 5 months,	June 13th at 6 %.

Find the true discount on a bill of

		Drawn.	Discounted.
8.	£5202. 4*s.* 3*d.*	Jan. 7th at 6 months,	April 28th at 10 %.
9.	£7597. 16*s.*	July 10th at 4 months,	Aug. 1st at $12\frac{1}{2}$ %.
10.	£1941. 3*s.* 9*d.*	May 5th at 6 months,	Aug. 27th at $7\frac{1}{2}$ %.

XXVII. PROFIT AND LOSS PER CENT.

197. SUPPOSING that A sells for £107 goods which cost him £100, and that B sells for £26 that which cost him £20, we see that A's profit is £7, whereas B's is only £6. At the same time it would be misleading to say that A's transaction was more profitable than B's; for A on an outlay of £100 made only £1 more than B made on an outlay of £20.

The clearest comparison is got by measuring the two *rates of profit.*

To do this we take some **standard outlay**, say £100.

If goods costing £100 are sold for £93, there is a **loss per cent.**, in this case 7 per cent.

In the instance mentioned above we see that A with an outlay of £100 made £7 profit.

∴ his *rate of profit* was 7 per cent.

B, spending £20, made £6 profit.

∴ at the same rate he would make a profit of £30 if his outlay were £100.

∴ B's *rate of profit* was 30 per cent.

198. ***It must be remembered that profit per cent. and loss per cent. are calculated on the outlay (or cost price), not on the selling price.***

100 % of the cost price is the same as the cost price.

A price which gives 3 % profit = 103 % of cost price.

" " 4 % loss = 96 % "

EXAMPLE 1. **A coal-merchant finds that his outlay per ton, including expenses, is 16s. for a certain sort of coal. At what price must he sell it to gain 25 per cent.?**

First Method.

	£.	s.	d.
The cost =	0 .	16 .	0
Profit = $\frac{25}{100}$ of cost =	0 .	4 .	0
∴ selling price =	1 .	0 .	0

Second Method.

What is bought for 100 is sold for 125.

What is bought for 16 is sold for how much?

The selling price $= \frac{125 \times 16}{100} = \frac{5 \times 16}{4} = 20$ shillings.

The 2^{nd} method has the advantage that it is applicable to a number of questions, such as the inverse one of finding the cost when the selling price and gain (or loss) per cent. are given; and again, to such questions as this—Given that selling at £35 causes a gain of 5 %, what is the gain per cent. in selling at £43?

EXAMPLE 2. **A man selling goods for £196 finds his profit to be 12 %. What did they cost him?**

He sells for 112 what cost him 100.

" 196 " how much?

The required cost $= 100 \times \frac{196}{112} = 100 \times \frac{7}{4} = 7 \times 25 =$ £175.

Verification. The cost = £175.

The profit $= \frac{12}{100}$ of $175 = \frac{3}{25}$ of $175 =$ £21.

∴ (by addition) the selling price = £196.

Example 3. **By selling goods for £35 I gain 5%. What do I gain per cent. by selling for £43?**

£35 is 105 per cent. of the cost.
£43 ,, x ,, ,,

$\therefore\ x=\frac{43}{35}\times 105=129.$

$\therefore$ £43 = 129 per cent. of the cost.

$\therefore$ the gain = 29 per cent.

The following is wrong. It is left to the student to discover why.

£35 gives a profit of 5 per cent.
£43 ,, ,, how much per cent.?

The required percentage $=\frac{5\times 43}{35}$

$=\frac{43}{7}=6\frac{1}{7}.$

Example 4. **If a dealer by selling goods for £74. 3s. 1d. gain 11%, at what price must he sell to gain 25%?**

£74. 3*s*. 1*d*. is 111 per cent. of the cost.
What price ,, 125 ,, ,,

$$\text{The required price is } \frac{£74.\ 3s.\ 1d.\times 125}{111}$$

$$=\frac{£2.\ 0s.\ 1d.\times 125}{3}$$

$$=\frac{£250.\ 10s.\ 5d.}{3}$$

$= £83.\ 10s.\ 2d.$ to the nearest penny.

Example 5. **If the manufacturer makes a profit of 25 per cent., the agent one of 8 per cent. and the shopkeeper one of 20 per cent., what is the cost to the manufacturer of an article which is sold in the shop for £32. 8s.?**

Here 25, 8 and 20 per cent. must not be added together; for they are percentages on different prices.

The principle to work upon is that of adding a percentage by multiplication.

If £x be the cost to the manufacturer, the cost to the agent is $x\times\frac{125}{100}$. The cost to the shopkeeper is $x\times\frac{125}{100}\times\frac{108}{100}$, and his selling price is

$$x\times\frac{125}{100}\times\frac{108}{100}\times\frac{120}{100}.$$

$$\therefore\ x\times\frac{125}{100}\times\frac{108}{100}\times\frac{120}{100}=32{\cdot}4,$$

$$i.e.\ \ x\times\frac{5}{4}\times\frac{27}{25}\times\frac{6}{5}=32{\cdot}4,$$

$$i.e.\ \ \frac{162}{100}\text{ of } x=32{\cdot}4.$$

$$\therefore\ x=\frac{100}{162}\text{ of } 32{\cdot}4=\frac{3240}{162}=20,$$

i.e. the cost to the manufacturer was £20.

EXAMPLE 6. **By selling an article for a certain price a man gains 5 %. If he had sold it for 3s. more, he would have gained 6 %. What was the cost price?**

The difference between 106 % of cost and 105 % of cost = 3 shillings.

$$\therefore\ 3 \text{ shillings} = 1\,\% \text{ of cost} = \tfrac{1}{100} \text{ of cost.}$$

$$\therefore\ \text{the cost} = 300 \text{ shillings} = £15.$$

EXAMPLE 7. **A mixture consisting of 3 lb. of tobacco costing 4s. a lb. and 5 lb. costing 3s. 6d. a lb. and 6 lb. of another sort costing 3s. 9d. a lb. is sold at 4d. an ounce. What profit per cent. is there?**

		£.	s.	d.
3 lb. at 4*s.*	cost	0	12	0
5 ,, 3*s.* 6*d.*	,,	0	17	6
6 ,, 3*s.* 9*d.*	,,	1	2	6
∴ 14 lb. of the mixture cost		2	12	0

The selling price of the 14 lb. $= 4d. \times 16 \times 14$

$$= 8d. \times 112 = 896d.$$

$$= 74s.\ 8d.$$

$$= £3.\ 14s.\ 8d.$$

£1. 2*s.* 8*d.* is the profit on £2. 12*s.*

What ,, ,, £100?

$$\text{The required profit per cent.} = 1\tfrac{2}{15} \times \frac{100}{2\frac{3}{5}}$$

$$= \frac{17}{15} \times \frac{500}{13} = \frac{1700}{39}$$

$$= 43{\cdot}59.$$

EXAMPLE 8. **A man bought 100 qrs. of wheat, and sold part at a profit of 8 %, part at a loss of 2 %. On the whole he gained 5 %. If the whole sold for £210, what was the cost of the wheat, and how many qrs. were sold at a loss?**

On the whole there was a gain of 5 %.

100 was the cost price of what was sold for 105.

What ,, ,, ,, 210?

The cost = £200, *i.e.* £2 per qr.

Suppose x qrs. were sold at 8 % profit.

,, y ,, ,, 2 % loss.

x qrs. of the 1st kind cost him £$2x$.

What cost 100 sells for 108.

,, $2x$,, how much?

$$\text{Selling price} = \frac{108 \times 2x}{100}.$$

y qrs. of the 2nd kind cost him £$2y$.

What cost 100 sells for 98.

,, $2y$,, how much?

$$\text{Selling price} = \frac{98 \times 2y}{100}.$$

Then $$2x \times \frac{108}{100} + 2y \times \frac{98}{100} = 210.$$

$$\therefore\ 108x + 98y = 10500.$$

Also $$108x + 108y = 10800. \quad (\text{Since } x + y = 100)$$

$$\therefore\ 10y = 300.$$

$$\therefore\ y = 30.$$

The wheat cost £2 a quarter; and 30 qrs. were sold at a loss.

Verification.

30 qrs. at £2 cost £60; loss on this $= \frac{1}{50}$ of £60 $=$ £1. 4*s*.
70 qrs. at £2 cost £140; profit on this $= \frac{8}{100}$ of £140 $=$ £11. 4*s*.
Total profit $=$ £10. $\therefore$ the whole was sold for £210.

EXAMPLE 9. **A grocer gains 20 % by selling at 2s. per lb. a mixture formed of 7 lb. of a common tea with 2 lb. of better. If he had mixed 7 lb. of the latter with 2 lb. of the former he would have lost 20 % by selling at that price. What did each kind of tea cost him?**

The cheaper cost him x shillings per lb.
,, dearer ,, y ,,

In the 1st case he gained 20 %.

What was sold for 120 cost 100.
,, ,, 2*s*. cost how much?
Cost $= \frac{200}{120} = \frac{5}{3}$ shillings per lb.

In the 2nd case he would lose 20 %.

What was sold for 80 cost 100.
,, ,, 2*s*. cost how much?
Cost $= \frac{200}{80} = \frac{5}{2}$ shillings per lb.

$$\left.\begin{aligned} \text{Thus}\quad 7x + 2y &= \tfrac{5}{3} \times 9 = 15 \\ \text{and}\quad 2x + 7y &= \tfrac{5}{2} \times 9 = \tfrac{45}{2} \end{aligned}\right\}.$$

$$\therefore\ 49x + 14y = 105$$

and $$4x + 14y = 45.$$

By subtraction we have $$45x = 60,$$

$$x = \tfrac{4}{3}.$$

$$\therefore \text{ also } \tfrac{8}{3} + 7y = \tfrac{45}{2},$$

$$7y = \tfrac{45}{2} - \tfrac{8}{3} = \tfrac{135 - 16}{6} = \tfrac{119}{6}.$$

$$\therefore\ y = \tfrac{17}{6};$$

i.e. the cheaper cost 1*s*. 4*d*. a lb., the dearer 2*s*. 10*d*.

EXAMPLES XXVII. (*Oral* 1–52.)

1. When a man buys and sells something, on which price is the gain or loss per cent. calculated?

2. Given the selling price and the gain per cent., how can we find the actual gain?

3. To increase a number by 17 per cent. of itself by means of one multiplication, what multiplier would you employ?

4. To decrease it by 6 per cent., what multiplier would you employ?

Find the gain or loss per cent. in the following cases:

5. Cost price 4 shillings, selling price 5 shillings.
6. „ 5 „ „ 4 „
7. „ 1 shilling, „ 1*s.* 6*d.*
8. „ 2*s.* 6*d.*, „ 2*s.*
9. „ £1, profit 1 shilling.
10. „ $2\frac{1}{2}$*d.*, „ $\frac{1}{2}$*d.*
11. „ £1, „ 2*s.* 6*d.*
12. „ 1 shilling, „ 1*d.*
13. „ 6*s.* 8*d.*, selling price 4*s.* 8*d.*
14. „ 12*s.*, „ 10*s.* 6*d.*

What selling price will give a profit of

15. 10 % on £23? 16. 5 % on £35? 17. $7\frac{1}{2}$ % on £120?

What is the gain or loss per cent. on goods

18. bought for 15*s.*, sold for 16*s.* 6*d.*?
19. „ 8*s.*, „ 10*s.*?
20. „ 10*s.*, „ 8*s.*?
21. „ £25, „ £28?
22. „ £20, „ £18?
23. „ £12. 10*s.*, „ £15?
24. „ £15, „ £12?

25. Increase £125 by 4 per cent.
26. „ £76 „ 10 „
27. „ £300 „ 3 „
28. „ £30 „ 3 „
29. „ £174 „ 10 „
30. Diminish £50 by $\frac{1}{2}$ per cent.
31. „ £80 „ 10 „
32. „ £4 „ 5 „

What is the ratio of the selling price to the cost price when there is

33. a gain of 11 %? 34. a gain of 12 %? 35. a gain of 20 %?
36. „ 25 %? 37. „ 15 %? 38. „ $8\frac{1}{2}$ %?
39. a loss of 5 %? 40. a loss of 7 %? 41. a loss of 20 %?
42. „ 25 %? 43. „ $12\frac{1}{2}$ %?

What is the selling price when

44.	the cost is £1, and the gain		15 % ?
45.	,,	£2. 10*s*., ,,	8 % ?
46.	,,	£5, ,,	6 % ?
47.	,,	6*s*. 8*d*., ,,	5 % ?
48.	,,	3*s*. 4*d*., and the loss	5 % ?
49.	,,	£6. 5*s*., ,,	8 % ?
50.	,,	£3. 2*s*. 6*d*., ,,	16 % ?
51.	,,	8*s*. 4*d*., and the gain	11 % ?
52.	,,	£12. 10*s*., and the loss	8 % ?

53. A man sold for 13*s*. 9*d*. something which cost him 11*s*. What was his profit per cent. ?

54. What gain per cent. is there in buying at 11*s*. 8*d*. and selling at 12*s*. 3*d*. ?

55. A chair cost the manufacturer 6 shillings and was sold by him for 7*s*. 6*d*. What profit per cent. did he make ?

56. What is the loss per cent. in buying for 21*s*. and selling for 17*s*. 6*d*. ?

57. What gain per cent. is there when these prices are interchanged ?

58. A merchant made a profit of £54 on an outlay of £720. How much per cent. was it ?

59. A bicycle which cost £4. 11*s*. 8*d*. was sold for 8*s*. 3*d*. less than this. What was the loss per cent. ?

60. A grocer gets sugar at 17*s*. 6*d*. per cwt. and retails it at $2\frac{1}{2}$*d*. per lb. What percentage of profit is he making ?

61. A horse, bought for £37. 10*s*., is sold at a profit of 24 %. What is the selling price ?

62. What does a person gain or lose per cent. by selling butter at 1*s*. 5*d*. a lb. which cost him £7 a cwt. ?

63. What is the gain per cent. when the cost price is 10*s*. 6*d*. and the selling price 12*s*. 9*d*. ?

64. Butter was bought at 5 guineas per cwt. and sold at $10\frac{1}{2}$*d*. per lb. What was the gain or loss per cent. ?

65. Some eggs were bought at the rate of 10 for a shilling and sold at 8 a shilling. What was the profit per cent. ?

66. A grocer buys tea at 12 guineas per cwt. and retails it at 2*s*. 9*d*. per lb. What does he gain per cent.?

67. What was the cost price of goods on which 20 % was lost by selling them for £64 ?

68. What was the cost price of an article sold for 24*s*. 6*d*. at a profit of $16\frac{2}{3}$ per cent. ?

69. What was given for a horse by a man who sold it at 28 % profit for £68 ?

70. Sugar sold at $2\frac{3}{4}d.$ per lb. gives a profit of $28\frac{1}{3}$ %. What was the cost per cwt.?

71. A dealer sold 30 tons of coal for £28. 10*s*., and so made 14 % profit. How much per ton did the coal cost him?

72. A piano was sold for £57. If this meant a loss of 5 %, what should have been the selling price to give a profit of 10 %?

73. If the profit on selling some eggs at 1*d*. each was 5 %, what would be the loss per cent. if they were sold at 7 for 6*d*.?

74. Apples are sold at the rate of 35 for 2 shillings, and the gain is 20 %. At what price were they bought?

75. Sugar costing 17*s*. 6*d*. per cwt. is retailed at 2*d*. per lb. What is the gain per cent.?

76. 100 oranges were bought for 2*s*. 6*d*. 10 were thrown away, and the rest were sold at $\frac{1}{2}d.$ each. Find the gain per cent.

77. A man sells 24 pears for the price which he gave for 35. Find his gain per cent.

78. An article sold for 2*s*. 11*d*. gives a loss of $12\frac{1}{2}$ %. At what price should it be sold to give 15 % gain?

79. If 4 % is lost by pricing some silk at 10*s*. a yard, what ought to be the price to give a profit of 5 %?

80. By selling envelopes at 2*s*. 8*d*. a thousand, a stationer gained 8 %. What would he have gained per cent. by selling at 2*s*. 9*d*.?

81. What addition must be made to a price of 4*s*. 4*d*. to convert a loss of 9 % into a gain of 5 %?

82. A merchant sells at an advance of 12 % on the cost price. How much per cent. of the selling price is profit?

83. Coal raised for 14 shillings a ton costs 25 % of that for carriage, and the dealer's profit on the whole is 20 %. At what price does he sell it?

84. If a profit of £39. 17*s*. 6*d*. is made by selling 1 ton 2 cwt. 3 qr. 4 lb. of tea at 2*s*. $9\frac{3}{4}d.$ a lb., what did it cost per lb., and what is the rate of profit?

85. A watch is offered for sale for £5. 15*s*.; and, if that price be reduced by 5 %, the seller will still make $9\frac{1}{4}$ % profit. How much did the watch cost him?

86. Postcards used to be sold at 6*d*. for 10; later at 11 for 6*d*. What was the decrease per cent. in price.

87. £773. 11*s*. is the selling price of goods to make 8 % profit. What alteration in the profit per cent. is caused by adding £14. 6*s*. 6*d*. to this price?

88. In one year a man increases his capital of £300 by 20 %, and in the next year diminishes his capital by 20 %. How much is his loss on the 2 years?

89. A grocer bought 3 cwt. of tea at £13. 6*s*. 8*d*. per cwt. and 5 cwt. at £16. He mixed them and sold the mixture to gain 12 %. What was the selling price of a lb.?

90. By selling at a certain price a dealer loses 22 %. If he sold at $\frac{1}{3}$ more, what would he gain per cent.?

91. A coal merchant sells 10 tons at 22*s.* 6*d.* a ton, 9 tons at 21*s.* 6*d.* and 5 tons at 20*s.* Each sort cost him 17*s.* 6*d.* a ton. Calculate his profit per cent.

92. Coffee and chicory, costing 1 shilling a lb. and 4*d.* a lb. respectively, are mixed in the ratio of 5 lb. to 1 lb., and the mixture is sold at a shilling a lb. What is the profit per cent.?

93. 90 oranges were bought for 2*s.* 6*d.* 20 were given away; half the rest were sold at 5 for 3*d.*, half at 5 for 2*d.* What was the gain per cent.?

94. The loss on a motor-car was 3 %. If it had been sold for 21 guineas more there would have been a gain of 4 %. What was the cost price?

95. A tobacconist mixes 17 lb. of tobacco costing him 4*s.* a lb. with 25 lb. costing 4*s.* 8*d.* a lb., and sells the mixture at 4*d.* an ounce. What is his gain per cent.?

96. What was the cost to a wholesale dealer of an article sold at a shop for 8*s.*, if the retail dealer makes a profit of 20 % and the wholesale dealer one of 28 %.

97. A dealer buys three sorts of tea at 1*s.* 6*d.*, 2*s.* 3*d.*, 1*s.* per lb., and makes a mixture containing 6 lb. of the 1st and 4 of the 2nd to every 7 lb. of the 3rd. What must be his selling price to gain 2 %?

98. A horse was sold at a loss of 20 %. If £12 more had been charged the profit would have been 10 %. What was the cost price?

99. Goods are imported at an expense equal to 35 % of the cost of production, and the importer makes 15 % on the whole outlay by selling them at £7. 15*s.* 3*d.* per ton. What was the cost of production of a ton?

100. A blend of 3 sorts of tea was made with 7 lb. at 1*s.*, 8 at 1*s.* 3*d.* and 9 at 1*s.* 8*d.* Put such a price on the lb. as to give $6\frac{1}{4}$ % profit.

101. A sold an article to B at a profit of $12\frac{1}{2}$ %, B to C at 15 % profit. If C paid £46. 11*s.* 6*d.* for it, what did it cost A?

102. In what ratio are coffee at 1*s.* and chicory at 4*d.* a lb. mixed together, if the price of 1*s.* a lb. for the mixture gives 20 % profit?

103. A dealer bought 56 sheep, and then sold $\frac{3}{8}$ of them at a profit of 15 %, $\frac{2}{7}$ at 10 % profit and the rest at 5 % loss, and thus gained £5. 14*s.* on the transaction. What did he pay for them?

104. A railway company, whose expenses were half the receipts from fares, reduced their fares, and thus added 8 % to these receipts, but increased the expenses by 10 %. What did the company gain or lose per cent. by this?

105. A tradesman sold 72 yds. of cloth for the sum which 83·52 yds. cost him. What was his profit per cent.?

106. An estate was bought, and sold for £625 less, the loss per cent. being $1\frac{1}{4}$. Find at what price it should have been sold to gain $12\frac{1}{2}$ %.

107. A milk-seller pays 13*d.* a gallon for milk, and adds water. By selling at 2*d.* a pint he clears 40 per cent. What is the ratio of milk to water?

108. A sells a house to B for £4550, losing 9 per cent. B sells to C at a price at which A would have gained 17 per cent. What was B's profit per cent.?

109. A man embarks his whole property in four successive ventures, increasing his property by 100 % in his first venture and losing 20 % in each of the others. What per cent. of his original property is his profit?

110. How much per cent. must a tradesman add on to the cost price of his goods that he may make 10 % profit after allowing his customers a reduction of 5 % on their accounts?

111. A tradesman buys an article and sells it at a gain of 10 %. If he had bought it at 10 % less and sold it for 6*d.* less, he would have made a profit of 20 %. Find the cost price.

112. In what ratio does a seller blend two sorts of spirit costing 14*s.* 6*d.* and 17*s.* 6*d.* a gallon if, by selling the mixture at 17*s.* 3*d.*, he gains $12\frac{1}{2}$ per cent.?

113. A bicycle was sold at a loss of 40 % on the cost price, and a second bicycle was bought with the proceeds when 5 guineas had been added. This second bicycle was sold at a loss of 35 per cent., and a third bicycle was bought with the proceeds after the addition of £5. 15*s.* 3*d.* This third bicycle cost £16. What was the price of the first?

114. A tradesman has 17 articles worth 7*s.* $2\frac{1}{2}$*d.* each, 12 worth 6*s.* $8\frac{1}{4}$*d.* each and 4 worth 8*s.* 1*d.* each. Find the average value of the articles, and what the profit per cent. would be if they were sold for 7*s.* 6*d.* each.

XXVIII. STOCKS AND SHARES.

199. IF a business is to be started, some capital must be provided by the owner or owners. In the case of a large business the proprietorship may be open to the public, as it is in the case of railway companies for instance.

Suppose a new railway is to be constructed. The public are invited to become partners in the concern by subscribing the necessary capital. One man may take a large number of shares of the capital, others may take fewer shares. The shares may

be £1 or £10, or possibly £100. It will be convenient to speak of £100 shares for the present.

Suppose the capital of the Company to be a million £. The holder of a £100 share has one ten-thousandth part of the capital, and is entitled to $\frac{1}{10000}$ of those profits which are distributed among the shareholders after the year's working.

The distributed money, which is called the *dividend*, is dependent on the amount of profit made by the Company.

The Company may raise additional capital by offering to the public a sort of share which bears a fixed rate of interest, and whose holder is more protected against loss of his capital in the event of the railway not doing well. What are called *Debentures* are an instance of this sort.

200. Governments and Corporations use the same means of raising money, promising to pay a certain rate, say £3 or £4 on every £100 *stock*, *i.e.* on every £100 lent to them.

Thus a man who wishes to invest money may buy shares of some Company, or Stocks of some Government or town.

It will be convenient to have a name to denote the £100 share or Stock of a Company, Corporation or Government. The name we shall adopt is a **Bond.**

By 1 bond we mean £100 Stock.

By £100 sterling we mean £100 actual money.

It is very necessary to distinguish between £100 stock and £100 sterling. Originally 1 bond may have cost £100 sterling; but the value of a bond keeps changing according to the success of the Company or the credit of the Government or Corporation in question.

A bond is represented by a piece of paper, a certificate held by the owner. It may be bought and sold over and over again on the Stock Exchange; and its value may vary from day to day according to the state of the market. There are some railways whose £100 stock is worth £140 or more; there are some companies whose £100 stock is worth £20 or less. As a case of enormous increase in value it may be mentioned that some years ago a £100 share in the New River Water Co. was sold by auction for £125.000.

201. When one speaks of "a 4 per cent. stock," as for instance the 4 % New Zealand Government Stock, it is to be understood that £4 *is the income paid every year on the* £100 *stock, not on* £100 *of sterling money invested in that stock.* The consolidated stock of our own kingdom is known by the name of *Consóls.* During the Great War (1914-1918) the Government had to raise large sums of money, and, to accomplish this, had to offer higher rates of interest. The National Debt was therefore increased by several stocks, known generally as War Loan, and issued at varying rates of interest.

202. It is essential, in working questions on this subject, that **it should be quite distinctly recognised when it is stock and when it is sterling money which is meant.**

For our unit of stock we shall employ 1 *bond*, *i.e.* £100 stock.* If the learner has a clear notion whether stock or actual money is meant, and if he also understands that bonds, or stocks, can be bought and sold just as tons of coal or any other commodity can, he will find many questions on stocks quite simple.

[The price mentioned is the price of 1 bond (£100 stock) unless it is otherwise stated in the question.]

When a stock is sold it is said to have *realised* such and such an amount, *e.g.* if £300 stock is sold for £384, it is said to "realise" £384, *i.e.* to fetch so much real money.

If the price of £100 stock is £100, it is said to be at **par.**
" " " £103, " " 3 % **premium.**
" " " £98, " " 2 % **discount.**

Example 1. **(a) Find the cost of £2375 L. & N.W. Railway stock at 140. (b) Find what this stock will realise when the price has risen to 143.**

(*a*) £100 stock costs £140 cash.
£2375 " £x "

$$\frac{x}{140}=\frac{2375}{100}=23\cdot75.$$

$$\therefore\ x=140\times23\cdot75=7\times475=£3325.$$

(*b*) £100 stock sells for £143 cash.
£2375 " " how much cash?

$$\text{The required value}=\frac{143\times2375}{100}=143\times23\cdot75$$

$$=3396\cdot25=£3396.\ 5s.$$

*There have been many changes in the price of stocks since 1914 but the principle on which problems dealing with them are based remains the same.

EXAMPLE 2. **Find the quantity of New South Wales 3 per cent. stock at 88 which can be bought for £3762.**

[Here the words '3 per cent.' are unnecessary for our purpose. We may regard them as the name of the stock. They would be required if we had to calculate the income obtained.]

£88 buys £100 stock.

£3762 ,, how much stock?

The quantity of stock $= \frac{100 \times 3762}{88} = \frac{100 \times 342}{8} = \frac{34200}{8} = £4275.$

EXAMPLE 3. **Find the yearly income from the possession of £3780 4% stock at 92.**

Here the £3780 is stock, not sterling money.

£4 is the income on £100 stock.

What ,, ,, £3780 stock?

The income $= \frac{4 \times 3780}{100} = 4 \times 37 \cdot 8 = 151 \cdot 2 = £151.\ 4s.$

[The mention of 92, the price of the stock, is unnecessary information.]

EXAMPLE 4. **Find the yearly income derived from investing £3937. 10s. in a 4% stock at $93\frac{3}{4}$.**

£4 is the income on £$93\frac{3}{4}$ cash.

What ,, ,, £$3937\frac{1}{2}$ cash?

$$\text{The income} = \frac{4 \times 3937\frac{1}{2}}{93\frac{3}{4}}$$

$$= \frac{15750 \times 4}{375} = \frac{5250 \times 4}{125}$$

$$= \frac{1050}{25} \times 4 = \frac{210}{5} \times 4 = £168.$$

EXAMPLE 5. **How much must be invested in the $2\frac{1}{2}$% Consols at 89 to produce an annual income of £422?**

£89 invested will buy £100 stock; and the income from this will be £$2\frac{1}{2}$.

£$2\frac{1}{2}$ is the income from £89 cash.

£422 ,, ,, how much cash?

$$\text{The cash required} = \frac{89 \times 422}{2\frac{1}{2}} = 168 \cdot 8 \times (90 - 1)$$

$$= 15192 - 168 \cdot 8$$

$$= 15023 \cdot 2$$

$$= £15023.\ 4s.$$

EXAMPLE 6. **If a man buys a 5% stock at 130, what percentage is he getting on his capital outlay?**

£5 is the income from £130 cash.

What ,, ,, £100 ,,

The percentage obtained $= \frac{500}{130} = \frac{50}{13} = 3 \cdot 846\%.$

EXAMPLE 7. **Which is the more paying investment, a 3 % stock at 86 or a 4% stock at 110?**

(1) The interest on £86 cash is £3.

$\therefore$,, ,, £100 ,, $=\frac{3\times100}{86}=\frac{150}{43}=3\frac{21}{43}=$ £3·48....

(2) The interest on £110 cash is £4.

. ,, ,, £100 ,, $=\frac{400}{110}=\frac{40}{11}=$ £3·63....

$\therefore$ the latter stock is the better investment.

EXAMPLE 8. **What increase of capital is caused by investing £8653 in a railway stock at 112 and selling out when the price has risen to 125?**

£13 is the profit on £112 cash.
What ,, ,, £8653 ,,

$$\text{The required increase}=\frac{13\times8653}{112}=\frac{112489}{16\times7}$$
$$=\frac{7030\cdot5625}{7}$$
$$=1004\cdot366$$
$$=£1004.\ 7s.\ 4d.$$

EXAMPLE 9. **Find the alteration of my income caused by transferring £3485 stock from a 3 % stock at 96 to a $4\frac{1}{2}$ % stock at 102.**

[Even if the word 'stock' after the £3485 were omitted, the meaning would still be that the £3485 was stock; for we are told that it has to be *transferred* from one stock to another.]

I hold £3485 stock.

The income from £100 stock is £3.

$$\therefore \text{ the income from £3485 stock}=\frac{3\times3485}{100}=104\cdot55$$
$$=£104.\ 11s.$$

I sell the stock at 96.

For £100 stock I receive £96 cash.
,, £3485 ,, ,, how much cash?

$$\text{The cash received}=\frac{96\times3485}{100}=£96\times34\cdot85.$$

This money I invest at 102.

Income from £102 cash is £$4\frac{1}{2}$.

Income from £$96\times34\cdot85$ cash is how much?

$$\text{Income}=\frac{4\frac{1}{2}\times96\times34\cdot85}{102}=\frac{4\frac{1}{2}\times16\times34\cdot85}{17}$$
$$=9\times8\times2\cdot05=9\times16\cdot4=147\cdot6$$
$$=£147.\ 12s.$$

The original income was £104. 11*s*.

$\therefore$ the increase of income is £43. 1*s*.

203. In questions concerning investment in various Companies we may meet with shares of £1, £5, £10 and so on, instead of bonds of £100 stock.

For instance, take the following question:

EXAMPLE. **I invest £728 in £5 shares of a gas company at £6. 10s. per share. How many shares do I buy? What do I realise by selling them at £6. 14s. per share?**

$£6\frac{1}{2}$ buys one share.

£728 „ how many?

$$\text{The number of shares bought} = \frac{728}{6\frac{1}{2}} = \frac{1456}{13}$$
$$= 112.$$

The money realised by the sale of these shares at £6. 14*s*.

$$= £6{\cdot}7 \times 112 = £750{\cdot}4 = £750.\ 8s.$$

EXAMPLES XXVIII. a. (*Oral.*)

Which would you choose, £100 cash or £100 stock, if the stock were

1. Bank of England stock at 258?
2. Canadian Pacific Railway stock at 187?
3. British Government Consols at 84?
4. Colne Valley Water Co. 10 % stock at 250?
5. Metropolitan Water Board 3 % stock at 90?

6.	What is the value of	£300	stock at	110?
7.	„ „	£400	„	90?
8.	„ „	£50	„	88?
9.	„ „	£1200	„	60?
10.	„ „	£600	„	92?
11.	„ „	£325	„	80?
12	„ „	£475	„	120?
13.	„ „	12 (£1)	shares at	$1\frac{1}{4}$?
14.	„ „	16	„	$2\frac{1}{4}$?

15.	How many £1	shares at	3	can be bought for	£90?
16.	„	„	$2\frac{1}{2}$	„	£50?
17.	„ £10	„	15	„	£300?

What profit is made by

18.	buying £200	stock at	87	and selling at	par?
19.	„ £300	„	par	„	107?
20.	„ £800	„	91	„	103?

21. buying £700 stock at 108 and selling at 114?
22. „ 30 (£5) shares at 4 „ $6\frac{1}{2}$?
23. „ 20 (£1) „ 2 „ $2\frac{1}{4}$?

24. How much cash can be obtained for £700 stock at 120?
25. „ „ £1100 „ 80?
26. „ „ £50 „ 140?
27. „ „ £400 „ 95?
28. „ „ £350 „ 60?
29. „ „ £450 „ 160?

30. What is the income derived from holding £600 of 5 % stock?
31. „ „ £300 „ 7 % „
32. „ „ £400 „ 3 % „
33. „ „ £600 „ $2\frac{1}{2}$ % „
34. „ „ £700 „ $3\frac{1}{2}$ % „
35. „ „ £50 „ 6 % „
36. „ „ £200 „ $4\frac{1}{4}$ % „
37. „ „ £150 „ 3 % „
38. „ „ £225 „ 4 % „
39. „ „ £375 „ 4 % „
40. „ „ £1700 „ 5 % „
41. „ „ £720 „ $2\frac{1}{2}$ % „
42. „ „ £2400 „ $5\frac{1}{2}$ % „
43. „ „ £880 „ $2\frac{1}{2}$ % „

44. What is the income derived from holding 7 (£10) shares, each producing 2*s.* 6*d.* income?

45. What is the income derived from holding 18 shares, each producing 12*s.* 6*d.* income?

What is the income derived from investing

46. £720 cash in a 5 per cent. stock at 120?
47. £800 „ 4 „ „ par?
48. £360 „ 3 „ „ 90?
49. £340 „ 2 „ „ 85?
50. £2160 „ 3 „ „ 72?
51. £1000 „ 7 „ „ 125?
52. £560 „ $4\frac{1}{2}$ „ „ 112?
53. £819 „ 3 „ „ 91?
54. £60 in shares costing £15 each and each producing £2 income?
55. £140 „ „ £20 „ „ „ £1 „
56. £48 „ „ £8 „ „ „ 5*s.* „

What percentage does a man get for his money if he invests in

57. a 4 per cent. stock at	80?			**58.** a 3 per cent. stock at	60?		
59. ,, 7 ,, ,,	200?			**60.** ,, 12 ,, ,,	300?		
61. ,, 6 ,, ,,	180?			**62.** ,, 3 ,, ,,	80?		
63. ,, 5 ,, ,,	125?						

64. By selling at 91 some stock which I bought at 80 I made £44 profit. How much stock did I sell?

65. How many shares producing £3. 10*s*. income must I hold to get an income of £17. 10*s*.?

66. How many of these shares must I have to get an income of £49?

Which is the more paying investment,

67. a 3 % stock bought at 81 or a 4 % at par?
68. a 6 % ,, ,, 126 ,, 5 % at 105?
69. a 4 % ,, ,, 80 ,, 6 % at 150?
70. a 10 % ,, ,, 300 ,, 3 % at 87?

At what price must a man buy

71. 3 % stock, to get 4 % on his money?
72. 4 % ,, 5 % ,,
73. 6 % ,, 3 % ,,
74. 9 % ,, 3 % ,,
75. 5 % ,, 4 % ,,
76. 4 % ,, 10 % ,,

Oral questions may be set from the following examples by requiring learners to state whether the sums mentioned are stock or sterling money. Other simple oral questions on the examples may be framed with a view to finding out whether the meanings of the examples are thoroughly understood.

EXAMPLES XXVIII. b.

Find the cost of the following and the income derived:

1. £700 New South Wales 3 % stock at 91.

2. £1250 City of Bombay 4 % stock at 98.

3. £8400 India $3\frac{1}{2}$ % stock at $95\frac{1}{8}$.

4. £650 G.W. Railway stock (dividend $5\frac{1}{2}$ %) at 128.

5. £425 Highland Railway stock (dividend 2 %) at 44.

6. £1970 L. and N.W. Railway stock (dividend $6\frac{1}{2}$ %) at 145.

7. £1360 Great Central Railway $4\frac{1}{2}$ % debentures at 124.

8. £436 Portuguese 3 % stock at 65.

Find the annual income derived from investing

9. £1320 in $2\frac{1}{2}$ % Consols at 88. **10.** £4830 in a 4 % stock at 115.

11. £2650 in a $3\frac{1}{2}$ % stock at 105. **12.** £1760 in a $2\frac{3}{4}$ % stock at 80.

Find the annual income derived from investing

13. £637. 10*s*. in 4 % Newfoundland stock at 102.

14. £627 in London County Council $2\frac{1}{2}$ % stock at 76.

15. What annual income is derived from a capital of £14875, if $\frac{4}{7}$ of it be invested in a $2\frac{1}{2}$ % stock at 85 and the rest in a 9 % stock at $212\frac{1}{2}$?

What percentage is obtained on capital invested in

16. a $2\frac{1}{2}$ % stock at 75?

17. 3 % Corporation stock at 85?

18. $2\frac{1}{2}$ % Consols at $87\frac{1}{2}$?

19. Cape 4 % at 103?

20. Natal 3 % at 84?

Which is the more paying investment,

21. a 7 % stock at 133 or a 5 % at 90?

22. a 4 % „ 110 „ $4\frac{1}{2}$ % at 120?

23. $8\frac{3}{4}$ % „ 210 „ $5\frac{1}{4}$ % at 126?

24. A man invests equal sums in 3 per cents. at 96 and 4 per cents. at 120. What percentage does he get for his money in each case, and what percentage on the whole?

25. Capital is invested, half in 3 % stock at 90 and half in 4 % at par, and the resulting income is £550. Which is the more profitable investment and what was the capital?

26. A sum of £3645 has to be invested in a 3 % stock at $91\frac{1}{8}$ or a $3\frac{1}{4}$ % at $101\frac{1}{4}$. Which is the more profitable? what is the difference in income?

What alteration of capital is caused by

27. investing £10200 at 85 and selling out at 88?

28. „ £9768 „ 88 „ „ 92?

29. „ £333 „ 148 „ „ 139?

30. „ £2168. 5*s*. „ $103\frac{1}{4}$ „ „ 107?

31. buying £3400 railway stock at 93 and selling at 101?

32. „ £725 Japanese stock at 91 and selling at $94\frac{1}{2}$?

33. „ £816 Consols at $88\frac{1}{2}$ and selling at 86?

34. „ £1020 Queensland 4 % at 103 and selling at $107\frac{1}{8}$?

35. I invest £8745, half at $99\frac{3}{8}$, half at $103\frac{1}{8}$. The whole is sold at $101\frac{1}{4}$. What is my gain or loss?

36. A man bought £3825 stock at 91, and by selling it gained £229. 10*s*. At what price did he sell?

37. By investing £1000. 10*s*. in stock at 87 and selling the stock I gained £40. 5*s*. At what price did I sell?

38. A man invests £2052. 15*s*. in a $3\frac{1}{2}$ % stock and receives an income of £73. 10*s*. At what price did he buy? If the price goes up £5, how much will he gain by selling out?

39. Enough $3\frac{1}{2}$ % stock at $93\frac{1}{2}$ is sold out to produce £9350. At what price must a 4 % stock be, so that this sum invested in it may give an increase of £90 in income?

[*Where the word 'transferring' is used, it implies that* **the sum** *mentioned is stock, not sterling money.*]

Find the change of income caused by transferring

40. £1900 from $2\frac{1}{2}$ % Consols at 90 to a 4 % stock at 114.

41. £6150 from a 3 % stock at 85 to a $3\frac{1}{2}$ % at 102.

42. £1875 from a 4 % stock at 96 to a $4\frac{1}{2}$ % at $112\frac{1}{2}$.

43. £3000 from a $3\frac{1}{2}$ % stock at 98 to a 4 % at 105.

44. £5565 from a $2\frac{1}{2}$ % stock at 87 to a 6 % at 145.

45. A buys 3 % stock at $89\frac{3}{8}$. He receives one half-year's dividend, and afterwards sells his stock at $94\frac{5}{8}$, and finds that he has altogether gained £54. What sum did he originally invest?

204. Brokerage. To buy or sell stock a stockbroker is employed. His payment, called **brokerage**, is calculated either on the amount of the stock bought (or sold), or else on the value of it in £ sterling; in other words, the brokerage is a percentage on the face-value of the stock or on the money paid for it (technically known as the consideration-money). The latter is the custom in many railway stocks; but in British and foreign Government stocks the broker's charge is made on the amount of stock, and is generally $\frac{1}{8}$ per cent.; *i.e.* the brokerage on 1 bond, or £100 *stock*, is half a crown.

When brokerage is reckoned in this way there is no difficulty in taking it into account in questions of investing or selling. If I buy Consols at 88, I must consider that they cost me $88\frac{1}{8}$; for the broker has to be paid as well as the vendor. If, on the other hand, I am selling this stock at 88, I must remember that $\frac{1}{8}$ has to go to the broker, so that I get for myself only $87\frac{7}{8}$.

In questions where brokerage is mentioned it must be calculated on the amount of stock, unless other directions are given.

If nothing is said about brokerage, it is not to be reckoned at all.

EXAMPLE 1. **Find the change of income if £4500 of 3 % stock at $95\frac{1}{8}$ is transferred to a 4 % stock at $113\frac{7}{8}$, brokerage being $\frac{1}{8}$ per cent. on each transaction.**

First income. The income from £100 stock is £3.

,, ,, £4500 stock is how much?

Income $= 3 \times 45 = £135$.

Proceeds of sale. Each £100 stock is sold for $95\frac{1}{8}$, and so brings £95 cash when the brokerage is allowed for.

£100 stock brings £95 cash.

£4500 ,, ,, how much cash?

The cash realised $=£95 \times 45$.

This is invested at $113\frac{7}{8}+\frac{1}{8}$, *i.e.* at 114.

£4 is the income from £114 cash.

What ,, ,, $£95 \times 45$ cash?

Second income. The income $=\frac{4 \times 95 \times 45}{114}=\frac{4 \times 19 \times 5 \times 3 \times 15}{19 \times 6}=£150.$

The original income was £135.

Increase of income $=£15$.

Inverse Problems in Stocks.

EXAMPLE 2. **By investing $\frac{1}{3}$ of my capital in a 5% stock at 110, $\frac{1}{4}$ in a 4% at par and the rest in a 3% at 80, I get an income of £618. 19s. 9d. What was the capital?**

Take £x to be the capital.

$$\text{From the 1}^{\text{st}}\text{ stock the income}=\frac{x}{3} \times \frac{5}{110}.$$

$$\text{From the 2}^{\text{nd}}\text{ the income}=\frac{x}{4} \times \frac{4}{100}.$$

$$\text{From the 3}^{\text{rd}}\text{ the income}=\frac{5x}{12} \times \frac{3}{80}.$$

$$\therefore \frac{x}{66}+\frac{x}{100}+\frac{5x}{4 \times 80}=618\tfrac{79}{80}.$$

Multiply through by 80.

Then
$$\frac{40x}{33}+\frac{4x}{5}+\frac{5x}{4}=49519;$$

$$\therefore \frac{2153}{660}x=49519;$$

$$\therefore x=\frac{49519 \times 660}{2153}=23 \times 660$$
$$=15180.$$

The capital was £15180.

		£	s.	d.
Verification. The income from the 5%	$=\frac{x}{66}$	$=230$	0	0
,, ,, 4%	$=\frac{x}{100}$	$=151$	16	0
,, ,, 3%	$=\frac{x}{64}$	$=237$	3	9
Total		618	19	9

EXAMPLE 3. **By selling £725 of 4 % stock at 124 and buying 3 % stock with the proceeds, I increase my income by £2 a year. What was the price of the 3 % stock?**

The original income (from £725 of 4 % stock) = £29.

I sell £725 stock at 124.

The cash realised = $7·25 \times 124 = £29 \times 31$.

This cash is invested in a 3 % stock at x.

£3 is the income from £x cash.

What ,, ,, £29×31 cash?

$$\therefore \text{ the new income} = \frac{3 \times 29 \times 31}{x}.$$

But the new income $= 29 + 2 = 31$;

$$\therefore \frac{3 \times 29 \times 31}{x} = 31;$$

$$\therefore x = 87.$$

EXAMPLES XXVIII. c.

1. Find the cost of, and the income derived from, £1820 Greek 5 % stock at $52\frac{3}{8}$. Brokerage $\frac{1}{8}$ %.

2. Find the cost of £154 Hungarian 4 per cent. stock at $93\frac{5}{8}$, and the income derived from it. Brokerage $\frac{1}{8}$ per cent.

3. Find the annual income derived from investing £1768. 16*s.* 3*d.* in Bank 9 per cent. stock at 272. Brokerage $\frac{1}{8}$ per cent.

4. A man invests £1958. 3*s.* 9*d.* in a 7 % stock at 206. Brokerage $\frac{1}{8}$ per cent. What is his annual income?

5. A $3\frac{1}{2}$ per cent. stock stands at $99\frac{1}{4}$. What sum when invested will produce £262. 10*s.* annually? Brokerage $\frac{1}{8}$ per cent.

6. If I sell £21750 stock out of the $3\frac{1}{2}$ per cents. at $92\frac{5}{8}$ and buy 3 per cents. at 87 with the proceeds, find (1) how much stock I shall obtain, and (2) my change of income, assuming that the broker charges $\frac{1}{8}$ per cent. for selling stock, but makes no charge for re-investment.

7. Find the change of income caused by transferring £3150 4 per cent. Debentures at $130\frac{1}{8}$ to French 3 per cent. Rentes at $90\frac{7}{8}$. Brokerage $\frac{1}{8}$ per cent. on both stocks.

8. A person buys for £1150 a house for which he has been paying £75 a year rent. For the purchase he sells out a sufficient amount of 3 % stock at 92. Determine the increase of his yearly income.

9. I invested £5000 in a 3 % stock at par, the income from this being subject to a tax of 6*d.* in the £. Another £5000 I invested at 104 in a $3\frac{1}{4}$ % stock free from tax. Which was the more profitable and by how much?

10. Find the income derived from investing £$11578\frac{1}{2}$ in a $4\frac{1}{2}$ % stock at $93\frac{3}{8}$ after deducting an income-tax of 1 shilling in the £.

11. A fund-holder sells £5000 3 % stock at $87\frac{1}{2}$ and invests the proceeds in railway shares at 105, which he sells again when they have fallen to $31\frac{1}{2}$, and re-invests in 3 % stock at $93\frac{3}{4}$. How much income does he lose?

12. By selling £2700 stock at $89\frac{5}{8}$ and re-investing in a 5 % stock at $111\frac{3}{4}$, what income do I get, if there is a brokerage of $\frac{1}{8}$ per cent. on each stock?

13. What change of income is caused by transferring £5200 from 3 per cents. at $86\frac{3}{8}$ to 4 per cents. at $114\frac{7}{8}$? Brokerage in both cases $\frac{1}{8}$ per cent.

14. A man, who has invested £6480 in $2\frac{1}{2}$ % Consols at 108, sells out at 112 and invests the proceeds in a 6 % stock, thereby increasing his income by £90. At what price did he buy the 6 % stock?

15. The possessor of some 3 % Consols sold them at $87\frac{1}{4}$, and invested the proceeds in railway shares at $174\frac{1}{2}$ for the £100 share. He thus increased his income from £120 to £200. What dividend per cent. on its stock did the railway pay?

16. By selling $2\frac{3}{4}$ % stock at $96\frac{1}{4}$ and investing the money in 4 % bonds, I increase my income by 5 %. Find the price of these bonds.

17. I sold 3 % stock at $81\frac{1}{8}$ and bought 5 % at $107\frac{7}{8}$, and by so doing increased my income by £66. 7*s.* 6*d.*, brokerage $\frac{1}{8}$ % being charged on each operation. How much stock did I sell?

18. Formerly I could buy 3 % Consols at 104 and had to pay an income-tax of 6*d.* in the £. If Consols, now $2\frac{1}{2}$ %, cost 95 and I have to pay a shilling in the £, by how much per cent. is the net income decreased?

19. By buying 3 % Consols I got $3\frac{3}{5}$ %, and derived a net income of £421. 4*s.* after paying 6*d.* in the £ income-tax. Find the amount of stock bought and the price.

20. A man sold £15000 railway stock (paying 5 % dividend) at 105. With the money he purchased 4 % stock at 90 and sold at 96. He re-invested in the railway stock which was still at 105 and was still paying 5 %. What was the change in his income?

21. A person holding £5250 3 % stock and £4750 $6\frac{1}{2}$ % Bank stock sells out, the price of the former being $91\frac{7}{8}$ and of the latter $185\frac{3}{8}$. With part of the proceeds he buys ninety 8 % railway bonds at $142\frac{3}{8}$ (brokerage $\frac{1}{8}$ per cent. on stock being paid in each of these three transactions) and obtains 4 % on the remainder. Find the change in his income.

22. A broker was instructed to buy eight £100 bonds at $272\frac{3}{8}$ after selling for the purpose £850 3 % stock at $95\frac{5}{8}$ and £1300 $4\frac{1}{2}$ % stock at $105\frac{3}{8}$, the brokerage being $\frac{1}{8}$ % on stock in each case. What was the broker's total fee?

23. In question 22, was there any margin of capital left after the purchase?

24. A man invested a certain sum of money, $\frac{3}{5}$ of it in $2\frac{3}{4}$ % Consols at $93\frac{3}{4}$, $\frac{1}{4}$ of it in a 4 % stock at 108 and the remainder in a 6 % stock at 144. Find the average rate per cent. on the whole investment.

25. I am investing £9256. 10*s.* partly in 5 % stock at 150 and partly in $2\frac{3}{4}$ % at 99. What sum must I invest in each stock to get the same income from each?

26. A sum of £3720 is to be invested, part in a $2\frac{1}{2}$ % stock at 89 and the rest in a 3 % at 97. Find the amount invested in each if the income derived is to be the same as from investing the whole in a $2\frac{3}{4}$ % stock at 93.

XXIX. REVISION PAPERS.

[*Answers in money should be calculated to the nearest penny.*]

XXIX. a.

1. A merchant buys 485 m. 70 cm. of silk, and pays for it 3764 fr. 20 c. Find the cost per metre in francs and centimes (correct to the nearest centime).

2. In an examination the average number of marks gained by 25 boys is 39. The first ten boys get an average of 63, and the last 6 an average of 17. Find the average of the others.

3. Find the interest on £19314. 11*s.* 8*d.* for 19 days at 4 per cent.

4. Find the difference between commercial and true discount on £350 due in 4 years at 5 per cent.

5. In a forest 5 per cent. of the trees are blown over in a gale, and after 3 per cent. of those remaining are cut down there still stand 55290 trees. How many were there in the forest before the gale?

6. A man buys $2\frac{1}{2}$ cwt. of tea at £8. 3*s.* 6*d.* per cwt. and $1\frac{1}{2}$ cwt. at £11. 7*s.* 6*d.* per cwt. He mixes the whole together, and sells the mixture at 1*s.* $10\frac{1}{2}$*d.* per lb. What is his gain per cent.?

7. The possessor of £7500 of a 3 % stock sells out and invests the proceeds in a 5 % stock at 120, increasing his income by £25. Find the price at which he sold out.

XXIX. b.

1. A motor-van travels for the first 20 minutes at the rate of 6 Km. 126 m. per hour, then, for 2 hours 20 minutes, at 30 Km. per hour, and afterwards, for 25 minutes, at 7 Km. 152 m. per hour. Find the total distance traversed by the van.

2. Three men and 5 women working together earn between them £4. 10*s.* in a week; but if one man and one woman were absent for half the week, the joint earnings would be only £3. 18*s.* What are the wages of each man and woman?

3. In 1900-1901 the national expenditure was £183,592,264. Of this the Army cost £91,710,000, the Navy £29,520,000 and the Civil Services £23,500,000. Express each of these as a percentage of the whole, to the nearest integer.

4. Find the amount of £250 in 3 years at 3 % compound interest.

5. What sum will amount to £1532. 5*s.* in 3 years at $4\frac{1}{2}$ per cent. simple interest?

6. A merchant sold goods for £75. 6*s.* more than he gave for them, and thereby gained 15 %. What did he give for them?

7. A man having £4000 to invest places £2000 in $2\frac{3}{4}$ per cents. at $98\frac{1}{2}$ and £2000 in India 3 per cents. at $98\frac{5}{8}$. Find to the nearest penny the income obtained.

XXIX. c.

1. Find the cost of gravelling a path 4 ft. 6 in. wide, surrounding a rectangular lawn 117 ft. by 54 ft., with gravel 3 in. deep at 8*s.* per cubic yd.

2. If 29 m. 75 cm. of cloth cost 190 fr. 40 c., what is the cost of 22 m. 50 cm.?

3. A begins business with £4000; after 4 months takes B as a partner with £300 and 2 months later C with £5000. The profit at the end of the year is 16 % on the whole capital. What are their shares of it to the nearest half-sovereign?

4. Find the difference between true and commercial discount on £264. 10*s.* due in 3 yrs. at 5 % simple interest.

5. At what rate is the simple interest on £2186. 13*s.* 4*d.* for $2\frac{1}{2}$ years equal to £246?

6. A grocer mixes 60 lb. of tea which cost 1*s.* 9*d.* a lb. with 80 lb. which cost 2*s.* 4*d.* a lb., and sells the mixture at 2*s.* 7*d.* a lb. What is his profit per cent. on the outlay?

7. What is the income derived from investing £1000 in a $2\frac{1}{4}$ % stock at $112\frac{1}{2}$? If the stock is sold at 105, what is the loss of capital?

XXIX. d.

1. A railway on leaving a station S rises uniformly for half a mile at a slope of 1 in 160; it then runs on the level for $\frac{3}{4}$ mile, and afterwards falls for the next mile at a slope of 1 in 110. Find at how many yards from S it is at the same level with S, the distance being measured along the railway.

A slope of 1 in 160 means that a train rises vertically 1 foot in travelling 160 feet.

2. A man buys eggs at 1*s.* 3*d.* a dozen and sells them at 11*s.* 8*d.* a hundred. Find his gain per cent.

3. If £401. 10*s.* amounts to £404. 5*s.* at $2\frac{1}{2}$ per cent. simple interest, what is the length of time?

4. If a sum of money at simple interest amounts in $1\frac{1}{2}$ years to £226. 2*s.* 8*d.* and would have amounted to £238. 18*s.* 8*d.* if the time had been 3 years instead, what is the rate per cent., and what is the principal?

5. A puts into a business £2400, B £2100, C £1800, D £2800. A takes his money out after 3 months, B after 4 months, C after 6 months; D's money remains till the end of the year, when a profit of £525 is divided. What share ought each to have?

6. A merchant sells goods to a customer at a profit of 44%; but the customer becomes bankrupt and pays only 14*s.* $4\frac{1}{2}$*d.* in the £. What per cent. does the merchant gain or lose by the transaction?

7. A man invests £5670 in $2\frac{3}{4}$% Consols at $94\frac{1}{2}$. Find the amount of money which he must invest in 3% Victoria Stock at $97\frac{3}{4}$ to get the same income from it.

XXIX. e.

1. A gram is the weight of a cubic centimetre of water and a decimetre is 3·937 inches. It being given that a cubic foot of water weighs 1000 oz., express the kilogram in lb. avoirdupois correctly to 2 places of decimals.

2. Find the cost of papering the walls of a room which is 61 ft. 5 in. long, 21 ft. 11 in. wide and 21 ft. 9 in. high, the width of the paper being 2 ft. 1 in. and its price 2*s.* 7*d.* per piece of 12 yds., allowing 625 sq. ft. for doors and windows.

3. A man pays annually for life insurance 10% of his income. This he legally deducts from his income, and pays on the remainder an income-tax of 1*s.* 4*d.* in the £. If he has £1071 a year left, what is his gross income?

4. Find the difference between commercial and true discount on a bill for £276. 17*s.* 6*d.* due 6 months hence at $3\frac{1}{2}$% per annum.

5. Find the compound interest on £318. 6*s.* for 3 years at $4\frac{1}{2}$%.

6. If cloth sold at 14*s.* 3*d.* realises a profit of 14%, at what price should it be sold to clear 12%?

7. A man buys £1275 stock at $5\frac{1}{2}$ per cent. below par and sells at $10\frac{1}{4}$ premium. What does he gain?

XXIX. f.

1. Given that 1 metre = 39·37 inches, express a kilometre as a decimal of a mile. What error is made in taking a kilometre to be 5 furlongs?

2. A hollow cube is formed of 6 plates, each $\frac{1}{2}$ inch thick, an edge of the internal surface being 15 inches. Find the ratio of the internal surface to the external.

3. A man bequeaths $\frac{1}{10}$ of his property to each of his 3 daughters, $\frac{1}{6}$ to each of his 2 younger sons and the rest to his eldest son. The eldest son's share is an amount of capital which at 4 % would produce an income of £211. 4*s*. Find the shares of each of the others.

4. A house is insured against fire at the rate of $\frac{1}{5}$ per cent. If the premium paid is £1. 17*s*. 6*d*., what is the amount for which the house is insured?

5. Find the true discount on £1350 due 7 months hence at 6 %.

6. The present income of a railway company would justify a dividend of 6 per cent. if all the shares ranked equally. But, as £400,000 of the stock consists of preference shares which receive $7\frac{1}{2}$ per cent. per annum, the ordinary shareholders receive only 5 per cent. What is the whole amount of stock?

7. By investing £1283. 16*s*. 3*d*. in a 3 % stock I get an income of £36. 18*s*. At what price did I buy the stock?

XXIX. g.

1. What area is represented by 1·76 sq. cm. on a map whose scale is 1 cm. to 4 kilometres?

2. In how many years will £1001. 6*s*. 8*d*. amount to £1076. 8*s*. 8*d*. at 3 % simple interest?

3. The population of a place increased 12 per cent. between 1891 and 1901. In 1901 it was 16,492. What was the population in 1891?

4. A, B and C are partners. A receives $\frac{2}{5}$ of the profits, B and C dividing the remainder equally. A's income is increased by £220 when the rate of profit rises from 8 to 10 per cent. Find the capital of B or C.

5. What sum will amount to £409. 10*s*. in 2 years at compound interest, the rate being 4 % for the 1st year and 5 % for the 2nd.

6. A man buys 120 oranges for 6*s*. 8*d*. He sells $\frac{1}{3}$ of them at $1\frac{1}{2}d$. each and $\frac{1}{5}$ at $\frac{1}{2}d$. each. For what price did he sell the rest if he made $52\frac{1}{2}$ % profit?

7. A person holding £16,000 of 3 per cent. stock sells out half when they are at $91\frac{1}{8}$, and invests in Greek 6 per cents. at 72. Find the ratio of his new to his old income.

XXIX. h

1. If the cost of enclosing a square containing 1·6 of an acre is £53, what is the cost of enclosing a rectangle of the same area whose breadth is 22 yds.?

2. A railway had 1407 miles open for traffic in one year and $1425\frac{1}{2}$ in the next year, the receipts being £59,578 in the former year and £62,310 in the latter. Find the increase in the average receipts per mile (to the nearest penny).

3. Find the amount of £975 for 3 years at $3\frac{1}{3}$ % compound interest.

4. In a business owned by 3 partners the profits amounted in one year to 10 % of the capital. Two of the partners drew £200 and £600 respectively ; £300, which was 2 % on the whole capital, was paid to the reserve fund. Find what was the original capital contributed by each partner.

5. A man buys a certain number of oranges at 3 a penny and one-third of that number at 4 a penny. At what rate must he sell them to gain 140 per cent.?

6. If the difference between simple interest and true discount on a sum of money at 8 per cent. for $1\frac{1}{4}$ years is £4. 18*s.*, what is the sum of money?

7. What sum invested in a 4 % stock at 103 gives a net income of £155. 5*s.* 8*d.* after a tax of 9*d.* in the £ has been paid?

XXIX. k.

1. The value of a certain house in 1910 shows an increase of 35 % since 1907. It was rated at $\frac{2}{3}$ of its value in 1907, and in 1910 is rated at $\frac{3}{5}$ of its value, the rate in the £ being unchanged. Compare the rate paid in 1907 with that in 1910.

2. An empty cistern has 3 pipes. A fills it in 3 hrs., B in 4, and C empties it in 1 hr. They are respectively opened at 1, 2, 3 o'clock. When will the cistern be empty?

3. Find the true discount on £142. 1*s.* 9*d.* due 18 months hence at $3\frac{1}{2}$ per cent.

4. The incomes of two men would be equal if one were increased by 7 % and the other diminished by $7\frac{1}{2}$ %, and the sum of their incomes = £418. 19*s.* What are the separate incomes?

5. At what rate does £188. 6*s.* 8*d.* amount to £198. 4*s.* 5*d.* in a year and a half?

6. By transferring £14,595 from a $3\frac{1}{2}$ % stock at $93\frac{3}{4}$ to a 3 % at $87\frac{1}{2}$ what change of income is made?

7. A farmer bought 6 oxen and 100 sheep for £336. Four sheep died, and the rest were sold at £2. 7*s.* 6*d.* each. 2 of the oxen fetched £15 each. Find the price at which the remaining 4 oxen had to be sold to give a profit of 5 % on the whole.

XXIX. l.

1. Find to the nearest franc the cost of fencing, at 3 fr. 25 c. per metre, a square field whose area is 7000 sq. metres.

2. Find the difference between simple and compound interest on £7545. 10*s.* for 2 years at 4 % per annum, interest being payable half yearly.

3. At what rate per cent. will £2500 amount to £2865. 12*s.* 6*d.* in $3\frac{1}{4}$ years at simple interest?

4. What sum will amount to £5780 in 3 years at 2 % compound interest?

5. Supposing the land under barley in England to be this year half as much again as that under wheat and the quantity under oats to be equal to the other two together; if the quantity under wheat next year be reduced by 25 % and the quantity under barley increased by 5 %, the whole quantity remaining the same as before, by how much per cent. will that under oats be increased?

6. A dealer was making a profit of 25 % by selling an article at 15*s*. Owing to a rise of 6*d*. in the cost to him he sold it at 15*s*. 6*d*. instead. What was his profit per cent. after the change?

7. How much stock at $97\frac{1}{2}$ must I sell out in order to realise £3900? If by investing this £3900 in a $4\frac{1}{4}$ % stock at 102 I increase my income by £25 per annum, what interest was I receiving on £100 of the former stock?

XXIX. m.

1. Find the value, correct to 2 decimal places, of

$$\frac{15+\sqrt{10}}{15-\sqrt{10}}+\frac{30-\sqrt{10}}{15+\sqrt{10}}.$$

2. A, B, C and D rent a field, using it only one at a time. B uses it twice the time that A does; C uses it 2 months longer than B; and D as long as the rest together. A pays £22. 4*s*. 6*d*., which is $\frac{1}{11}$ of the rent. Find the amount of rent due from each of the rest.

3. One gallon of spirit which contains 11 % of water is added to 3 gallons containing 7 % of water, and to this mixture half a gallon of water is added. Find the percentage of water in the mixture.

4. Two sums of money amounting together to £3892. 10*s*. were put out at simple interest, the smaller at 4 %, the other at $4\frac{1}{2}$ % per annum. At the end of 18 months the interest altogether was £250. 9*s*. 9*d*. What were the two sums?

5. In what time will £977. 13*s*. 4*d*. amount to £1063. 4*s*. 3*d*. at $2\frac{1}{2}$ % simple interest?

6. A buys a carriage and sells it to B at a profit of 5 per cent., B sells it to C at a profit of 5 per cent. and C sells it to D for £49. 12*s*. 3*d*., making a profit of $12\frac{1}{2}$ per cent. What did it cost A?

7. A person invests £1842. 15*s*. in a 4 % stock at $102\frac{3}{8}$; he afterwards sells out at 105 and re-invests in 5 per cents. at 126. Find his change of income.

XXIX. n.

1. A sum of £1250 was borrowed on April 1st and repaid on Aug. 25 of the same year, with interest at $3\frac{3}{4}$ % per annum. What sum had to be paid?

2. On what sum will the simple interest for $1\frac{1}{2}$ years at $2\frac{1}{2}$ % be £47. 1*s*. 6*d*.?

3. How much tea which cost 1*s*. $5\frac{1}{2}$*d*. per lb. must be mixed with 1 cwt. which cost £9. 16*s*. 8*d*. in order that 15 per cent. may be gained by selling the mixture at 1*s*. 11*d*. per lb.?

4. A bill for £6132. 10*s*., drawn on March 14th and payable 9 months after date, was discounted by a banker on May 12th at $3\frac{1}{4}$ %. How much did the banker give for the bill? [Banker's Discount.]

5. On Monday a score of eggs cost as many pence as half a dozen chickens cost shillings. On Tuesday the price of eggs had fallen $12\frac{1}{2}$ %, and the price of chickens had risen 25 %; and on that day a chicken and a score of eggs could be bought for 4*s*. 6*d*. What was the price of a chicken on Monday?

6. A shopkeeper marks his goods with a price from which he can deduct 15 % for ready money and still have $10\frac{1}{2}$ % profit. What is the marked price of an article which cost him £3. 19*s*. 2*d*.?

7. On a certain day $2\frac{1}{2}$ per cent. Consols were at $96\frac{1}{4}$ and Egyptian 3 per cents. at $102\frac{1}{2}$. A man, who had a certain sum to invest, calculated that if he invested in one of these stocks he would have a larger income by £5. 4*s*. than if he invested it in the other. What sum had he to invest?

XXIX. o.

1. Shew that $\sqrt{2}$ lies between $1\frac{5}{12}$ and $1\frac{12}{29}$, and is nearer to the latter than the former.

2. A railway company declares the following dividends on its stock for a half year at the rate of

4 %	per annum	on	£15,100,406 Guaranteed Stock.
4 %	„	„	£27,873,631 Preference Stock.
$6\frac{3}{4}$ %	„	„	£42,887,723 Ordinary Stock.

Calculate the total amount of dividend distributed.

3. A and B are partners in a business, and A contributed £10,000 of the capital. At the end of a certain year there is a profit of £1152 for division. If A's share is £675, find how much of the capital was contributed by B.

4. A man, having a bill for £500 payable 7 months hence, gets it cashed at a (commercial) discount of 5 % per annum. How much cash does he receive?

5. A grocer has 2 kinds of tea which cost him 2*s*. 8*d*. and 2*s*. 1*d*. per lb. respectively. Find how many lb. of the former kind he must mix with 56 lb. of the latter to gain 25 % by selling the mixture at 2*s*. 11*d*. a lb.?

6. I buy £680 of $5\frac{5}{8}$ % stock at $126\frac{3}{4}$ and sell out at 139. What profit do I make if I pay $\frac{1}{8}$ % brokerage on each operation?

7. Two-thirds of a man's capital is invested at $4\frac{1}{2}$ %, the other third at 3 % per annum. The income-tax at 11*d*. per £ on the whole annual interest amounts to £38. 10*s*. Find the capital invested.

XXIX. p.

1. The area of a rectangular space is an acre, and its length and width are in the ratio of 5 : 2 ; around this space on the inside is a path whose width at any point is $\frac{1}{20}$ of the distance to the opposite path. Find how many bricks will pave the path, allowing 45 bricks for a sq. yd.

2. A and B can do a piece of work in $6\frac{2}{3}$ days, A and C in $5\frac{1}{3}$ and A, B and C in $3\frac{3}{4}$ days. In how many days can A do it alone?

3. A man, who has £764, gets $3\frac{1}{2}$ % on part of it and $4\frac{1}{2}$ % on the rest. How much had he at each rate, if he received equal incomes from the two parts?

4. If a sum of money amounted in 2 years at compound interest to £81·12 and in 3 years to £84·3648, what was the sum and what the rate per cent.?

5. A bill, having 3 months to run, is discounted by a bill-broker for £379. 4*s.*, commercial discount being allowed at 5 % per annum. For what sum was the bill drawn?

6. If the manufacturer makes a profit of 20 %, the agent 10 % and the shopkeeper 25 %, what is the cost to the manufacturer of an article which is sold in the shop for £15. 8*s.*?

7. A man invests in $2\frac{1}{2}$ % Consols at 108, sells out at 112 and invests the proceeds in a 6 % stock at 168. If he thereby increases his income by £90, how much did he originally invest?

XXIX. q.

1. An express train travelled a kilometre in 35·5 seconds. How many miles, correct to 3 decimal places, could it travel in an hour at that speed. [1 metre = 39·37 inches.]

2. A works 7 hours a day for 6 days, B 4 hours a day for 9 days and C 5 hours a day for 7 days. If the wages paid to A and B together amount to £7. 16*s.*, what do A, B and C each receive?

3. The interest for one year on £3060, after a tax of 6*d.* per £ is deducted from it, is £99. 9*s.* Find the rate per cent.

4. Upon a debt, which is paid a year and a half before it is due, true discount of £11. 4*s.* is allowed at 4 % per annum. What is the amount of the debt?

5. A man purchases a farm which, on being let at £2 per acre, pays him $3\frac{1}{3}$ per cent. on the investment; he purchases another, which he lets for 2 guineas an acre, and it pays him $3\frac{3}{4}$ per cent. Compare the prices paid per acre for the two farms.

6. A dealer buys spirit at 3*s.* $1\frac{1}{2}$*d.* a gallon, and adds so much water that he can sell the watered spirit at a profit of 32 % when he charges 3*s.* a gallon for it. Find the ratio of water to spirit.

7. A man invests £9075 in the 3 per cents. at $90\frac{3}{4}$, and, on their rising to 91, transfers it to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$. What is the increase of his income?

XXIX. r.

1. The fare for a certain railway journey is 18 fr. 70 c.; while for another journey, 87 kilometres longer than the former, the fare is 24 fr. 50 c. Find the length of each journey, the rate per kilometre being the same for both.

2. Three persons contribute £250, £500, £750 respectively towards a venture, on the understanding that profits shall be so divided that the *rate* of interest which each receives shall be proportional to the amount of his contribution. If the profits for the year amount to £245, how much will each receive?

3. A tradesman, having promised me $12\frac{1}{2}$ % reduction from a bill of £16. 16*s.* 8*d.*, takes off only 10 %. By what sum am I entitled to further diminish his bill? What per cent. is this of the (reduced) account rendered by him?

4. A man left £9656 to be divided amongst his three sons in proportion to their ages 25, 22 and 21. What difference would it have made to each of them if he had lived a year longer? Show what each did get and what each would have got.

5. The interest on £125 for $3\frac{1}{2}$ years is £19. 13*s.* 9*d.* What is the rate?

6. Find, to the nearest £, the present value of £10,000 due 3 yrs. hence at 5 % compound interest.

7. By selling Railway $3\frac{1}{2}$ % stock at 96 and buying Tramway $4\frac{1}{4}$ % stock at 108, a man increased his income by £25. What amount of stock did he sell?

XXIX. s.

1. A square field contains 78,961 sq. yds. What is the cost of fencing it at $9\frac{3}{4}$*d.* per yd.?

2. Standard gold contains 11 parts of pure gold to one of alloy, and 20 lb. Troy of standard gold make 934 sovereigns and 1 half-sovereign. A gold ornament weighing 1 oz. is bought for £4. 10*s.* Allowing 25 % of this for workmanship and profit, find the ratio of gold to alloy in the ornament, the alloy being supposed worthless.

3. The difference between commercial and true discount for 1 yr. at 5 % on a sum of money is £12. 7*s.* What is the sum?

4. In a working union of 3 railway companies their shares of net profits are to be 26·88 %, 35·73 % and 37·39 %. Find to the nearest £ the share of the first named in a net profit of £6,167,422.

5. A could do a piece of work in $7\frac{1}{2}$ hours, B in $8\frac{1}{3}$, C in 10. The work was done by them all working together, except that B left off half an hour before its completion. In what time was it done?

6. A man gets $3\frac{3}{4}$ % on part of his capital and $4\frac{1}{4}$ % on the rest. His income is the same from each of these two investments. How much capital has he in each if his whole capital is £3000?

7. A invests $\frac{1}{3}$ of his capital in Bank Stock, $\frac{1}{6}$ in Consols and the rest in a railway. Selling out he makes a profit of 5, 3, 2 % respectively on these investments, and realises £12,380. What was his capital originally?

XXIX. t.

1. If a cubic ft. of water weighs 1000 oz., find the weight of a cubic metre of water in kilograms correct to 3 decimal places; having given that a centimetre = ·0328 ft., and a gram = ·0353 oz.

2. A sum of money put out at $2\frac{1}{2}$ % simple interest amounts in 3 yrs. to £172. What would be the amount if the rate were $3\frac{1}{4}$ % and the time $2\frac{1}{3}$ yrs.?

3. What sum becomes at compound interest £2475. 2*s.* 3*d.* in 2 yrs. at 5 %?

4. A man buys goods, and finds that the cost of carriage is 4 % on the cost of the goods. He is compelled to sell at a price which causes a loss of 5 % on his total outlay. If he had received £3. 5*s.* more than he did, he would have gained $2\frac{1}{2}$ %. What was the original cost?

5. I invest capital so that on $\frac{1}{3}$ of it I make 3 %, on $\frac{1}{5}$ I lose 4 % and on the rest I make 6 %. What is the average rate?

6. I buy some 4 % stock at 94, receive a half-year's dividend and then sell at par. If my total gain is £72, what was the original sum invested?

7. A man bequeathed £1000 to his widow and £1500 in legacies to institutions, and directed that $\frac{1}{3}$ of the residue of his property should go to his widow and the remaining two-thirds be equally divided amongst his 3 children. If the widow received altogether £4000 more than each child, what was the value of the property which the man left?

XXIX. u.

1. Find to the nearest integer the number of sq. cm. in a sq. ft., given that 1 m. = 39·37 inches.

2. If 7 men and 10 boys earn £6 a week and 3 men and 5 boys earn £11 in 4 weeks, in what time will 5 men and 4 boys earn £14?

3. Sugar passes through the hands of three dealers, each of whom makes 10 % profit on his outlay, and is sold by the last of the three at $2\frac{3}{4}$*d.* per lb. Find the original cost per cwt.

4. £9000 invested in a certain 5 % stock produces approximately £139. 0*s.* 4*d.* annually more than if it had been invested in a $2\frac{3}{4}$ % stock at 112. Find the price of the 5 % stock.

5. A railway company reduced passenger rates 10 % on 1st class fares, 20 % on 2nd, 30 % on 3rd class. The increase in the number of passengers per mile was 15 % in the 1st class, 25 % in the 2nd and 40 % in 3rd. How much did the company gain or lose per cent. on each class?

6. The manufacturer of a machine sells it to a wholesale dealer at 20 % profit; the wholesale dealer sells it to a shopkeeper at a profit of 5 %; and the shopkeeper makes 12 % profit by selling it for £79. 0*s*. 3*d*. What was the cost to the manufacturer?

7. A man has an income of £415 derived from capital invested in 4 % stock. He sells this stock at 119 and re-invests the proceeds in a 5 % stock. What price must he pay for this latter stock if his new income is £425?

XXIX. v.

1. 1120 square feet of paper will just cover the walls of a room which is 8 feet longer than it is wide. If the height is 10 feet, what are the other dimensions?

2. If A, B and C could reap a field in 18 days; B, C and D in 20 days; C, D and A in 24 days; and D, A and B in 27 days; in what time would it be reaped by them all together?

3. If the present number of inhabitants of a city is 512,000 and the annual increase is 25 in a thousand, what will the population be in 3 years time?

4. A Parliamentary grant is made at the rate of 5*s*. per head for all the children at elementary schools. If the grant is distributed at the rate of 5*s*. 9*d*. per child in town and 3*s*. 3*d*. per child in country schools, what percentage of the total number of children is in each class of school?

5. If 5 francs = 4 shillings and 176 pints = 100 litres, at how many francs per decalitre must beer be sold, which cost 2*s*. per gallon, that the seller may gain 20 %?

6. What principal will amount to £409. 9*s*. in 4 years at 4 per cent. compound interest?

7. A man invests £1911 in the purchase of Stock and sells out so as to gain £150, the price having increased $6\frac{1}{2}$. Find the price originally paid, allowing $\frac{1}{8}$ % for brokerage on each transaction.

XXIX. w.

1. Find to the nearest tenth of a franc the cost of fencing a square field, of which the area is 3792 sq. metres, at 4 fr. 15 c. per metre.

2. Equal numbers of men and boys are to be employed to complete in 24 days some work which would require 4 men and 13 boys for 39 days, or 12 men and 6 boys for 26 days. How many must be engaged?

3. What principal amounts to £696 in 1 yr. 15 days at 5 %?

4. A and B engage in trade, their capitals being in the ratio 7 : 11. At the end of 3 months A withdraws $\frac{1}{3}$ of his capital, and a month afterwards B adds twice as much as A had withdrawn. How should a profit of £337. 17*s*. 6*d*. be divided at the end of the year?

5. The true discount on a sum of money due 3 years hence is £30. and the commercial discount is £33. 12*s.* Find the sum and rate of interest.

6. By selling out of 3 per cents. at 81 and buying into 4 per cents. at 92 a man made an alteration of £60 a year in his income. How much stock did he sell?

7. A ship-owner wishes to insure his cargo, worth £91,500, at $8\frac{1}{2}$ per cent. For how much must he insure it so as to recover his premium as well as the value of the cargo in case of total loss?

XXX. MISCELLANEOUS PROBLEMS. CLOCKS, EXCHANGE, RACES, GAMES OF SKILL, RELATIVE VELOCITY, ETC.

Clock Problems.

205. In all clock problems the fact to observe, and use, is that the minute hand travels 12 times as fast as the hour hand.

We can express this in several ways.

(1)	While the	hour	hand sweeps through		1	minute	space,
	,,	minute	,,	,,	12	,,	spaces.
(2)	,, ,,	hour	,,	,,	1	,,	space,
	,,	minute	,, gains		11	,,	spaces on the hour hand.
(3)	,, ,,	hour	hand sweeps through		x	,,	spaces,
	,,	minute	,,	,,	$12x$	,,	,,
(4)	,, ,,	hour	,,	,,	$\frac{x}{12}$	,,	,,
	,,	minute	,,	,,	x	,,	,,
(5)	,, ,,	hour	,,	,,	x	,,	,,
	,,	minute	,, gains		$11x$	,,	,, on the hour hand.

(6) **In x minutes the minute hand gains $x - \frac{x}{12}\left(=\frac{11x}{12}\right)$ minute spaces on the hour hand.**

EXAMPLE 1. **At what time between 3 and 4 o'clock are the minute and hour hands of a clock together?**

If the hands are together at x min. past 3,

since 3 o'clock the minute hand has swept through x minute spaces

and ,, hour ,, ,, ,, $\frac{x}{12}$,, ,,

Also since 3 o'clock the minute hand has gained 15 minute spaces on the hour hand.

$$\therefore\ x-\frac{x}{12}=15, \quad i.e.\ \frac{11x}{12}=15.$$

$$x=\frac{12\times 15}{11}=\frac{180}{11}=16\tfrac{4}{11}.$$

$\therefore$ the hands are together at $16\frac{4}{11}$ min. past three.

EXAMPLE 2. **At what times between 4 and 5 o'clock are the hands of a clock 10 minute spaces apart?**

(1) Let the minute hand be 10 min. spaces *behind* the hour hand at x minutes past 4.

At 4 o'clock the minute hand was 20 min. spaces behind the hour hand.

At x min. past 4 it has gained 10 min. spaces.

In x min. the hour hand has swept through $\frac{x}{12}$ minute spaces.

$$\therefore\ x-\frac{x}{12}=10, \quad i.e.\ \frac{11x}{12}=10.$$

$$x=\frac{120}{11}=10\tfrac{10}{11}.$$

$\therefore$ at $10\frac{10}{11}$ min. past 4 the minute hand is 10 min. spaces behind the hour hand.

(2) Let the minute hand be 10 min. spaces in *front* of the hour hand at y min. past 4.

The minute hand has then gained 30 min. spaces on the hour hand.

$$\therefore\ y-\frac{y}{12}=30, \quad i.e.\ \frac{11y}{12}=30.$$

$$y=\frac{360}{11}=32\tfrac{8}{11}.$$

$\therefore$ at $32\frac{8}{11}$ min. past 4 the minute hand is 10 min. spaces in front of the hour hand.

EXAMPLES XXX. a.

At what time between 6 and 7 o'clock are the hands of a clock

1. together?

2. at right angles, the minute hand being behind the hour hand?

3. at right angles, ,, in front of ,,

At what time between 1 and 2 o'clock are the hands of a clock

4. together? **5.** ten minute spaces apart?

6. pointing in opposite directions?

7. Between 1 and 2 o'clock the hands of a clock are 17 minute spaces apart. What is the time?

8. How many minute spaces are there between the hands of a clock at 12 minutes to four?

9. How many minute spaces are there between the hands of a clock at 24 min. past ten?

10. At what times between 10 and 11 o'clock do the hands of a clock include an angle of 150°?

11. A clock which gains $7\frac{1}{2}$ minutes in 24 hours is twelve minutes fast at midnight on Sunday. What o'clock will it indicate at 4 o'clock on Wednesday afternoon?

12. A watch was 5 minutes fast at 9 a.m. on Monday, and 10 minutes slow at 12 noon on the following Wednesday. Find when it was exactly right, assuming that it lost time uniformly.

13. A clock, set right at noon, loses 4 minutes per hour; what will be the correct time on the same day when the clock indicates 7 p.m.?

14. Two clocks sound the first stroke of 12 o'clock at the same instant; one clock allows an interval of 20 secs. between each stroke and the next, and the other allows 25 secs. How many strokes of the slower clock remain after the quicker one has finished striking, and what time will elapse between the 12th stroke of the quicker one and the following stroke of the slower one?

15. Two clocks take respectively 25 secs. and 30 secs. to strike *twelve*; and the former begins to strike 15 secs. after the latter has begun. Show that there is no interval between the 4^{th} stroke of the former and the 9^{th} stroke of the latter.

Exchange.*

206. If an English merchant buys goods in France, he has to pay for them in French money. The amount he has to pay depends upon the **Exchange** value of French and English money.

When £1 was worth 25 francs, he would pay £4 to discharge a debt of 100 francs, for £4 $= 4 \times 25 = 100$ francs.

If the exchange value of £1 were 25 francs, and English money were **at a premium**, say of 10 per cent., this would mean that

$$\text{£}1 = 25 \text{ francs} + 10 \text{ per cent. of } 25 \text{ francs}$$
$$= (25 + \tfrac{1}{10} \text{ of } 25) = 25 + 2\cdot5 = 27\cdot5 \text{ francs.}$$

In this case, he would pay £4 to discharge a debt of $4 \times 27\cdot5$, *i.e.* 110 francs.

All examples on Exchange can be worked by the Unitary Method.

* See table on page xii.

EXAMPLE 1. **What is the exchange value of 500 francs in English money, to the nearest penny, at the rate of 25·2 francs for £1?**

$$25{\cdot}2 \text{ francs} = £1.$$

$$\therefore \quad 1 \text{ franc} = £\frac{1}{25{\cdot}2}.$$

$$\therefore \; 500 \text{ francs} = £\frac{500}{25{\cdot}2}.$$

The reduction is left to the student.

EXAMPLE 2. **English money being at a premium of 2 per cent., what English money, to the nearest penny, must a man remit to discharge a debt of 1000 francs, exchange being at the rate of 25·24 francs at par?**

$$£1 = \frac{25{\cdot}24 \times 102}{100} \text{ francs.}$$

$$\therefore \; £\frac{100}{25{\cdot}24 \times 102} = 1 \text{ franc.}$$

$$\therefore \; £\frac{100 \times 1000}{25{\cdot}24 \times 102} = 1000 \text{ francs}$$

The reduction is left to the student.

EXAMPLES XXX. b.

1. Find the value in French money of £3. 10*s.* at the rate of 25 francs 24 centimes for £1.

2. Find, to the nearest penny, the value of 72 dollars in English money at the rate of 10 dollars for £2. 1*s.* 1*d.*

3. On landing at Calais from Dover I change 17*s.* 9*d.* into French money, at the rate of 25 francs for £1. How much do I get, to the nearest centime?

4. Find the value of 720 francs in English money, to the nearest penny, at the rate of 25·24 francs for £1.

5. Find the value of £10 in French money, to the nearest centime, at the rate of 10 francs for 7*s.* $11\frac{1}{4}d.$

6. If 100 Turkish piastres are worth 18 English shillings, find the value of 7325 piastres in English money.

7. If a Japanese 20 yen piece is worth £2. 1*s.*, find the value of £100 in yens correct to two decimal places.

8. Find the value of £74. 5*s.* in German marks at the rate of 21 marks for £1.

9. 10 German marks are worth 9*s.* $9\frac{1}{2}d.$ Find the value of £10 in German money to the nearest pfennig. (1 mark = 100 pfennige.)

10. How many roubles must a man remit from St. Petersburg to discharge a debt of £67. 13*s.* 9*d.* in London, the rate of exchange being 3*s.* 2*d.* for one rouble?

11. A man in London wishes to send 3000 francs to Paris through his Bank, exchange being at the rate of 24·25 francs for £1. How much English money must he pay into his Bank?

12. What is the exchange value of 600 francs in English money, to the nearest penny, when £1 = 25·25 francs at par, and English money is at a premium of 10 per cent.?

13. How much English money must a man pay into his Bank in order to discharge a debt of 1000 dollars at New York, the rate of exchange being 4*s.* $2\frac{3}{8}d.$ for one dollar?

14. If a franc is worth $8\frac{3}{4}d.$ and a mark is worth $10\frac{1}{2}d.$, what sum of money in francs will discharge a debt of 534 marks?

15. English money being at a premium of 5 per cent., what is the exchange value of 300 francs in English money, to the nearest penny, when £1 = 25·4 francs at par?

16. If the exchange between London and Amsterdam be 36*s.* Flemish per £1 sterling, and between Paris and Amsterdam 2 francs per 3*s.* Flemish, how much money sterling must be remitted to Paris, by way of Amsterdam, to discharge a debt of 20,000 francs?

17. If 31 Napoleons (20-franc pieces) contain 180 grams, and 6231 sovereigns 704,103 grains of gold, find the smallest whole number of Napoleons that are equal to a whole number of sovereigns, 5 lb. Avoirdupois being equivalent to 2·268 kilograms.

18. A man bought a substance at 1 franc per kilogram; how much in English money, to the nearest penny, did 100 lb. cost him? Take £1 = 25 francs 30 centimes, and 1 kilogram = 2·205 lb.

19. A person in London owes another in Petersburg a debt of 460 roubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 23 francs for £1, and between Paris and Petersburg 2 francs for 1 rouble. The remittance is delayed until the rates of exchange are 24 francs for £1 and 3 francs for 2 roubles. What does the broker lose or gain by the transaction?

If the following are the values of £1 in various countries:

Switzerland 25·25 fr., Germany 20·45 M., Russia 6·30 roubles*, U.S. 4·86 $, find the value of

20. 275 marks 50 pf. in English money.

21. 372 dollars 60 cents ,,

22. £64 6*s.* 8*d.* in roubles and kopecks. (100 kopecks = 1 rouble.)

23. 82 dollars 35 cents in Swiss money.

24. Before entering Norway I changed £56 at the rate of 18 krone 30 ore for £1, and I spent 960 kr. 75 ore. What did I get for the remainder in English money at the same rate? (100 ore = 1 krone.)

25. If a Dutch 10 florin piece is worth 16*s.* 6*d.*, and £1 = 20 65 marks, express 70 florins in marks.

* In Nov. 1930 the rouble was officially quoted as equal to 2*s.* $0\frac{1}{2}d.$

26. A man changes £63 into U.S. money when the dollar is worth 4*s.* 2*d.*, and has to change back what he has received into English coinage when the exchange is 4*s.* $1\frac{1}{4}d.$ for the dollar. Find his loss.

27. When the exchange between London and Paris is 25·30 francs for £1 and that between Paris and Berlin is 81 marks for 100 francs, find the arbitrated rate of exchange between London and Berlin (*i.e.* the exchange calculated through Paris).

28. If the exchange between London and Amsterdam is 12·14 florins for £1, and between London and Paris 25·38 fr. for £1, what is the arbitrated exchange for 100 fr. into Dutch money ?

29. If the exchange between London and Paris is £1 = 25 fr. 30 c., between Paris and Berlin 100 fr. = 80 M. 71 pf. and between London and Berlin £1 = 20 M. 38 pf., how much more profitable would it be to exchange £100 into German money through Paris than direct?

30. A bill for £630 passes through Berlin to New York. The exchange being 20·60 Mk. for £1 and $4·70 for 20 Mk., find the value of the bill in dollars.

31. If £1 = 25·35 francs, what is the (commercial) present value in English money of a bill of 19012·50 francs due in 3 months at 4 %?

32. A field of 21,870 sq. metres was sold in Germany for 7560 Mk. Supposing that £1 = 25 fr., 200 fr. = 161 Mk. and 1 ac. = ·4047 Hectare, find the price in £ *s.* *d.* per acre.

207. EXAMPLE 1. **In a race of 100 yds., A can beat B by 7 yds. and B can beat C by 5 yds. (also in 100 yds.); by how much can A beat C in 100 yds.?**

Whilst A runs 100 yds., B runs 93 yds.

$\therefore$ „ $\frac{100}{93}$ „ „ 1 yd.

$\therefore$ „ $\frac{100\times100}{93}$ „ „ 100 yds.

$\therefore$ „ $\frac{100\times100}{93}$ „ C „ 95 „ [for C runs 95 yds. whilst B runs 100 yds.]

$\therefore$ „ 100 „ C „ $\frac{95\times93}{100}$

(= 88·35 yds.).

$\therefore$ A beats C by 100 − 88·35, *i.e.* 11·65 yds.

EXAMPLE 2. **Two men start at the same instant in opposite directions round a circular course of 440 yds. If they run respectively at the rates of 7 and 8 miles an hour, how long will it be before they meet (1) for the first time, (2) for the fourth time ?**

(1) The two men travel a total distance of 15 miles in 1 hour.

$\therefore$ „ „ „ 1 mile „ $\frac{1}{15}$ hour (= 4 min.).

$\therefore$ „ „ „ 440 yds. „ 1 minute,

i.e. they meet at the end of 1 minute.

(2) At the end of 1 minute they meet. In the next minute they travel a total distance of 440 yds. again.

$\therefore$ at the end of 2 minutes they meet again, and so on.

$\therefore$ they meet for the fourth time at the end of 4 minutes.

EXAMPLE 3. **Two trains travel at 45 and 60 miles per hour in opposite directions on parallel rails. If they are respectively 300 and 316 ft. long, how long do they take to completely pass one another?**

$$45 \text{ miles per hour} = \frac{45 \times 1760 \times 3}{60 \times 60} = 66 \text{ ft. per sec.}$$

$$60 \quad ,, \quad = \frac{60 \times 1760 \times 3}{60 \times 60} = 88 \quad ,,$$

If x secs. is the required time, the first train travels $66x$ ft. and the second $88x$ ft. in that time.

$$\therefore\ 66x + 88x = \text{the sum of the lengths of the trains}$$
$$= 616.$$
$$\therefore\ x = \tfrac{616}{154} = 4.$$

The required time is 4 seconds.

EXAMPLES XXX. c.

1. In a quarter-mile race A can beat B by 22 yds. and B can beat C by 22 yds. over the same distance. By how much can A beat C over a quarter-mile?

2. In a race of 100 yds. A beats B by 10 yds. and C by 19 yds.; by how much will B beat C in 100 yds.?

3. A and B ride a race of 31 miles on bicycles. The driving wheel of A's machine makes 3410 revolutions per hour and has a circumference of 168 inches; that of B makes 3520 revolutions per hour and has a circumference of 162 inches: which will win and by how much?

4. An autocar and a dogcart start together from the same place to go in the same direction. The autocar takes 6 minutes to perform the first mile, but it subsequently travels at the rate of 18 miles an hour. The dogcart goes at a uniform rate of 12 miles an hour. Find which of them first reaches the end of the second mile, and how many yards it then is in front of the other carriage.

5. Two men run a race of 100 yds. One man is given 5 yds. start and is beaten by 3 yds. If the winner covers the distance in $10\frac{3}{4}$ seconds, how long would the loser take to run the same distance?

6. A can give B 20 yards and can give C 41 yards start in a race of a quarter of a mile, and B can give C 3 seconds over the same distance. How long does each take to run a quarter of a mile?

7. In a race of one mile A can give B 20 yards and B can give C 88 yards. What can A give C?

8. In a game of Fives, out of 15 points A can give B 3; also A can give C 7 points. How many points can B give C so as to make an even match?

9. How long will a train 600 ft. long be in completely passing two telegraph posts 280 feet apart, if it travels at the rate of 40 miles an hour?

10. A train travelling 30 miles an hour takes 5 secs. to pass a man walking alongside the line of railway in the same direction at 4 miles an hour. Find the length of the train.

11. A train 308 feet long takes 5 secs. to pass a man walking along the line of railway in the same direction at 3 miles per hour: find the rate of travelling of the train, in miles per hour.

12. Two trains of equal length, each travelling at 45 miles per hour, but in opposite directions, take 4 secs. to pass one another. Find the length of each train.

13. A man bicycles at a steady pace of 12 miles per hour, and 20 minutes afterwards a motor car starts from the same place in the same direction and runs at a steady pace of 18 miles per hour. How far from the start does the car catch the man up?

14. Two men run round a circular course 440 yds. in circumference, starting from the same point, in the same direction, at the same time. If they run respectively at 8 and 9 miles an hour, how long will it be before the faster runner gains one lap on the slower?

15. At what o'clock will a train, which leaves London for Crewe at 11.0 a.m. and goes at the rate of 45 miles an hour, meet a train which leaves Crewe for London at 11.40 a.m. and goes at the rate of 35 miles an hour, the distance between London and Crewe being 158 miles?

16. Two men were walking, each at 3 miles per hour, in opposite directions at the side of a railway, and a train passed one of the men in 5 seconds and the other in 6 seconds. Find the length of the train.

17. Two men were walking, each at 3 miles per hour, in opposite directions along a railway, and a train passed one of them in 9 seconds and the other in $11\frac{1}{4}$ seconds. How long was the train, and how fast was it going?

18. A train takes 50 minutes longer to do a journey when it is running 27 miles an hour than when it is running 36 miles an hour. Determine the length of the journey.

19. The railway between two stations A and B ascends at a gradient of 1 in 275 from A to a place C, distant $16\frac{1}{4}$ miles from A, and then descends at a gradient of 1 in 165 from C to B. If the station at B is 28 ft. lower than that at A, find the distance from A to B, measured along the railway.

[A gradient of 1 in 7 means that the train rises vertically 1 ft. in travelling 7 ft.]

20. Two bicyclists start simultaneously from two places 63 miles apart to meet one another. If they ride at the rates of 8 and 10 miles an hour respectively, find when and where they meet. Solve the problem graphically and otherwise.

21. Two bicyclists start at the same time from two towns 65 miles apart, and meet in $2\frac{1}{2}$ hours. If one rides 3 miles an hour faster than the other, where do they meet?

22. A train leaves Waterloo at 11.45 a.m. and travels at an average rate of 45 miles an hour; at what time is it due at Exeter, if it meets the 12.15 p.m. from Exeter—which averages 54 miles an hour—at a place 90 miles from Waterloo? How far is it from Waterloo to Exeter?

23. A man travels 60 miles in 3 hours, partly by rail and partly by coach. If he had gone all the way by rail he would have arrived at his destination an hour earlier, and would have saved two-fifths of the time he was on the coach. How far did he travel by coach?

24. A train 400 feet long, travelling 40 miles an hour, catches up another, travelling at 30 miles an hour on parallel rails. If the second train is 480 feet long, how long does the first take to pass the second?

25. In a quarter-mile race A has 20 yds. start and runs at the rate of 350 yds. in 50 secs. B runs at the rate of 450 yds. per minute. Which wins, and by how many yards?

26. A and B start from two places P and Q respectively at the same time, and walk to and fro between these places at the rates of $3\frac{3}{4}$ and $3\frac{7}{8}$ miles per hour. How long will it be before they are both at P at the same instant, if the distance from P to Q is one mile? Prove that they are never simultaneously at Q.

27. Two trains run in opposite directions with equal speeds round a circle which they traverse in 70 minutes. Ten minutes after a meeting of the trains the steam-pressure in one falls so that half its speed is lost. In how many minutes after this meeting does the next meeting occur?

28. In a race of 100 yds. A can give B 5 yards. If in another race over the same distance A runs with $\frac{9}{10}$ of his former speed and B with $\frac{17}{19}$ of his (B's) former speed, by how much does A beat B?

29. Two boats start to row a race at 5 p.m. The race is over at $6\frac{2}{5}$ minutes past 5, the losing boat being 40 yards behind at the finish. At 4 min. past 5 this boat was 700 yards from the winning post. Find the speed of each boat in miles per hour.

30. Messengers travelling 10 miles an hour are sent out every 6 minutes to meet a person approaching at 5 miles an hour: at what intervals of time will they meet him?

31. A tricyclist, going at the rate of 5 miles an hour, passes a milestone, and 14 minutes afterwards, a bicyclist, going in the same direction at the rate of 12 miles an hour, passes the same milestone; find when and where the bicyclist will overtake the tricyclist.

32. A train leaves London for Brighton at 9 a.m., travelling at a uniform rate of 15 miles an hour. An express train leaves Brighton for London at 10 a.m. and travels at a rate of 40 miles an hour. At what time will they pass each other and at what distance from London, the distance from London to Brighton being 50 miles?

33. A and B start at the same time from London to Blisworth, A walking 4 miles an hour, B riding 9 miles an hour. B reaches Blisworth in 4 hours, and immediately rides back to London. After 3 hours' rest he starts again for Blisworth at the same rate. How far from London will he overtake A, who has in the meantime rested for 6 hours?

34. In an examination B made 5 per cent. more marks than A, C made 10 per cent. more than B, D made 15 per cent. more than C and E made 25 per cent. more than D. E made 5313 marks. How many did A make?

35. When 52 lb. of coffee are worth as much as 12 lb. of tea, 22 lb. of tea are worth as much as 572 lb. of sugar, a cask of sugar costs 2 guineas and 1 cwt. of coffee costs 8 guineas; what is the weight of a cask of sugar?

36. A merchant mixes wine at 25*s.* a dozen with wine at 30*s.* a dozen, sells the mixture at 36*s.* a dozen and gains 26 per cent.; in what proportion are the wines mixed?

37. A man leaves Southampton at 10 a.m. on Saturday for New York, which is 3080 miles away. At 5 p.m. on the same day a detective leaves Liverpool for Quebec, a distance of 2630 miles, and on landing at Quebec proceeds to New York, which he reaches 12 hours later, just in time to meet the other on his arrival. If the detective's boat travelled at 18 miles an hour, find how fast the other boat travelled.

38. A and B have money in the proportion of 7 to 9; B pays one-third of his money to A and receives back one-seventh of what A then has, thus leaving A with £1. 12*s.* more than B. How much had each at first?

39. The beam of a defective balance is horizontal when 15 oz. are placed in one scale and 14 oz. in the other. A tradesman, placing a pound weight alternately in the two scales of this balance, sells what he imagines to be pounds of a certain commodity at an advance of 4 per cent. on the cost price. What is his actual gain per cent.?

40. A builder undertakes a contract for building a house at a total cost of £2500 and calculates that he will realise a profit of 10 per cent. on his outlay. During the progress of the work wages rise, so that at the end he finds that he has paid on the average $6\frac{1}{2}d.$ an hour to each workman instead of $6d.$ Assuming that the contractor calculates that $\frac{3}{5}$ of his outlay will be spent on labour, find his actual profit to the nearest penny.

41. An armourer undertakes to supply 2000 swords at 17*s.* 3*d.* each. He estimates that if 5 per cent. fail to stand the required test and are worthless, the profit will be 15 per cent. on his whole outlay. At the trial 35 per cent. prove worthless. If worthless swords are not replaced, how much does the armourer lose on the contract?

42. A owes B £1577. 15*s*. 6*d*. payable 18 months hence. B owes A £1494. 11*s*. 4*d*. payable 8 months hence. If they agree to settle their account by a ready money payment, what sum should be paid over, reckoning the rate of true discount at 6 per cent.?

43. A manufacturer makes cloth at a cost of 2*s*. $10\frac{1}{2}$*d*. a yard, and sells it in England at 3*s*. 6*d*. a yard and in France at 5 francs a metre. If he incurs an additional expense of $1\frac{1}{2}$*d*. a yard by selling in France, which of these prices gives him the higher rate of profit per cent. on his expenditure? State the percentage profit on sales in England and in France.

Take 25 francs = £1, 1 yard = 0·9144 metre.

44. A, B, C start a business, contributing to the capital in the proportion of 7, 10, 3 respectively. At the end of the first year A withdraws £2000 of capital, which is replaced by B. At the end of the second year B's share of the profit for the two years is £2730 and C's £780. Assuming that the same profit was made in each year, find the amount of capital originally invested by each.

45. An orange-woman sells half her oranges at 3 for twopence and the remainder at 20 for a shilling. If the oranges cost her 5*d*. per dozen, what is her gain per cent.?

46. A man has a lease of a house, which has 3 years to run, and the rent is £150 a year, to be paid at the end of each year. In consideration, however, of a sum of money paid down he gets the rent reduced to £100 for the rest of his term. What sum of money (to the nearest £) ought he to pay, the current rate on investments being 3 per cent. (simple interest)?

47. Goods are sold so that when 12 per cent. is allowed off the sale price a profit of 10 per cent. is made. How much per cent. is the sale price higher than the cost price?

48. After a week's exhibition of pictures the entrance fee was lowered by 25 per cent., which resulted in an increase of receipts during the next week by 8 per cent. By what percentage was the weekly number of visitors altered?

49. If 5 litres of a liquid which weighs 1·13 times as much as water are mixed with two litres of another liquid which weighs 0·87 times as much as water, find how many times as heavy as water the mixture will be.

50. The value of a certain house had by 1880 increased 35 per cent. since 1877. The house was rated in 1877 at two-thirds of its value, and in 1880 it was rated at three-fifths of its value, the rate in the £ remaining the same. Compare the rate paid in 1877 with that paid in 1880.

51. Two horses can plough in a given time as much as 3 oxen, but the cost of 4 oxen is only equal to that of 3 horses, the daily cost of a horse being 3*s*. A certain field can be ploughed by 3 horses in 8 days. What would be the cost of ploughing it by oxen in 6 days?

gallon of spirit which contains 11 per cent. of water is three gallons containing 7 per cent. of water, and to this half a gallon of water is added. Find the percentage of the mixture.

0 lb. Troy of standard gold can be coined into 1869 sovereigns, the proportion of pure gold to alloy in standard gold being 22 to 2. What weight of gold, to the nearest grain, is there in a sovereign?

54. There is a well containing 750 gallons of water; two pumps raising 20 and 30 gallons per minute respectively are employed to empty it; while it is constantly supplied by a spring which can refill it in half an hour. The two pumps work together for 15 minutes, when that of larger capacity ceases work for 10 minutes; the two pumps then work together until the well is empty. How long will each pump have been employed?

55. By buying 3 per cent. Consols at a certain price I find I obtain $3\frac{3}{5}$ per cent. for my money, and derive a net income therefrom, after paying an income-tax of 6*d*. in the £, of £421. 4*s*. Find the amount of stock and the price at which I bought it.

56. A person gave £75 for 20 casks of oil, each containing 30 gallons. He sold 5 casks at 3*s*. per gallon; one cask was stove in and the whole of its contents lost, and 15 gallons were also lost by ordinary leakage. He then sold the remainder at a price per gallon which made his gain amount to 20 per cent. on the whole transaction. What was his selling price per gallon at the second sale?

XXXI. LOGARITHMS.

Indices.

[*A full stop (.) may be used for the sign of multiplication where there is no risk of its being mistaken for a decimal point.*]

208. It has been explained in Chapter I. that $a^2 = a \,.\, a$ and $a^3 = a \,.\, a \,.\, a$.

Consequently $\quad a^2 \times a^3 = a \,.\, a \times a \,.\, a \,.\, a = a^5$.

Similarly $\quad a^n \,.\, a^p = a^{n+p}$.

These results depend upon the definition "a^n is the product of n factors a"; and this definition evidently requires that n should be a positive integer.

If we wish to employ fractional or negative indices, we must give a definition which is applicable to them.

DEF. ***Fractional and negative indices are defined as being such that they obey this law:—To multiply powers of a quantity add the indices.***

To find the meaning of $a^{\frac{1}{n}}$.

$a^{\frac{1}{2}}$ when multiplied by itself becomes $a^{\frac{1}{2}+\frac{1}{2}}$, *i.e.* a; $\therefore\ a^{\frac{1}{2}}=\sqrt{a}$.

Similarly $a^{\frac{1}{3}}$ when cubed becomes $a^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$, *i.e.* a; $\therefore\ a^{\frac{1}{3}}=\sqrt[3]{a}$.

$a^{\frac{1}{n}}$ when raised to the n^{th} power becomes a; $\therefore\ a^{\frac{1}{n}}=\sqrt[n]{a}$.

We therefore denote $\sqrt{a}$ by $a^{\frac{1}{2}}$, $\sqrt[3]{a}$ by $a^{\frac{1}{3}}$, $\sqrt[n]{a}$ by $a^{\frac{1}{n}}$.

To find the meaning of a^0.

$$a^0=\frac{a^0\,.\,a^n}{a^n}=\frac{a^{0+n}}{a^n}=\frac{a^n}{a^n}=1.$$

To find the meaning of a^{-n}.

$$a^{-n}=\frac{a^{-n}\,.\,a^n}{a^n}=\frac{a^{-n+n}}{a^n}=\frac{a^0}{a^n}=\frac{1}{a^n}.$$

EXAMPLES XXXI. a.

ORAL EXAMPLES ON INDICES.

Read off the results in the following :

1. $a^{\frac{1}{3}}\times a^{\frac{2}{3}}$. **2.** $a^{\frac{3}{4}}\times a^{\frac{1}{4}}$. **3.** $a^{\frac{5}{8}}\times a^{\frac{11}{8}}$. **4.** $a^{\frac{4}{3}}\div a^{\frac{1}{3}}$. **5.** $a^{\frac{5}{4}}\div a$.

6. $a^{-2}\times a^3$. **7.** $a^3\div a^{-2}$. **8.** $10^2\times 10^{-2}$. **9.** $10^{\frac{9}{8}}\div 10^{\frac{1}{8}}$.

Express with indices

10. $\sqrt{a^3}$. **11.** $\sqrt[3]{a^6}$. **12.** $\sqrt[3]{a^5}$. **13.** $\sqrt[4]{a^8}$.

14. $\sqrt{a}\times\sqrt{a}\times\sqrt{a}$. **15.** $\sqrt{a^{\frac{1}{2}}}$. **16.** $\sqrt[3]{a^2}$. **17.** $\sqrt[4]{a^9}$.

Express

18. 16 as a power of 2. **19.** $\sqrt{8}$ as a power of 2.

20. $2\sqrt{8}$ „ 2. **21.** 125 „ 5.

22. $\sqrt{125}$ „ 5. **23.** $\sqrt[3]{100}$ „ 10.

24. ·0001 „ 10. **25.** $10^{\frac{1}{2}}\times 10^{\frac{1}{4}}$ „ 10.

Logarithms.

209. DEF. If one number be chosen as base, the logarithm of any number n to this base is the index of the power to which the base must be raised to be equal to n.

The logarithm of n to the base a is written $\log_a n$.

Thus if $a^p = n$, $p = \log_a n$.

Logarithms calculated to the base 10 are called *common* logarithms.

$$10^1 = 10 \quad \therefore \text{ by definition } \quad \log_{10} 10 = 1,$$
$$10^2 = 100 \quad \therefore \quad ,, \quad \log_{10} 100 = 2,$$
$$10^3 = 1000 \quad \therefore \quad ,, \quad \log_{10} 1000 = 3.$$

210. $a^0 = 1$; $\therefore$ $\log_a 1 = 0$, *i.e.* whatever the base, the logarithm of 1 is zero.

211. *To prove* $\log_a mn = \log_a m + \log_a n$.

Let $\log_a m = x$, and $\log_a n = y$.

Then $$m = a^x,\ n = a^y;$$
$$\therefore\ mn = a^x . a^y = a^{x+y};$$
$$\therefore\ \log_a mn = x + y = \log_a m + \log_a n.$$

i.e. **the logarithm of a product = the sum of the logarithms of its factors.**

Thus $$\log 20 = \log 10 + \log 2 = 1 + \log 2;$$
$$\log 500 = \log 100 + \log 5 = 2 + \log 5.$$

Here $\log 2$ means $\log_{10} 2$, and similarly for the others.

212. *To prove* $\log_a \frac{m}{n} = \log_a m - \log_a n$.

Let $\log_a m = x$, $\log_a n = y$.

Then $$m = a^x,\ n = a^y;$$
$$\therefore\ \frac{m}{n} = \frac{a^x}{a^y} = a^{x-y};$$
$$\therefore\ \log_a \frac{m}{n} = x - y = \log_a m - \log_a n.$$

i.e. **the logarithm of a fraction = log numerator – log denominator,**

or **the logarithm of a quotient = log dividend – log divisor.**

Thus $\log \cdot 02 = \log \frac{2}{100} = \log 2 - \log 100 = \log 2 - 2$.

213. *To prove* $\log_a n^r = r \log_a n$.

Let $\log_a n = x$, then $n = a^x$;
$$\therefore\ n^r = (a^x)^r = a^{rx};$$
$$\therefore\ \log_a n^r = rx = r \log_a n,$$

i.e. **the logarithm of any power of a number is the product of the logarithm of the number and the index of the power.**

Thus $\log 10000 = 4 \log 10 = 4$; $\log 16 = \log 2^4 = 4 \log 2$.

Art. 211 shows that to obtain the product of numbers we may use the sum of their logarithms. Here we learn that raising a number to a given power depends upon multiplying the logarithm by the given index.

Thus square root is obtained by halving the logarithm and finding the number of which the result is the logarithm.

DEF. If x is the logarithm of n, then n is called the **antilogarithm** of x.

Hence the rule given above takes a simpler form, viz. the square root of a number is obtained by halving its logarithm and finding the antilogarithm of the result.

In Arithmetical operations *common logarithms*, as those to base 10 are called, are the ones employed.

Characteristic and Mantissa.

214. DEF. *The integral part of a logarithm is called its* **characteristic**, *the decimal part its* **mantissa.**

$$10^4 = 10000, \ 10^5 = 100000;$$

$\therefore$ the common logarithm of a number between 10,000 and 100,000 is between 4 and 5, and is therefore $4 +$ a decimal.

$\therefore$ the characteristic of the logarithm of a number of 5 integral figures is 4.

Similarly in other cases.

Thus the logarithm of a number n has for characteristic a number less by 1 *than the number of integral digits in n.*

215. $$a^0 = 1; \quad \therefore \ \log_a 1 = 0.$$

Thus $\log_{10} 1 = 0$; $\therefore$ the logarithm of a number less than 1 is less than 0, *i.e.* it is negative.

If a logarithm is negative, its mantissa is usually made positive by changing the characteristic.

$$\log_{10} \cdot 3 = \log 3 - \log 10 = \cdot 4771 - 1.$$

This might be written $-\cdot 5229$; but here the mantissa would be negative. The proper way of writing it is $\bar{1}\cdot 4771$, where the bar over the 1 indicates that the characteristic is negative and the rest positive.

$\log \cdot 0001 = \log \dfrac{1}{10^4} = \log 10^{-4} = -4$. Similarly $\log \cdot 001 = -3$.

∴ the logarithm of any number beginning with a point followed by 3 zeros is between -4 and -3. It must therefore be $-4+$ a decimal. Thus the characteristic is -4.

A similar argument proves that ***when we take the logarithm of any decimal less than 1, the characteristic is negative and is numerically one more than the number of zeros before the first significant figure.***

[The student has already learnt that the significant figures of a number are those which remain when all zeros at the beginning and end have been removed; *e.g.* the significant figures of ·0032016 and of 32016000 are 32016.]

It should be noticed that

$$\log \cdot 0032016 = \log \frac{32016}{10^7} = \log 32016 - 7\,;$$

$$\text{and } \log 32016000 = \log 32016 + \log 1000 = \log 32016 + 3.$$

So both these logarithms have the same mantissa as log 32016.

The mantissae of the logarithms of numbers are the same, provided the numbers have the same significant digits.

$$\log \cdot 0003 = \bar{4}\cdot 4771.$$

Suppose it is required to multiply or divide this by a number, say 5.

$$\bar{4}\cdot 4771 \times 5 = (-4 + \cdot 4771) \times 5 = -20 + 2\cdot 3855 = \overline{18}\cdot 3855.$$

$$\frac{\bar{4}\cdot 4771}{5} = \frac{-5 + 1\cdot 4771}{5} = -1 + \cdot 2954 = \bar{1}\cdot 2954.$$

The Principle of Proportional Parts.

216. *For numbers differing by small quantities, i.e. by small fractions of themselves, the differences of the logarithms are nearly proportional to the differences of the numbers.*

This principle is most important in the construction, and to some extent in the use, of tables.

If we know log 213 and log 214, by this principle we can find log 213·7.

$$\log 213 = 2\cdot 3284$$

$$\text{and } \log 214 = 2\cdot 3304.$$

Here the numbers differ by 1, and their logarithms differ by ·0020.

If the numbers differed by $\frac{1}{4}$ of 1, their logarithms would differ by $\frac{1}{4}$ of $\cdot 0020$; and similarly for other cases.

If we wish to obtain $\log(213+x)$ from $\log 213$, we may call it $2\cdot 3284+y$, where we recognise that y is the same fraction of $\cdot 0020$ as x is of 1.

Thus $\qquad \log 213\cdot 7=\log(213+\cdot 7)=2\cdot 3284+y,$

where $\qquad \dfrac{y}{\cdot 0020}=\dfrac{\cdot 7}{1}, \qquad$ *i.e.* $y=\cdot 0014.$

Hence $\qquad \log 213\cdot 7=2\cdot 3284+\cdot 0014=2\cdot 3298.$

Mathematical Tables.

217. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17

The quotation given above from a table of 4-figure logarithms will show the method of reading the logarithm of any number. For instance, suppose that log 1756 is required. Look along the line beginning 17 until you reach the figures below the 5 which occurs in the top line. The figures are 2430. For the final 6 take the figures below 6 in the columns on the right of the page, viz. 15. The total result is 2445. In reality, the figures are $\cdot 2430$ and $\cdot 0015$, giving a total of $\cdot 2445$ as the mantissa of the logarithm of a number whose significant figures are 1756.

Add the proper characteristic, and the logarithm of 1756 is $3\cdot 2445$.

Also $\quad \log 17\cdot 56=1\cdot 2445$ and $\log \cdot 01756=\bar{2}\cdot 2445$;

for the logarithms of numbers have the same mantissa if the numbers themselves have the same significant digits.

By the principle of proportional parts, if we required $\log 1756\cdot 7$ we should have to add $\frac{1}{10}$ of the difference for 7; *i.e.* $\dfrac{\cdot 0017}{10}$, *i.e.* $\cdot 0002$.

Thus $\log 1756\cdot 7=3\cdot 2447$.

Antilogarithms.

218. The reverse process, that of finding the antilogarithm of a set of figures (*i.e.* the number whose logarithm is the given set of figures), can be accomplished by searching in the columns for the given set of figures or the set next less than these, and making, in the latter case, the proper allowance for the difference by means of the right-hand columns. Labour is saved, however, by using tables of **antilogarithms**, which are read in a similar manner to the tables of logarithms.

It must be remembered that the mantissa only is given in the table and only the **significant digits** of the antilogarithm. The position of the decimal point in the antilogarithm must be determined by the given characteristic.

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4

Find the antilogarithm of ·2445 and of 3·2445.

The significant digits are found by adding to 1754 the figure under 5 in the right-hand columns. [See line beginning ·24.]

Thus the antilogarithm of ·2445 is 1·756, since the significant digits are 1756 and the characteristic 0 shows that there is *one* integral figure.

So also 3·2445 is the logarithm of 1756.

219. The tables of logarithms may be used in such a manner as to obtain the proper characteristic without direct reference to the rules given for writing down the characteristic. The method itself explains how the characteristic occurs. Any logarithm as it appears in the tables, viz. with 0 for characteristic, is the logarithm of a number containing one integral digit. This is regarded as the **standard form**; and all numbers, whose logarithms

are required, may be expressed in terms of this standard form by multiplying or dividing by some power of 10.

For instance, $\log 7{\cdot}253 = 0{\cdot}8605.$

$$\log 7253 = \log(7{\cdot}253 \times 10^3) = {\cdot}8605 + 3 = 3{\cdot}8605.$$
$$\log {\cdot}0007253 = \log(7{\cdot}253 \times 10^{-4}) = {\cdot}8605 - 4 = \overline{4}{\cdot}8605.$$

EXAMPLES XXXI. b. (*Oral.*)

[*The common base*, 10, *is used in all these examples.*]

What is the characteristic of

1. $\log 136$?   **2.** $\log 2050$?   **3.** $\log 2{\cdot}63$?

4. $\log 73000$?   **5.** $\log 0{\cdot}65$?   **6.** $\log 0{\cdot}7254$?

7. $\log {\cdot}0005$?   **8.** $\log {\cdot}00365$?   **9.** $\log 17{\cdot}8924$?

Given that $\log 2 = {\cdot}3010$, read off the value of

10. $\log 20$.   **11.** $\log 2000$.   **12.** $\log {\cdot}2$.

13. $\log {\cdot}0002$.   **14.** $\log 2 \times 10^5$.   **15.** $\log\left(\dfrac{2}{10^3}\right)$.

Given that $\log 2364 = 3{\cdot}3736$, read off the value of

16. $\log 2{\cdot}364$.   **17.** $\log 236{\cdot}4$.   **18.** $\log 236400$.

19. $\log {\cdot}2364$.   **20.** $\log {\cdot}002364$.   **21.** $\log(2{\cdot}364 \times 10^5)$.

220. Before attempting examples involving logarithms, the student should have some oral practice in the use of logarithm and antilogarithm tables.

E.g. **Read off $\log 62{\cdot}37$, $\log 620{\cdot}9$, $\log {\cdot}0271$, and so on.**

Read off the numbers whose logarithms are

$3{\cdot}235$, $1{\cdot}067$, ${\cdot}0824$, $\overline{1}{\cdot}6258$, and so on.

Example 1. **A cube contains 3 c. ft. 904 c. in. Find the length of its edge.**

3 c. ft. 904 c. in. = 6088 c. in.

$$\log \sqrt[3]{6088} = \frac{1}{3}\log 6088 = \frac{3{\cdot}7845}{3} = 1{\cdot}2615.$$

Looking at the table of antilogarithms, we find that corresponding to ${\cdot}261$ are the figures 1824, and from the columns on the right we see that for the final 5 we must add 2.

$\therefore$ the significant figures required are 1826.

The characteristic 1 (in $1{\cdot}2615$) shows that there are 2 integral figures;

$$\therefore \text{antilog } 1{\cdot}2615 = 18{\cdot}26.$$
$$\therefore \sqrt[3]{6088} = 18{\cdot}26.$$
$$\therefore \text{the length of edge is 1 ft. } 6{\cdot}26 \text{ in.}$$

After a little practice it would only be necessary to write down

$$\log \sqrt[3]{6088} = \frac{1}{3} \log 6088 = \frac{3 \cdot 7845}{3} = 1 \cdot 2615 = \log 18 \cdot 26.$$

Length of edge = 18·26 in. = 1 ft. 6·26 in.

EXAMPLE 2. **Find by logarithms the product of 2·413 and ·6052.**

The logarithm of the product $= \log 2 \cdot 413 + \log \cdot 6052$
$= \cdot 3825 + \bar{1} \cdot 7819 = -1 + 1 \cdot 1644$
$= \cdot 1644.$

From the table of antilogarithms we find antilog ·1644 = 1·460.

∴ the required product = 1·460 to 3 decimal places.

EXAMPLE 3. **Find the value of 2·644 ÷ ·2863.**

The logarithm of the quotient $= \log 2 \cdot 644 - \log \cdot 2863$
$= \cdot 4223 - \bar{1} \cdot 4569 = 1 \cdot 4223 - \cdot 4569$
$= \cdot 9654 = \log 9 \cdot 235.$

∴ the required quotient = 9·235 (correct to 3 decimal places).

EXAMPLE 4. **Find the value of (£1. 3s. 6d.) × ·784.**

£1. 3*s*. 6*d*. = £1·175.

$$\log (1 \cdot 175 \times \cdot 784) = \log 1 \cdot 175 + \log \cdot 784 = \cdot 0701 + \bar{1} \cdot 8943$$
$$= \bar{1} \cdot 9644 = \log \cdot 9212.$$

∴ the required value = £·9212 = 18*s*. 5*d*.

EXAMPLE 5. **Find the amount of £76. 3s. in 11 years at 6 per cent. compound interest.**

Every year £1 is converted into $1 + \frac{6}{100}$, *i.e.* into £1·06.

∴ each year the principal is multiplied by 1·06.

∴ the amount of £76·15 in 11 years $= 76 \cdot 15 \times (1 \cdot 06)^{11}$.

$$\log [76 \cdot 15 \times (1 \cdot 06)^{11}] = \log 76 \cdot 15 + 11 \times \log 1 \cdot 06 = 1 \cdot 8817 + 11 \times \cdot 0253$$
$$= 2 \cdot 1600 = \log 144 \cdot 5.$$

∴ the amount = £144. 10*s*.

EXAMPLE 6. **In how many years does a sum of money double itself at $4\frac{1}{2}$ per cent. compound interest?**

Each year the principal is multiplied by $1 + \frac{4 \cdot 5}{100}$, *i.e.* by 1·045.

∴ by the end of n years the principal has been multiplied by $1 \cdot 045^n$.

It is required to find n so that this multiplier may be 2.

$$1 \cdot 045^n = 2.$$
$$\therefore\ n \log 1 \cdot 045 = \log 2.$$
$$\therefore\ n \times \cdot 0191 = \cdot 301.$$
$$\therefore\ n = \frac{\cdot 301}{\cdot 0191} = \frac{30 \cdot 1}{1 \cdot 91} = 15 \cdot 76.$$

∴ **16 is the number of complete years required.**

EXAMPLES XXXI. c. (*Oral* 1–7.)

The student is supposed to have a table of logarithms. The following are quoted because they are of very frequent occurrence:

$$\log 2 = \cdot 3010, \quad \log 3 = \cdot 4771, \quad \log 7 = \cdot 8451.$$

[*The logarithms are to the base* 10.]

1. Find $\log 1000$, $\log 100{,}000$, $\log 1$, $\log \cdot 01$ and $\log \cdot 0001$.

2. Given $\log 2$, find $\log 5$, $\log 500$ and $\log 25$.

3. Find the characteristic of $\log 317$, of $\log 1234$, of $\log 12 \cdot 3$, of $\log \cdot 623$ and of $\log \cdot 048$.

4. Given that $a = 10^5 . b$, find the difference between $\log a$ and $\log b$.

5. Simplify $\log 2 + \log 5$, $\log 2 + \log 3$, $\log 12 - \log 3$, $\log 12 - 2\log 2$.

6. In the logarithms of 6043, 32, 801, 89567, ·0017, ·32, 3·241, ·0000386, 56·24 give the characteristic.

7. Explain why $\log 3251$ and $\log 9999$ have the same characteristic.

8. Prove that $\log 723$ and $\log 7 \cdot 23$ have the same mantissa.

9. Add together $1 \cdot 2864$, $2 \cdot 1572$, $\bar{3} \cdot 7632$.

10. Add together $\bar{5} \cdot 6391$, $1 \cdot 5534$, $\cdot 7431$.

11. Subtract $1 \cdot 2345$ from $\bar{2} \cdot 6387$.

12. Subtract $\bar{2} \cdot 5461$ from $1 \cdot 0386$.

13. Multiply $\bar{2} \cdot 4771$ by 5.

14. Multiply $\bar{3} \cdot 6990$ by 2.

15. Divide $\bar{1} \cdot 5423$ by 3.

16. Divide $\bar{2} \cdot 3184$ by 4.

17. Divide $\bar{3} \cdot 3075$ by 5.

18. How many zeros follow the decimal point in $\cdot 2^{20}$?

19. Find $\log 2592$, given $\log 2$ and $\log 3$. Hence find $\log 2 \cdot 592$, $\log 259200$, $\log \cdot 002592$.

20. How many digits are there in 2^{20}?

21. " " " 2^{25}?

22. What are the significant figures in 203500, 20·35, ·00176, 1760, ·01308?

23. Give the characteristic of $\log 1760$, of $\log 1 \cdot 76$ and of $\log \cdot 00176$.

24. Calculate these logarithms from $\log 2$ and $\log 11$.

25. Find $\log 5^3 . 3^4 . 7^2$, given $\log 2$, $\log 3$, $\log 7$.

26. Find $\log \dfrac{2^3 . 3^2}{5^2}$ from $\log 2$ and $\log 3$.

27. Find $\log 3528$ from $\log 2$, $\log 3$ and $\log 7$.

28. Hence find $\log 35 \cdot 28$, $\log 352800$ and $\log \cdot 03528$.

29. Find $\log 3762 - \log 37 \cdot 62$ without tables.

From tables write down

30. log 26. **31.** log 2600. **32.** log 265. **33.** log 2658.

34. log 2·658. **35.** log 265·8. **36.** log ·002658.

37. Given antilog ·6153 = 4·124, find antilog 2·6153.

38. Find antilog ·3851. **39.** Find antilog 1·3851.

40. Find antilog 2·3851.

EXAMPLES XXXI. d.

Reduce each of the following (in questions 1 to 5) to *standard form* and find their logarithms.

1. 36840 and 368·4 **2.** 1567 and ·01567.

3. 428·6 and 4286000. **4.** 2113·5 and ·0021135.

5. 386·5, ·05713, 7641000, ·7648.

Find approximately the results of the following by four-figure logarithms:

6. $2{\cdot}3 \times 1270$. **7.** $\cdot2413 \times 6{\cdot}052$. **8.** $4{\cdot}951 \times 2{\cdot}836$.

9. $1{\cdot}414 \times 1{\cdot}732$. **10.** $\cdot3463 \times \cdot3973$. **11.** $\cdot746 \times \cdot6235$.

12. $3{\cdot}72^4$. **13.** $\cdot407 \times 40{\cdot}3 \times \cdot006$. **14.** $\cdot0438 \times 937$.

15. $48{\cdot}25 \div 634{\cdot}9$. **16.** $2{\cdot}644 \div \cdot2863$. **17.** $74{\cdot}25 \div 8{\cdot}89$.

18. $8{\cdot}475 \div 14{\cdot}36$. **19.** $\cdot07644 \div 147$. **20.** $\cdot86751 \div 24{\cdot}3$.

21. $8{\cdot}30676 \div 3596$.

22. A fourth proportional to 9·28, 10·19, 1·23.

23. £3. 7*s*. 6*d*. $\times 1{\cdot}01^3$.

24. The reciprocal of 4·97. (*i.e.* $1 \div 4{\cdot}97$.)

25. " " 24·5.

26. " " 623.

27. " " ·784.

28. The volume of a rectangular solid 64·3 cm. by 27·2 cm. by 35·5 cm. (Work in decimetres.)

29. $\sqrt[3]{567}$. **30.** $\sqrt[3]{1972}$. **31.** $\sqrt[3]{26\frac{3}{4}}$.

32. The area of a path $3\frac{3}{4}$ yds. wide, bounding a rectangular lawn 85·25 yds. by 56·3 yds.

33. The length of the edge of a cube whose volume is 880 c. cm.

34. Find the compound interest on £621 for 5 years at 3 p.c. to the nearest £.

35. Find the compound interest on £50 for 10 years at 4 p.c. to the nearest £.

36. Find at compound interest the amount of £825 in 6 yrs. at $3\frac{1}{2}$ p.c. to the nearest £.

37. Find at compound interest the amount of £120. 17*s.* 6*d.* in 7 yrs. at 4 p.c. to the nearest £.

38. Find at compound interest the amount of £30. 5*s.* 10*d.* in 14 yrs. at 5 p.c. to the nearest £.

39. In what time does a sum of money double itself at 5 p.c. compound interest?

40. In what time at 4 p.c.?

41. Find the amount of £10 in 40 years at 5 p.c. compound interest.

42. If the birth rate in a place be 76 in 1000 and the death rate 48 in a thousand, in how many complete years will the population be doubled?

43. Find the amount of £6000 in 6 yrs. at 5 p.c. compound interest to the nearest £10.

44. Money is put out to compound interest at $2\frac{1}{2}$ per cent.; show that it will have more than doubled itself in 29 years.

45. A sum of £1000 is lent on condition that it shall bear compound interest at 5 p.c. per ann. for the first 5 years, 10 p.c. per ann. for the next 5 years, and afterwards 15 p.c. per ann. What will the debt amount to in 20 years?

46. Given that 1 metre = 3·281 ft., find the nearest whole number of cub. yds. that there are in 1000 c. metres.

XXXII. RECURRING DECIMALS.

General Treatment of Recurring Decimals.

221. The following examples will afford practice in converting vulgar fractions into decimals which do not terminate.

Example 1. **Express $\frac{32}{41}$ as a decimal.**

$$
\begin{array}{r l}
 & \cdot\dot{7}804\dot{8} \\
41\,) & 32{\cdot}0\ldots\ldots(a) \\
 & \underline{28\;7} \\
 & \;\;3\;30 \\
 & \;\;\underline{3\;28} \\
 & \;\;\;\;200 \\
 & \;\;\;\;\underline{164} \\
 & \;\;\;\;\;360 \\
 & \;\;\;\;\;\underline{328} \\
 & \;\;\;\;\;\;320
\end{array}
$$

320 The quotient now begins to repeat itself. See line (a).

$$\therefore\ \frac{32}{41} = \cdot\dot{7}804\dot{8}.$$

EXAMPLE 2. **Express $\frac{159}{550}$ as a decimal.**

Here we shall use factors. $550 = 50 \times 11$.

$$\begin{array}{r|l} 50 & 159 \\ \hline 11 & 3{\cdot}180000\ldots \\ \hline & {\cdot}289090\ldots \end{array}$$

$$\therefore \frac{159}{550} = {\cdot}28\dot{9}\dot{0}.$$

In dividing by factors it is advisable to divide by 2's and 5's **first**; for the quotients then terminate.

Fractions which produce Recurring Decimals.

222. Every terminating decimal is equivalent to a vulgar fraction with some power of 10 as denominator; and there are no prime factors except 2 and 5 in a power of 10.

Therefore any fraction which, in its lowest terms, contains a prime factor other than 2 or 5, must fail to produce a terminating decimal.

Now, if we convert such a fraction as $\frac{1}{23}$ into a decimal by division, we see that 22 is the largest possible remainder in the course of the division; and consequently there cannot be more than 22 possible remainders.

But when a remainder appears which has occurred before, the quotient begins to repeat itself.

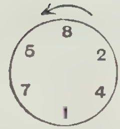
Therefore the decimal corresponding to a fraction with 23 as denominator cannot have more than 22 figures in the *recurring period* or *repetend.*

It will be found that the denominator 23 does actually produce a repetend of 22 figures; whereas 11 produces 2 figures, 13 produces 6 figures, and 37 produces 3 figures.

There are certain primes which have the property of producing a repetend of the maximum number of figures, *e.g.* 7, 17, 19, 23, 29, 47, etc.; and we may find it useful to note the following fact about them.

With such a denominator, whatever be the numerator, the repetend has the same figures in the same circular order, but beginning in various places.

Thus $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$. Circular order as in the figure.
$\frac{2}{7} = \cdot\dot{2}8571\dot{4}$. Circular order the same.



If the fraction be $\frac{3}{7}$, the first figure is got by dividing 30 by 7.

Thus $\qquad \frac{3}{7} = \cdot\dot{4}2857\dot{1}$.

$\qquad\qquad \frac{4}{7} = \cdot\dot{5}7142\dot{8}$.

Again, $\qquad \frac{1}{17} = \cdot\dot{0}58823529411764\dot{7}$.

$\qquad\qquad \frac{2}{17} = \cdot\dot{1}17647058823529\dot{4}$, etc.

One curious property of *these* repetends is that, if the first half be put under the other half and added to it, the sum consists entirely of nines.

$$\begin{array}{r} 857 \\ \underline{142} \\ 999 \end{array} \qquad \begin{array}{r} 94117647 \\ \underline{05882352} \\ 99999999 \end{array}$$

It is useful to remember the repetend corresponding to 7 so as to recognise at once that $\cdot\dot{4}2857\dot{1} = \frac{3}{7}$, and so on.

EXAMPLES XXXII. a.

[*Perform the division by factors where possible.*]

Convert the following into decimals :

1. $\frac{1}{13}$. 2. $\frac{5}{13}$. 3. $\frac{10}{13}$. 4. $\frac{3}{13}$. 5. $1\frac{13}{41}$. 6. $3\frac{4}{135}$.
7. $4\frac{13}{675}$. 8. $5\frac{61}{84}$. 9. $10\frac{1}{21}$. 10. $7\frac{1}{405}$. 11. $9\frac{13}{720}$.

223. EXAMPLE 1. **To express a recurring decimal as a vulgar fraction.**

Let $x = \cdot\dot{4} = \cdot 4444\ldots$.

Multiplying these equal quantities by 10, we have

$$10x = 4\cdot 4444\ldots.$$

Also $\qquad x = \ \ \cdot 4444\ldots.$

$\therefore$ by subtraction, $\quad 9x = 4. \quad \therefore\ x = \frac{4}{9}$,

i.e. $\quad \cdot\dot{4} = \frac{4}{9}$.

Again, let $x = \cdot 52\dot{3} = \cdot 523333\ldots$.

Multiplying these equal quantities by 1000, we have

$$1000x = 523\cdot 333\ldots.$$

Also multiplying both by 100, $\quad 100x = \ 52\cdot 333\ldots.$

$\therefore$ by subtraction, $\quad 900x = 471.$

$$\therefore\ x = \tfrac{471}{900},$$

i.e. $\quad \cdot 52\dot{3} = \frac{471}{900}$. $\quad$ (*N.B.* $\frac{471}{900} = \frac{523-52}{900}$.)

The above vulgar fraction may be reduced to lower terms.

EXAMPLE 2. **Express $\cdot 72\dot{3}\dot{4}$ as a vulgar fraction.**

$$\text{Let}\quad x = \cdot 72343434\ldots.$$

$$\therefore\ 10000x = 7234\cdot 3434\ldots.$$
$$100x = \quad 72\cdot 3434\ldots.$$

$$\therefore \text{ by subtraction,}\qquad 9900x = 7234 - 72.$$

$$\therefore\ \cdot 72\dot{3}\dot{4} = x = \frac{7234-72}{9900} = \frac{7162}{9900}.$$

We can reduce this vulgar fraction to lower terms.

EXAMPLE 3. **Express $2\cdot 91\dot{3}0\dot{5}$ as a vulgar fraction.**

We need only deal with the decimal part.

$$\text{Let}\quad x = \cdot 91305305305\ldots.$$

$$100000x = 91305\cdot 3053053\ldots.$$
$$100x = \quad 91\cdot 3053053\ldots.$$

$$\therefore\ 99900x = 91305 - 91.$$

$$\therefore\ x = \frac{91305-91}{99900} = \frac{91214}{99900}, \text{ etc.}$$

An examination of the above results gives us the following rule:

A recurring decimal fraction is equal to a vulgar fraction whose numerator is the decimal portion as far as the end of the first repetend minus the portion which does not repeat itself, and whose denominator consists of as many 9's as there are recurring figures, followed by as many ciphers as there are non-recurring figures.

The integral part (if there is one) remains unaltered.

EXAMPLE 4. **Express $\cdot 01\dot{6}2\dot{3}$ as a vulgar fraction.**

$$\text{Let}\quad x = \cdot 01623623623\ldots.$$

$$100000x = 1623\cdot 623623\ldots.$$
$$100x = \quad 1\cdot 623623\ldots.$$

$$\therefore\ 99900x = 1623 - 1.$$

$$\therefore\ x = \frac{1622}{99900}$$
$$= \frac{811}{49950}.$$

224. Multiplication and Division of recurring decimals, when the repetends consist of *very* few figures, may be performed by conversion into vulgar fractions and back again; otherwise the ordinary methods of approximate Multiplication and Division are used.

To divide a recurring decimal by a terminating decimal, proceed as in ordinary division.

EXAMPLE 1. Divide $3\cdot0\dot{6}\dot{9}$ by $2\cdot4$.

Here we shall use factors.

$$\frac{3\cdot0\dot{6}\dot{9}}{2\cdot4}=\frac{30\cdot6969\ldots}{24}=\frac{30\cdot6969\ldots}{3\times8}.$$

$$\begin{array}{r|l} 3 & 30\cdot69696969\ldots \\ \hline 8 & 10\cdot23232323\ldots \\ \hline & 1\cdot2790404\ldots \end{array}$$

$\therefore$ the quotient $=1\cdot279\dot{0}\dot{4}$.

EXAMPLE 2. Divide $7\cdot47\dot{5}$ by $3\cdot19$.

$$\begin{array}{r} 2\cdot343\ldots \\ 3\cdot19\,)\,7\cdot47555\ldots \\ 6\,38 \\ \hline 1\,095 \\ 957 \\ \hline 1385 \\ 1276 \\ \hline 1095 \end{array}$$

$\therefore$ the quotient $=2\cdot\dot{3}\dot{4}$.

EXAMPLES XXXII. b.

Prove that

1. $\cdot\dot{9}=1$.
2. $\cdot3\dot{9}=4$.
3. $1\cdot\dot{9}=2$.
4. $\cdot82\dot{9}=\cdot83$.
5. $\cdot071\dot{9}=\cdot072$.

Convert the following decimals into vulgar fractions in their lowest terms, *and use the rule to check your result* :

6. $\cdot\dot{2}\dot{7}$.
7. $\cdot\dot{3}\dot{6}$.
8. $\cdot2\dot{5}$.
9. $1\cdot3\dot{4}$.
10. $\cdot0\dot{2}\dot{4}$.
11. $3\cdot0\dot{5}\dot{1}$.
12. $\cdot\dot{1}4285\dot{7}$.
13. $\cdot\dot{4}2857\dot{1}$.

Express the following decimals as vulgar fractions in their lowest terms :

14. $2\cdot0\dot{1}8\dot{5}$.
15. $\cdot24\dot{3}2\dot{4}$.
16. $\cdot0\dot{1}3\dot{5}$.
17. $3\cdot0\dot{1}6\dot{2}$.
18. $6\cdot645\dot{3}$.
19. $\cdot\dot{7}317\dot{0}$.
20. $\cdot\dot{5}3846\dot{1}$.

Divide

21. $90\cdot\dot{6}$ by 24.
22. $42\cdot1\dot{4}3\dot{6}$ by $3\cdot5$.
23. $35\cdot744\dot{9}\dot{6}$ by $\cdot96$.
24. $372\cdot8\dot{2}0\dot{7}$ by $\cdot125$.

XXXIII. REVISION PAPERS.

English money results should be calculated to the nearest penny.

XXXIII. a.

1. Among how many men must £79. 17*s.* 6*d.* be divided that half of them may have 10*s.* 7*d.* each and the other half 7*s.* 2*d.* each?

2. A farmer bought some stacks of hay for £522. 10*s.*, calculating that he was paying £2. 15*s.* per ton. The weight turned out to be $9\frac{1}{2}$ tons more than he had expected, and he sold the hay at £3. 5*s.* per ton. What was his profit?

3. From 3·7 m. are cut off as many lengths of 7·1 cm. as possible. What length remains?

4. If £1 = 25·25 francs and £1 = 11·75 gulden, find the value of 354 gulden in francs and centimes to the nearest centime.

5. A strip of building land on the side of a straight road has a depth of 180 ft. If it is sold at £5. 5*s.* per foot of frontage, what does it cost per acre?

6. A property was sold for £8963, and the owner received from the agent £8626. 17*s.* 9*d.* What percentage of the selling price did the agent get as commission?

7. A bankrupt's debts are 47350 fr., and the amount for distribution among the creditors is 35275 fr. 75 c. Find how much a creditor for 100 fr. will receive.

XXXIII. b.

1. There are two maps, one to the scale of 2 inches to the mile, the other to the scale of half an inch to the mile. The area of an estate on the first map is 1·40 sq. inches. What is the area of this estate on the second map?

2. If 454 grammes = 1 lb. Av., express 8399 Kg. in tons, etc.

3. By what prime number is 5885 divided if the remainder is 131?

4. A bankrupt's estate realises £1178. 12*s.* 6*d.* before expenses are deducted. The expenses amount to 10 % of this. If his debts amount to £1571. 10*s.*, what does he pay in the £ to his creditors?

5. Find the simple interest on £2020. 10*s.* at $4\frac{1}{2}$ % for 9 months.

6. Taking a kilometre as 1093·6 yds., express 91·4 m. in yds. correct to the nearest yd.

7. A rectangular grass plot 126 yds. by 100 yds. is surrounded by a path 6 ft. wide. Find the cost of repairing the path at $1\frac{3}{4}d.$ per sq. yd.

XXXIII. c.

1. If a metre $=39{\cdot}37$ inches, express in inches (correct to 2 places of decimals) the diagonal of a square metre.

2. If $a=\frac{3}{11}$ of b, $b=\frac{22}{7}$ of c and $c=4\frac{3}{8}$ of d, find what fraction d is of a.

3. A person's yearly income after the deduction of a tax of a shilling in the £ was £952. 17*s*. What was his gross income?

4. In what time will £537. 16*s*. 8*d*. amount to £591. 12*s*. 4*d*. at $2\frac{1}{2}\%$ simple interest?

5. Goods are sold so that when 10 % is allowed off the sale price a profit of $12\frac{1}{2}\%$ is made. How much per cent. is the sale price higher than the cost price?

6. At what o'clock will a train, which leaves London for Crewe at 10.0 a.m., and goes at the rate of 50 miles an hour, meet a train which leaves Crewe for London at 10.30 a.m., and goes at the rate of 45 miles an hour, the distance between London and Crewe being 158 miles?

7. A person invests £6030 in India 3 per cents. at $100\frac{1}{2}$ and pays income-tax at 1*s*. 2*d*. in the £; on the stock rising to 102 he sells out and invests the proceeds at par in railway stock paying 5 per cent. free of income-tax. Find the change in his net income.

XXXIII. d.

1. If 1 c. ft. weighs $62{\cdot}35$ lb. av., find the error in calculating the weight of 1000 c. ft. on each of the following approximate assumptions: (1) 1 c. ft. weighs 1000 oz. av.
(2) 1 c. fathom weighs 6 tons.

2. The external length, breadth and height of a rectangular wooden box, with a lid, are 18, 10, 6 inches; and the thickness is $\frac{1}{2}$ inch. The box when empty weighs 15 lb., when filled with sand 100 lb. Compare the weights of equal bulks of wood and sand.

3. At the siege of Sebastopol it was found that a certain length of trench could be dug by the soldiers and navvies in 4 days, but that when only half the navvies were present it required 7 days to dig the same length of trench. Show that the navvies did 6 times as much work as the soldiers.

4. If $\frac{6}{19}$ of A's capital $=\frac{2}{3}$ of B's, and if the capitals of B and C together $=\frac{10}{11}$ of A's, find what part of B's capital $=$ C's.

5. Find, correct to 3 decimal places, the radius of a circle whose area $=$ the difference of the areas of two circles whose radii are 17 and 13.

6. If a man gain 8 % by selling eggs at 1*s*. 3*d*. a score, how much per cent. would he gain by selling them at 1*s*. a dozen?

7. Find the sum of money on which the simple interest for 4 years at $2\frac{3}{4}$ per cent. is £312. 8*s*.

XXXIII. e.

1. Find the rent of 9 ac. 2 ro. 36 sq. po. at £20. 3*s.* 4*d.* an acre.

2. If 62 m. 50 cm. of silk cost 293 fr. 75 centimes, find the price of a metre.

3. Find the simple interest on £1374. 10*s.* 3*d.* for 8 months at $4\frac{1}{4}$ per cent. per annum.

4. Find the cost of papering the walls of a room 19 ft. long, 15 broad, and 11 high, allowing 100 sq. ft. for door, window and fire-place. The paper costs 2*s.* $8\frac{1}{2}$*d.* per piece, each piece being 12 yds. long and 27 inches wide.

5. A bankrupt's liabilities amount to £4987 and his assets realise £924. 13*s.* $5\frac{1}{2}$*d.* How much in the £ will the creditors receive?

6. Find in yards the length of fence required to enclose a square field containing 7 ac. 2984 sq. yds.

7. A man rides to a certain place at the rate of 15 miles an hour and returns at 10 miles an hour. At what uniform rate could he have done the journey there and back in the same time?

XXXIII. f.

1. Find the cost of 82 Kg. 125 g. of butter at 3 fr. 44 c. per Kg.

2. Find the rent of 24 ac. 3 ro. 26 sq. po. at £3. 18*s.* 4*d.* per acre.

3. If 8 labourers can dig a trench measuring 150 yds. long and 4 ft. deep in 4 days of 9 hours each, in how many days of 8 hours each will 12 men dig a trench 400 yds. long, 3 ft. deep, and of the same width as the former?

4. A bag contains £26. 5*s.* in half-crowns, florins and shillings. There are three times as many florins and four times as many shillings as half-crowns. Find how many coins of each kind the bag contains.

5. A wall, whose height is 6 times its thickness, and length 10 times its height, contains 5625 cub. ft. Find its thickness.

6. A and B can together do a piece of work in 8 hrs., A and C in $7\frac{1}{5}$ hrs., B and C in $10\frac{2}{7}$ hrs. In what time would each do the work alone?

7. If the population of a country would increase annually by 5 % were it not for emigration, which takes away annually ·5 %, what is the increase per cent. in the population in 4 years (correct to 2 decimal places)?

XXXIII. g.

1. The area of a circle is 27 times that of a second circle. If the diameter of the latter is 5 ft., show that the diameter of the first is very nearly 26 ft.

2. Compute to 3 significant figures without logarithms

$$3{\cdot}214 \times 0{\cdot}7423 \div 7{\cdot}912.$$

Check by logarithms.

3. An iron girder has a length of 6·25 m., and at right angles to the length a uniform section whose area is 326 sq. cm. Find the weight of the girder in kilogrammes, assuming that a c. cm. of iron weighs 7·76 grammes.

4. To catch a train I have to go 2 miles 528 yards in 26 minutes. How far must I run at 8 miles per hour so that by walking the rest of the way at $4\frac{1}{2}$ miles per hour I shall be just in time?

5. Find the compound interest on £1256. 14*s.* for 2 years at 4 % per annum payable half-yearly.

6. Two men were walking each at 4 miles an hour in opposite directions along a railway, and a train passed one of them in 6 seconds and the other in $7\frac{1}{2}$ seconds. How long was the train and how fast was it going?

7. How much stock must be sold at 80 to pay now a bill of £1170. 15*s.* 3*d.* due 9 months hence at 3 %, if true discount be reckoned?

XXXIII. h.

1. As many pieces as possible, measuring ·00189 yd. each in length, are cut from a rod ·0976 yd. long. What length remains?

2. Express in hectares, ares and centiares the area of a rectangular piece of land whose length and breadth are 2·81 Km. and 73·2 m. respectively. Find how many hectolitres of water would be required to flood it to an average depth of 7·5 cm.

3. Find the cost of turfing at 3*d.* per sq. yd. a garden 100 ft. square all except a path 4 ft. wide round it at a distance 4 ft. from the boundary.

4. A man buys two horses for £65 and £85 respectively. He sells the first at a gain of 15 % and the second at a loss of 20 %. He buys a third horse for £70 and sells it so that he neither gains nor loses on the three. What does he get for it?

5. A farmer sells to one man 9 horses and 7 cows for £300, and to another, at the same prices, 6 horses and 13 cows for the same sum. What was the price of each?

6. A rectangular field of $7\frac{1}{2}$ acres is 3 times as long as wide. How long would it take to walk round it at 3 miles an hour?

7. A man directs that his property shall be divided amongst his sons A, B, C, D so that A shall have half as much again as B, B half as much again as C and C half as much again as D. If A's share exceed D's by £2869, find how much each receives.

XXXIII. i.

1. What is the price of ribbon per metre when 303 m. 20 cm. cost 197 fr. 8 c.?

2. Find two numbers between 900,000 and 1,000,000 which are exactly divisible by 37,259.

3. How many bricks, 9 in. long, $4\frac{1}{2}$ in. wide and 3 in. thick, will be required for a wall 50 yds. long, 5 ft. high and 1 ft. $1\frac{1}{2}$ in. thick?

4. Find the expense of painting, at 1*s.* 6*d.* a sq. yd., the inside of a cubical box with a lid, if the thickness of material is $\frac{1}{2}$ inch and the outer edge is 2 ft. 7 in.

5. When are the hands of a clock together between 4 and 5 o'clock?

6. At what rate per cent. simple interest will £755 amount to £821. 1*s.* 3*d.* in $2\frac{1}{2}$ years?

7. The difference between the true and commercial discount on a certain sum due in 5 months at 4 % is 6*s.* 8*d.* Find the sum.

XXXIII. j.

1. How much will it cost to paint the walls of a room 19 ft. $10\frac{1}{2}$ in. long, 16 ft. $1\frac{1}{2}$ in. wide and 10 ft. 3 in. high, at $9\frac{1}{2}$*d.* per sq. yd.?

2. Find all the numbers less than 1500 which are divisible by each of the numbers 15, 28, 35, 42.

3. Find, to 2 decimal places, the value of $2{\cdot}1463 \times 2{\cdot}345 \div {\cdot}1234$. [Use contracted methods or logarithms.]

4. A man bought oranges at 5*s.* 4*d.* a hundred, and sold them at 15 for a shilling. How much per cent. did he gain?

5. Find the compound interest on £724 for 3 years at 4 %.

6. A bill for £1356. 12*s.* 6*d.* drawn on Nov. 1 for 3 months was discounted commercially at 4 % on Dec. 18. How much was paid for the bill?

7. A and B can run at the rate of $12\frac{1}{2}$ and $12\frac{7}{16}$ miles an hour respectively. If A give B 10 yds. start, in what time will he overtake him? When A has run a mile, will he be in front of or behind B?

XXXIII. k.

1. What is the cost of butter per Kg. when 7·125 Kg. cost 22 fr. 80 c.?

2. By contracted multiplication find the square of 0·98214 correct to 4 decimal places.

3. Calculate $\frac{1}{70} + \frac{2}{70^2} + \frac{3}{70^3} + \frac{4}{70^4} + \dots$ correct to 5 decimal places.

4. Some men agree to pay equally for the use of a boat, and each pays 15 pence. If there had been 2 more men in the party, each would have paid 10 pence. How much was the hire of the boat?

5. A and B have between them £400; A receives a legacy of £350 and then has twice as much as B. Find how much each had at first.

6. A farmer buys 100 sheep at 35*s.* each, and sells them in lots of 30 and 70, getting for each of the 30 five-sixths of the price got for each of the 70. Find these prices, his profit being 14 %.

7. How much $3\frac{3}{4}$ % stock do I hold if I get £16. 6*s.* 3*d.* as dividend after paying a tax of 8*d.* in the £?

XXXIII. l.

1. Find to the nearest penny the value of 10·05 of 70·873 kilogrammes at 30 grammes for $1\frac{1}{2}d$.

2. A square field has an area of 25,340 sq. metres. Find to the nearest metre the length of boarding, 22 cm. wide, required for a fence 1·2 m. high round the field.

3. At what rate is £195. 10*s*. the simple interest for 8 years on £575?

4. A car moving uniformly travels an integral number of feet every second, and an integral number of miles every hour. What is the least speed at which it can be travelling?

5. What sum of money must be invested in $3\frac{3}{4}$% debentures at $109\frac{3}{8}$ to yield an income of £268. 16*s*.?

6. Three-sevenths of the passengers on a steamer are men, nine-elevenths of the remainder are women, and there are 40 children. How many passengers are there?

7. A pays income-tax at 1*s*. in the £ on £150 and at 9*d*. in the £ on the rest of his income. B pays at 1*s*. in the £ on £350 and at 9*d*. in the £ on the rest. If B pays altogether £4 more than A, by how much does his taxable income exceed that of A?

XXXIII. m.

1. Find to the nearest shilling the cost of fencing a square field of 6 acres with an iron fence at 4*s*. 3*d*. per yard.

2. Find the weight in Kg. of a beam of wood 5·4 m. long, 0·35 m. wide and 0·28 m. thick, if the wood is $\frac{3}{4}$ as heavy as an equal volume of water, taking 1 c. cm. of water to weigh 1 gramme.

3. Divide £4050 among A, B, C, D, E, so that A may have half as much as B, B $\frac{4}{5}$ as much as C, C $\frac{5}{6}$ as much as D and D $\frac{3}{4}$ as much as E.

4. Two railroads cross each other at right angles at a point P. Two trains A and B are moving on them with velocities 30 and $22\frac{1}{2}$ miles per hour respectively. At a given instant A's front is 32 yds. short of P and B's front is 100 yds. past P. Find how far the two fronts are apart at the end of $\frac{3}{5}$ of a minute from the given instant.

5. The interest on a certain sum of money for 2 years is £143. 13*s*. 3*d*., and the discount for the same time at the same rate is £127. 14*s*., simple interest being calculated in each case. Find the rate and sum of money.

6. A man buys fruit at the rate of £2. 13*s*. 4*d*. per cwt., and sells it at 1*s*. per basket. What must be the weight of fruit in each basket in order that he may make 68 per cent. profit?

7. I receive the true Present Worth of £18,782. 8*s*. due in 1 year at 4%. This I invest in $2\frac{1}{2}$% Consols at 86. What is my yearly income from this investment after the deduction of an income-tax of 1*s*. 2*d*. in the £?

XXXIII. n.

1. Given that a metre is 39·3704 inches, find, to the 10,000th part, the length of the side of the square whose area is 11,300 sq. metres, (*a*) in metres, (*b*) in yards.

2. Half a mile of road is measured as a police trap. A motor car takes 1 min. $6\frac{1}{2}$ sec. to go this distance. Express its speed in miles per hour correct to 2 decimal places.

3. A rectangular field is twice as long as it is broad, and it would cost £92. 8*s*. to put a fence round it at 3*s*. 6*d*. per yard of fence. Find the area of the field in acres.

4. The weights of equal volumes of iron and water are in the ratio of 775 to 100. Find in kilogrammes the weight of a cube of iron, the length of one side of which is 2·26 decimetres.

5. A man bought 20 bicycles at £7. 10*s*. each. He sold 8 at a gain of 40 %, and the remainder at half the first selling price. What was his loss on the transaction?

6. By investing in a 5 % stock at $134\frac{3}{4}$, what percentage do I receive on my outlay after a deduction of 9*d*. in the £ has been made for income-tax?

7. The front seats at a concert cost 2*s*. 6*d*. each, and the others a shilling. If 400 tickets are sold and the receipts are £30. 10*s*., how many front seats are taken?

XXXIII. o.

1. A carpet $19\frac{1}{2}$ ft. by $15\frac{3}{4}$ ft., costing 8*s*. a sq. yd., is laid down in a room 23 ft. by 17 ft., and the rest of the floor is covered with floor-cloth at $5\frac{1}{4}$*d*. a sq. ft. Find the total cost.

2. A piece of work can be done by 3 men and 4 boys in 6 days, by 3 men and 1 boy in 8 days, by 2 women and 4 boys in 10 days. How long would a woman take to do the work single-handed?

3. A bill for £225. 12*s*. 6*d*., drawn November 1 for 4 months, was discounted (commercially) at 7 % on December 6. What was paid for the bill?

4. A bag contains sixpences, shillings and half-crowns. The 3 sums composed of the 3 sorts of coins are all equal, and the total number of coins is 102. How many are there of each sort?

5. The interest on a certain sum for 2 years is £287. 6*s*. 6*d*., and the interest on £255. 8*s*. for the same time at the same rate is £31. 18*s*. 6*d*. Find the sum and the rate.

6. A man invests equal sums of money in a $2\frac{3}{4}$ % stock at 102 and in the $2\frac{1}{2}$ % Consols at $93\frac{1}{2}$. If the difference in the incomes derived from the two investments is £3. 15*s*., find the sums invested.

7. If the price of candles $8\frac{1}{2}$ inches long be 9*d*. per half-dozen, and that of candles of the same thickness and quality $10\frac{1}{4}$ inches long be 11*d*. per half-dozen, which is the cheaper sort? What is the ratio of the prices?

XXXIII. p.

1. Reproduce the multiplication of which the following is all that is not rubbed out:

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 285
  **
----
 *5*
****
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*9**
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2. If 44 hectares are sold for 92,000 francs, find the price per acre in pounds and shillings to the nearest shilling, taking 5 acres as equal to 2·02 hectares, and 25 francs 25 centimes as the value of £1.

3. Find the cost of papering a room 12 m. long, 6·2 m. broad and 4·25 m. high, with paper of which the breadth is 70 cm. and the price is 2 fr. per piece of 8 m., 21 sq. m. being allowed for 4 windows and doors.

4. If two consecutive whole numbers are neither of them divisible by 3, show that their sum must be divisible by 3.

5. A sum of money amounts in 2 years at compound interest to £4321·8 and in 4 years to £4764·7845. Find the sum.

6. A grocer mixes tea at 1*s.* $4\frac{1}{2}$*d.* a lb. with tea at 2*s.* $0\frac{1}{4}$*d.* to sell at 2*s.* a lb. and make 4*d.* a lb. profit. How does he mix them?

7. An estate is divided among 3 persons proportionally to 4, 6 and 7. Find the value of the estate if £135 added to the largest share would make it equal to half the whole.

XXXIII. q.

1. A number when divided by 9 gives remainder 8. What is the remainder when the square of the number is divided by 9?

2. Find correct to 2 places of decimals:

$$31{\cdot}547 \times 15{\cdot}624 \div 410{\cdot}156.$$

Check by logarithms.

3. A gravel path 4 ft. wide surrounds a rectangular lawn which is 84 ft. long and 43 ft. broad. How many loads (cubic yards) of gravel would cover the path to a uniform depth of 3 inches, and how much would the gravel cost at 4*s.* 6*d.* a load?

4. Three bicyclists A, B and C ride a race, B receiving 240 yds. start and C 720 yds. If A travels at the rate of 18 miles an hour and B at the rate of 16 miles an hour, find at what rate C must travel in order that B may overtake him at the instant when A overtakes B.

5. In a place whose population is 5106 the annual death-rate is 27 in 1000 and the birth-rate 42 in 1000, and there is no increase from other causes. What will be the increase in 5 years?

6. Divide £562. 10*s.* amongst A, B, C, D, E, so that
A's share : B's = 1 : 2, B's : C's = 4 : 5, C's : D's = 5 : 6 and D's : E's = 3 : 4.

7. Find by logarithms what principal amounts to £2279 in 4 years at 5 % compound interest.

XXXIII. r.

1. A rectangular piece of ground has an area of 1 acre, and its length is twice its breadth; find the length and breadth to the nearest tenth of a yard. Verify your result by multiplying the length by the breadth, and explain any discrepancy that you may find.

2. There are two areas A and B, and A contains just as many sq. yds. as B contains sq. metres. Find the ratio of A to B as a decimal, assuming that a metre = 3·281 ft.

3. A man visits Switzerland with £56, which he changes into Swiss money at 25·25 francs for £1. He is there 30 days and spends at the rate of 37·5 francs a day. What has he left?

4. The work of 3 men = that of 5 women or 6 boys. If 3 men, 3 women and 8 boys do $\frac{2}{3}$ of a certain work in 7 days working $8\frac{1}{2}$ hours a day, how long will it take 4 men, 11 women and 14 boys to finish it, working 7 hours a day?

5. A man sells a bicycle at a loss of $3\frac{1}{2}$ %. If he had received 8*s.* 6*d.* more, he would have gained 5 %. What was the cost price to him?

6. Two trains A and B are moving uniformly in opposite directions along parallel rails. A's pace is $\frac{9}{10}$ of B's pace. Find the rates at which they are moving, if 10 minutes after passing each other they are $14\frac{1}{4}$ miles apart.

7. What must be the price of $2\frac{3}{4}$ % stock in order that, after deduction of income-tax at the rate of 1*s.* 2*d.* in the £, it may yield interest at the rate of $3\frac{2}{3}$ %?

XXXIII. s.

1. Given $\sqrt{105625} = 325$, find $\sqrt{10582009}$.

2. A barters some sugar with B for flour worth 2*s.* 3*d.* a stone, but uses a false stone-weight of $13\frac{1}{2}$ lb. What value should B set on his flour that the exchange may be fair?

3. How many complete years will elapse before a sum has trebled itself at $3\frac{1}{2}$ % compound interest? [Logarithms.]

4. Gold is sold to the Mint at £3. 17*s.* 9*d.* per oz. It is mixed with alloy worth 5*s.* 2*d.* per oz. in the ratio 11 : 1. If sovereigns be coined of this mixture each weighing 5 dwts. 3·247 gr., what is the Mint profit on 100 sovereigns?

5. A train whose length is 77 metres, moving at the rate of 56·4 Km. per hour, overtakes a second train moving at 48 Km. per hour. How long does the first train take to pass a passenger in the second?

6. A rectangular box, without a lid, is to be made of wood $\frac{1}{2}$ inch in thickness. In outside dimensions it is to be 14 inches long, 12 inches broad and $7\frac{1}{2}$ inches deep. What length of wood, $3\frac{1}{2}$ inches wide, will be required?

7. If by investing in $2\frac{1}{2}$ % Consols I obtain, after allowing for 1*s.* in the £ income-tax, a net interest of $2\frac{5}{7}$ % on the money invested, what is the price of Consols?

XXXIII. t.

1. A brass tube 8 ft. long has an outside diameter of 3 in. and inside 2·8 in. If a cubic inch of brass weighs 0·3 lb., what is the weight of the tube?

2. Two trains start simultaneously from stations A and B, 20 miles apart, and travel towards each other at 32 and 28 miles an hour, and wait 10 minutes on the completion of the outward journey. Find graphically where they meet on the return journey?

3. A merchant buys two kinds of tea at 1*s.* $11\frac{3}{4}$*d.* and 1*s.* 5*d.* per lb. In what ratio must he mix them so as to gain $37\frac{1}{2}$ % by selling the mixture at 2*s.* $3\frac{1}{2}$*d.* per lb.?

4. A school of boys and girls contains altogether 449 scholars, and the number of girls is almost exactly ·39 of the number of boys. Find the number of boys.

5. Find the rate per cent. so that the true discount on £2573 due in 1 yr. 73 days may be £93.

6. The capital of a firm consists of £713. 3*s.*, £964. 17*s.*, £2391. 3*s.* subscribed by 3 partners. Divide £2231 among them in proportion to their several capitals.

7. A man buys a number of £20 railway shares at £25. 15*s.* each. The purchase money comes to £1982. 15*s.* How many shares does he buy, and what yearly income does he get from them if the dividend is at the rate of 6 % on the original £20 shares?

XXXIII. u.

1. A brickmaker pays £260 a year rent and 11*s.* in the £ on this in rates and taxes. Labour, firing and materials come to 15*s.* per thousand. How many thousand bricks a week must he turn out to make 10 % on the total cost, if he gets 32*s.* a thousand for them?

2. What is the percentage error, correct to 2 decimal places, in the following rough rule for finding the area of a circle? Take $\frac{7}{9}$ of the square of the diameter and add 1 per cent.

3. Find the length of the diagonal of a square park containing 219 acres 2922 sq. yds.

4. In what ratio must coffee at 10*d.* and chicory at 3*d.* a lb. be mixed so as to give a profit of 20 % when the mixture is sold at $10\frac{1}{2}d.$ a lb.?

5. If the present value of £218 due 2 years hence be £200, what is the present value of £1000 due 6 years hence at the same rate?

6. A man holds 3 % stock which produces for him £300 a year. He sells out at 92 and invests the proceeds in South Devon Railway when a £50 share is worth £23. What dividend per cent. per annum must the South Devon Railway pay so that he may increase his income £50 by the operation?

7. A train travelling at the rate of 40 miles an hour whilst inside a tunnel meets another train of half its length travelling at 60 miles an hour and passes it completely in $4\frac{1}{2}$ seconds. Find the length of the tunnel if the first train passes completely through it in 4 min. $37\frac{1}{2}$ seconds.

XXXIII. v.

1. To how many decimal places is $\frac{218}{89}$ an accurate approximation to $\sqrt{6}$?

2. If 25 fr. = £1 and 1 Kg. = 2·204 lb., find to the nearest penny the cost in £. *s.* *d.* of 1 oz. Av. of an article which costs 3 fr. 50 c. per gramme.

3. A rectangular room is 6 m. 25 cm. high, 15 m. 60 cm. long and 7 m. 8 cm. broad. Find, correct to a penny, the cost of carpeting the floor at 4*s.* a sq. metre, and of papering the walls at 6*d.* a sq. metre.

4. A tradesman has been selling an article at a profit of 45 %. The cost to him is reduced 1*d.* per article and he lowers the selling price 1*d.* per article, thereby increasing his profit to 50 %. At what price per article is he now selling it?

5. A, B and C have £5000, £3000 and £2000 invested in a business. A and B receive 20 % and 10 % respectively of annual profits for managing the business, the remainder of the profit being divided between A, B and C according to their capital. If at the end of a year A receive altogether £600 more than B, what does each receive?

6. First, second and third class fares being 2*d.*, $1\frac{1}{4}d.$, 1*d.* per mile, a person who travels third class pays 6*s.* 3*d.* less than one who travels equal portions of the same distance by each of the three classes. Find the distance.

7. A man invests his money equally in two stocks, a 3 % at 88 and a 4 % at 96. Had he with his money purchased equal amounts of the two stocks, his annual income would have been 25*s.* more. How much did he invest?

XXXIII. w.

1. A woman's pay per day is $\frac{4}{7}$ of that of a man, but the work done only $\frac{3}{5}$ of that done by a man. If it costs £3. 10*s.* to hire a man to do a piece of work, what will a woman receive for the same piece, both being paid by time?

2. A man sells 126 yds. of silk for the cost price of 147 yds. What is his gain per cent.?

3. A man whose gross income is £880 pays income-tax at the rate of 9*d.* in the £ on earned income and 1*s.* in the £ on unearned income. If the total income-tax paid is £34. 16*s.* 8*d.*, find his gross unearned income.

4. A dairyman buys his milk at 4*s.* per 5 gallons, dilutes it with water, and sells the mixture at 1*s.* per gallon, thereby making a profit of $37\frac{1}{2}$ per cent. What proportion of water has he added?

5. A square field of grass is surrounded by a path of uniform breadth; the field and path together occupy 8·41 hectares, and the path alone ·2 of a hectare. Find the breadth of the path to the nearest decimetre.

6. A train from A to B stops at two intermediate stations. At the first of these one half of its passengers leave it and 135 new ones enter it. At the second stopping place $\frac{1}{3}$ of those who arrive by the train leave it, and 110 new passengers get into it. The train arrives at B with 350 passengers. With how many did it leave A?

7. A person derived an income of £462 from a $2\frac{3}{4}$ % stock. He sold out at $101\frac{1}{4}$, and invested the proceeds in 4 % railway stock, thereby increasing his income by £69. 11*s.* 3*d.* What was the price of the railway stock?

XXXIII. x.

1. Find in litres the capacity of a closed rectangular box whose external dimensions are 2 m. 30 cm., 1 m. 56 cm. and 104 cm., the thickness of material being 3 cm.

2. In making an excavation it is found that a navvy can cut 2 c. yds. of material a day and a horse can remove $10\frac{1}{2}$ tons a day. The material weighs 147 lb. per c. ft. How many horses should be employed when 80 navvies are at work?

3. A clock set right at 12 noon indicates 5 minutes past 7 at 7 o'clock the same evening. What is the true time when the clock shows 10 the same evening?

4. From a certain sum I took away a third part and put in its stead £50; from the resulting sum I took away one-fourth and put in its stead £70; I then counted the money and found £120. What was the original sum?

5. A and B enter partnership, A investing £5000, B £3000. They agree that at the end of each year they shall draw from the profits 4 % on the amounts they have invested as interest on capital, that B shall receive in addition 20 % of total profits for acting as manager, and that the remainder of the profits shall be divided equally between them. If in a certain year A received in all £750, what did B receive?

6. An article passes successively through the hands of 3 dealers, each of whom in selling adds as his profit 10 % of the price at which he bought it. What did the first dealer pay for goods which the third sells for £11. 1*s.* 10*d.*?

7. The true discount on a sum of money is £17. 10*s.*; the interest on the same sum for the same time at the same rate is £19. 19*s.* Find the sum.

XXXIII. y.

1. Find the side of a square equal in area to a rectangle measuring 513 yds. 1 ft. 11 in. by 1628 yds. 11 in.

2. A gallon contains 277·27 c. in. A cubic ft. of water weighs 62·42 lb. Find the weight of a pint of water in lb. correct to 2 places of decimals.

3. A rectangular box measures externally (when the lid is down) 4 ft. long, $2\frac{1}{4}$ ft. wide, 1 ft. 4 in. high; and it is made of wood an inch thick. Taking account of the wood only, find the ratio of the weight of the lid to the weight of the whole box.

4. The true discount on £678. 8*s.* due in $1\frac{1}{2}$ yrs. is £38. 8*s.* Find the rate per cent.

5. The sides of a rectangle are measured to the nearest millimetre and are found to be 2·621 and ·849 metres. Find the area of the rectangle in sq. metres. To how many decimal places is your result trustworthy? Give reasons for your answer.

6. A travels from P to Q, a distance of 30 miles, and back again at the rate of 9 miles an hour. On his way back he meets B, who travels at the rate of 6 miles an hour, and who started at the same time from P. Find graphically the distance of their meeting-point from P.

7. A works for a week at 8 hrs. a day. B works 5 hrs. on Monday and increases 1 hr. each subsequent day. A can do as much in 4 hrs. as B in 5 hrs. If the total sum paid to A and B as wages for the week be £2. 2*s.*, how much should each receive?

XXXIII. z.

1. A number of ladies lunching together spent as many pence each as there were ladies. The total spent was £3. 0*s.* 9*d.* How many were there?

2. What is the least number of ounces of standard gold worth £3. 17*s.* 10½*d.* per oz. which can be coined into an exact number of sovereigns?

3. A can beat B by 1 ft. if he gives B 3 yds. start in 100. If B gives C 8 yds. in 120, C wins by 28 inches. If A gives C 10 yds. in 100, who will win and by how much?

4. Two trains start simultaneously from A and B and proceed towards each other at 35 and 45 miles per hour. When they meet, one train has gone 17½ miles more than the other. What is the distance between A and B? Check your result graphically.

5. Find the side of a square field 3 acres larger than one which measures 91 yds. by 91 yds.

6. A man bought certain goods of which he sold ⅓ at a profit of 14 %, ⅗ at a profit of 17½ % and the remainder at a profit of 20 %. What was his profit per cent. on the whole?

7. I sell £4560 of 2½ % Consols at 85, and with the proceeds purchase 4 % stock at 114. After deducting income-tax (at the same rate in both cases) I find my income better by £20. 18*s.* What is the income-tax per £?

XXXIII. aa.

1. Through an error in pointing $\sqrt{\cdot 17}$ has been found instead of $\sqrt{1\cdot 7}$, which was required. What multiplier will give the necessary correction?

2. Two men measure a rectangular box: one finds its length, breadth and depth in inches to be 8·54, 5·17, 3·19. The other finds them to be 8·50, 5·12, 3·15. What is the percentage difference of either volume from the mean of the two volumes?

3. A merchant buys 1260 qrs. of corn, and sells ⅕ at a gain of 5 %, ⅓ at 8 % and the rest at 12 % gain. If he had sold the whole at a gain of 10 % he would have obtained £23. 2*s.* more. What was the cost price per qr.?

4. The price of gold is £3. 17*s.* 10½*d.* per oz. A composition of gold and silver weighing 18 lbs. is worth £637. 7*s.*; but if the proportions of gold and silver were interchanged it would be worth only £259. 1*s.* Find the proportion of gold and silver, and the price of silver per oz.

5. In a piece of coal there was found to be 11·30 lb. of carbon, 0·92 of hydrogen, 0·84 of oxygen, 0·56 of nitrogen, 0·71 of ash. There being nothing else, state the percentage composition of the coal.

6. Two trains start at the same time, one from A for B, the other from B for A. When they meet, the train from A has travelled 80 miles further than that from B; and the former will reach its destination 4 hours, the latter 9 hours later. How far is it from A to B?

7. A person who buys some $4\frac{1}{2}$ % stock finds that he gets $\frac{1}{2}$ % less interest on his money than his friend who bought the stock at $\frac{8}{9}$ of the price. What was the price?

XXXIII. bb.

1. Find a whole number between 1000 and 2000 which is a perfect square and is divisible by 13.

2. A starts 3 minutes after B for a place $4\frac{1}{2}$ miles off. B reaches the place, returns, and after a mile meets A. A's speed being a mile in 8 minutes, find B's speed. Check the result graphically.

3. A rectangular piece of ground 70 ft. by 28 ft. is surrounded by wire netting and divided by wire netting into 10 equal squares. What is the cost of the netting, its price being 6*s.* 9*d.* per roll of 50 yds. and 2*d.* a yd. for odd lengths.

4. Two solid lead spheres, of radii 2 in. and 4 in. respectively, are melted together and recast as a solid right circular cylinder of height 6 in. Show that the surface exposed is unaltered.

Vol. of a sphere $=\frac{4}{3}\pi r^3$, surface of a sphere $=4\pi r^2$, where r is its radius. Vol. of a cylinder $=\pi r^2 h$, surface of a cylinder $=2\pi r(r+h)$, where r is the radius of its base and h is its height.

5. A farmer finds that a boy can attend to 36 sheep, a man to 15 cows, and a man and 2 boys to 60 sheep and 17 cows. If a man receives 3*s.* 4*d.* a day, what should a boy's wages be?

6. A builder borrows £1261, to be paid back with compound interest at 5 % per annum by the end of 3 years in 3 equal yearly instalments. Show that each instalment is £463. 1*s.*

7. I invest a certain sum, half in a 4 % stock at par, half in a 5 % at 120. If I had invested $\frac{1}{4}$ in the first and $\frac{3}{4}$ in the second, I should have had £5 more annual income. What sum had I to invest?

XXXIII. cc.

1. Find the least cube which is divisible by 15, 18, 25 and 49.

2. A number is divided by 315 by short division, the factors 5, 7, 9 being used in that order; and the remainders in order are 3, 1, 4. What would be the remainders if the divisors were used in the order 9, 7, 5?

3. A and B have money in the ratio of 5 : 7. B pays half of his money to A, and receives back $\frac{1}{3}$ of what A then has, and is thus left with 6 shillings more than A. How much had each at first?

4. A, B and C begin a piece of work together on Monday morning. By Tuesday night half is done, when A leaves. By Thursday night $\frac{3}{4}$ of the remainder is done, when B leaves; and C finishes it by Saturday night. In how many days could each finish it alone?

5. By investing equal sums in a $4\frac{1}{2}$ % stock and a 6 % stock I get $5\frac{1}{2}$ % for my money. The $4\frac{1}{2}$ % is at 75. At what price is the 6 % ?

6. A man, who gives a boy 400 yds. in a race over a mile course, overtakes him in 4 minutes and beats him by 150 yds. How long is the man in running the mile?

7. Two steamers travel between two ports by the same route, one in 10 days, the other in 5. The cost of coals per voyage varies as the square of the speed, and the general expenses as the duration of the voyage. If on the faster voyage the expenditure for coals is £1000 more, and the general expenses £250 less, than on the slower, find the cost of running the vessels per annum, the faster vessel making 40 voyages and the slower 20.

XXXIII. dd.

1. A can just give B a start of 20 yds. and C a start of 27 yds. in a $\frac{1}{4}$ mile race. How much can B give C in 480 yds.?

2. Find the present value of £3087 due 3 yrs. hence at 5 % per annum compound interest.

3. Two casks originally contain 60 gallons of wine and 30 gallons of water respectively. On three successive occasions 12 gallons of liquid are drawn from each cask and placed in the other. Express in gallons and decimals of a gallon the quantity of wine now in each cask.

4. A box without a lid, whose internal length, breadth and depth are 28 in., 16 in. and 9 in., is made from board 1 in. thick, such that a square foot of board weighs 4 lb. What is the weight of the box?

5. A man sells out £2000, $2\frac{1}{2}$ per cent. Consols at 108 and invests the proceeds partly in land which yields $1\frac{1}{2}$ per cent. and partly in railway 5 per cent. stock at 150. What sum must he invest in each in order that his income may be unchanged by the transaction?

6. A man borrows £25,220, which he undertakes to pay back with compound interest at the rate of 5 % per annum in 3 equal yearly instalments at the end of one, two and three years. How much is each instalment?

7. A model of a machine in wood weighs 24·2 lb. The machine itself is to be made of iron, and every line in it is to be 8 times as long as the corresponding line in the model. If a cubic foot of the wood weigh 44 lb. and a cubic foot of iron 490 lb., what will be the weight of the machine in tons?

XXXIII. ee.

1. A specimen of bronze consisted of 85 per cent. of copper, 10 per cent. of zinc and 5 per cent. of tin; whilst a specimen of bell-metal contained copper and tin only. The two specimens were

fused together, and the mixture was found to contain 84 per cent. of copper, 8 per cent. of zinc and 8 per cent. of tin. Find the proportion of copper and tin in the bell-metal.

2. A and B are running in opposite directions along a straight road, and B's pace is $\frac{4}{5}$ of A's. At a certain instant they face each other and are 200 yds. apart; in the course of the next 3 minutes they pass each other, and at the end of the 3 minutes they are 1069 yds. apart. Find A's pace in yards per minute.

3. 40 bicycles were bought at £7. 10*s*. each. The purchaser sold a certain number at a gain of 40 % and the remainder at half the first selling price, and his whole profit on the transaction was £78. How many were sold at the reduced price?

4. A man invested £5173. 11*s*. 3*d*. partly in Consols at $101\frac{1}{8}$ and the remainder in Railway stock at $146\frac{1}{2}$. When the Consols were at $102\frac{1}{4}$ and the Railway stock at $143\frac{1}{8}$ he sold out, and received altogether the sum he had originally invested. How much did he invest in Consols?

5. A runner starts in a mile race at the rate of 6 yds. a second. How far from the end must he quicken to 7 yds. a second to run the mile in 4 min. 45 secs.?

6. A merchant buys two kinds of tea, one at 1*s*. 6*d*., the other at 1*s*. $7\frac{1}{2}$*d*. per lb. In what ratio should he mix them, so that by selling the mixture at 1*s*. 9*d*. per lb. he may gain $12\frac{1}{2}$ per cent.?

7. The square of the time of a planet's revolution round the sun is proportional to the cube of its distance from the sun, and the distances of the Earth and Mercury from the sun are as 91 to 35. Find in days the time of Mercury's revolution, assuming that the Earth's revolution takes 365 days.

XXXIII. ff.

1. Having given that a sphere of stone, 10 feet in diameter, weighs 1413180 ounces and that $\pi = 3{\cdot}1416$, find the weight of a cubic foot of the stone to the nearest ounce.

2. Two trains, each 80 yds. long, pass each other on parallel lines. If they are going in the same direction, the faster takes one minute to pass the other completely; if in different directions, they completely pass each other in 3 seconds. Find the rate of each train in feet per second.

3. Having given that a cubic foot of water weighs 62·5 lb. and a c. cm. of water 1 gramme, and that 1 lb. $= {\cdot}453$ Kilogramme, use logarithms to express the yard in centimetres.

4. In a year in which the income-tax is at 1*s*. in the £, the profit from a certain business belonging to two partners is such that £70 is allowed to be deducted from it before income-tax is charged. The following year the profit is the same, but the rate of the tax is

reduced from 1*s.* to 9*d.*, and the partners find that, by dividing the profit into two equal portions, £160 can be deducted from each portion before income-tax is charged. The result is that £16. 10*s.* less is paid for income-tax in the latter than in the former year. What was the profit of the business in each of the two years?

5. A sum of £3720 is invested partly in $2\frac{1}{2}\%$ stock at 89, and the remainder in 3 % stock at 97. Find the amount invested in each if the income derived is what would be derived from investing the whole in $2\frac{3}{4}\%$ stock at 93.

6. A cistern partly full, containing 7000 gallons, into which a steady stream is flowing, has a number of equal outlets which may be opened or shut. If now 10 are opened the cistern is emptied in 20 minutes; if 12 had been opened the cistern would have been emptied in 14 minutes. Show that, in order to empty the cistern, at least 6 outlets must be opened, and find the amount of water steadily flowing into the cistern.

7. The railway line between two towns A and B is $55\frac{1}{4}$ miles long. The 4.42 p.m. express train from A arrives at B at 6.3 p.m.; and the 4.30 p.m. express from B arrives at A at 5.42 p.m. Assuming that each train travels at a uniform rate, find the time at which they meet one another and the distances of the meeting place from the two towns. Illustrate the solution of this graphically.

XXXIII. gg.

1. When x and y are small, we may take $\dfrac{1+x}{1+y}$ as being very nearly equal to $1+x-y$. What is the error in this when $x=0{\cdot}02$ and $y=0{\cdot}03$?

2. Two railway termini are 199 miles apart, and a train travels from one to the other in $4\frac{3}{4}$ hours, calling at 4 intermediate stations, at each of which it waits for 3 minutes. Find the full speed of the train, assuming it to be uniformly maintained except for 1 mile before stopping at, or after starting from, a station, over which distances its average speed is 20 miles an hour.

3. A and B buy coal, each buying as many tons as he pays shillings per ton. How much do they buy, if A spends £3. 12*s.* more than B, and the price of coal is not below 12 shillings or above £1 per ton?

4. If 23 oz. of a 53·2 % solution of a certain salt be mixed with 20 oz. of a 35·32 % solution of the same salt, calculate to 2 decimal places the percentage of salt in the mixture.

5. I find that if I invest a certain sum in shares of a company paying 12*s.* per share, when the £10 share is at $13\frac{1}{5}$, I get £10. 16*s.* a year more for my money than if I invest in 3 % stock at 93. What sum have I to invest?

6. A man's income in 1901 was £20 more than in 1900. £120 of his income in 1900 was exempt from income-tax, and he paid 1 shilling in the £ on the remainder; £70 of his income in 1901 was exempt from income-tax, and he paid 14*d.* in the £ on the remainder; also he paid £8. 17*s.* more income-tax in 1901 than in 1900. What did he pay in 1900?

7. A man signed a contract on June 1 to buy a house for a certain sum of money. One-tenth of the price was paid on June 1, and the remainder, together with interest on it at 5 % per annum from June 1, was paid on the completion of the purchase, which took place on Oct. 1 following; and the sums paid on June 1 and Oct. 1 amounted altogether to £1294. 2*s.* 6*d.* How much was paid as interest?

CONSTANTS.

1 metre = 100 cm. = 1000 mm. = 0·001 Km.

1 yard = 3 ft. = 36 in. = 0·9144 metre. 1 mile = 8 furlongs = 80 chains = 320 poles = 1760 yds.

1 hectare = 100 ares = 10000 sq. metres.

1 acre = 4 roods = 10 sq. chains = 4840 sq. yds. = 0·4047 hectare.

1 litre = 1 c.dm. = 100 cl. = 0·01 hl. = 0·001 c. metre.

1 gallon = 4 qt. = 8 pt. = 0·1606 c. ft. = 4·546 litres. 1 quarter = 8 bush. = 32 pk. = 64 gal.

1 kilogram = 1000 g. = 0·001 tonne.

1 lb. = 16 oz. = 7000 grains = 453·6 grammes. 1 ton = 20 cwt. = 80 qr. = 2240 lb.

1 litre of water weighs 1 Kg. 1 c. ft. of water weighs 62·3 lb.

A circle of radius r has a circumference of length $2\pi r$ and has an area of πr^2, where $\pi = 3{\cdot}1416$.

ARITHMETIC PAPERS.

SET IN VARIOUS COMPETITIVE EXAMINATIONS CONDUCTED BY THE CIVIL SERVICE COMMISSIONERS.

PAPER I.—Time allowed, Two Hours.

You are not restricted to arithmetical methods. For various data see page 372. For full credit the working must be fully shown and the answers given only to the degree of accuracy that is asked for or is justified by the data.

1. A bale of cotton contains 480 lb. Find the cost of 150 bales of cotton at 5·47*d.* per lb.

2. In the year 1892 there were 1208 Trades Unions in the United Kingdom with a total membership of 1,502,358; in 1900 there were 1272 Unions with a membership of 1,905,116. Find the average number of members per Union for each of these years, and by how much per cent. the average number of members had increased from 1892 to 1900. Give each result to the nearest integer.

3. Find the weight in pounds per mile, and the cost in shillings per mile, of copper wire No. 10 B.W.G. The diameter of No. 10 wire is 0·134 inch. Take the weight of a cubic foot of copper as 555 lb. and the price of copper as £62 a ton.

4. Silver coins weigh 87 grains per shilling, and are made from an alloy $\frac{37}{40}$ of which is pure silver. Find the actual value of the silver in £100 worth of silver coins when the price of silver is $26\frac{3}{8}$*d.* per troy ounce (= 480 grains).

Find also, to the nearest penny, what the price of silver per troy ounce should be if the silver in the coins was worth £100.

5. A lake has an area of 1200 acres. Find how many million gallons of water must be added to it to raise the level of the water by 3 inches.

PAPER II.

(To Vulgar Fractions and Decimals, Interest and Mensuration.)

For full marks results must be given only to the degree of accuracy that is asked for or is justified by the data. Algebraic methods may be used. For various data see the list of Constants printed on page 372.

1. Correct, justify or complete the following statements :

(*a*) 0·07 is greater than $\frac{1}{15}$ and less than $\frac{1}{14}$.

(*b*) 30 inches = ... centimetres (to nearest centimetre).

(*c*) 15 miles per hour = ... feet per second.

(*d*) 17*s*. $6\frac{1}{2}d.$ = £0·877.

(*e*) In the specification of wire of a certain material, 35·6 feet per pound is equivalent to ...... pounds per mile.

2. A garden roller, of 13 inches radius and 3 feet long, is $\frac{4}{5}$ filled with water to make it heavier. Find the weight of water, given that the internal volume of a roller of r inches radius and l inches long is $\pi r^2 l$ cubic inches.

3. By the following method determine which of the following express runs is the fastest :

(1) 36 miles in 39 minutes.

(2) $25\frac{1}{4}$ miles in 28 minutes.

(3) $48\frac{1}{2}$ miles in 55 minutes.

(4) $34\frac{1}{8}$ miles in 36 minutes.

Take a straight line OX parallel to the long edge of a page of your manuscript book. Mark off from this line Op to represent 39 minutes ; at p draw a line pq perpendicular to OX of a length

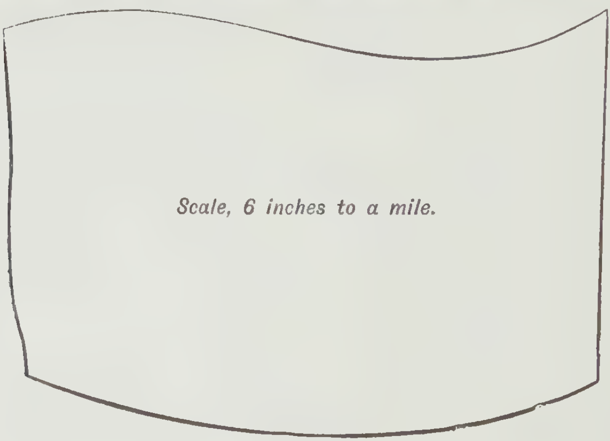


Fig. 39.

to represent 36 miles. Join Oq. Treat each of the other runs in the same way.

Scales--1 cm. to represent 2 minutes.
1 cm. „ 4 miles.

4. Fig. 39 is the plan of an estate; draw a rectangle over it having the same area as nearly as you can judge. Prick this rectangle through into your book, against the length and breadth write their measured values and use them to determine the acreage of the estate.

5. A tradesman's takings for a week are £25; what is his net profit if he marks his goods 20% above cost price and pays for rent, labour, etc., 10% of his takings?

PAPER III.—Time allowed, Two Hours.

You are not restricted to arithmetical methods. For various data see page 372. For full credit all your working must be shown, and your results must be given only to the degree of occuracy that is asked for or is justified by the data.

1. A dressmaker buys the following materials for a dress: $10\frac{1}{2}$ yards of cloth at 2*s.* 8*d.* a yard; $6\frac{3}{4}$ yards of lace at 4*s.* 6*d.* a yard; $5\frac{1}{2}$ yards of silk at 2*s.* 3*d.* a yard. Linings and sundries cost her 8*s.*, and she estimates that the labour will cost her 15*s.* If she charges 6 guineas for the dress, what profit does she make?

2. At the end of April 1907 there were employed in the Transvaal mines 91,824 natives and 53,588 Chinese, at the end of April 1908 the figures were 130,991 natives and 24,059 Chinese. Find at each date, to the nearest whole number, how many Chinese there were to every 100 natives, and the increase or decrease per cent. in the numbers of (a) natives, and (b) Chinese employed at the end of April 1908 as compared with the numbers for April 1907.

3. The area of the County of London is 75,000 acres. How many square inches does this cover on a map drawn to a scale of an inch to 2 miles?

4. A cloth manufacturer prices some cloth at *either* 4 francs a metre *or* 2*s.* 11*d.* a yard. If he intends these prices to be the same, find how many francs, to the nearest hundredth, he has taken as equal to £1.

5. A manufacturer sells jam in 1-pound and 3-pound pots. The inside of the 1-pound pot measures 2·9 inches across and 3·6 inches in depth, the corresponding measurements for the 3-pound pot are 3·9 inches and 5·8 inches. A pot measuring inside a inches across and b inches deep has a capacity of $0·7854 \times a \times a \times b$ cubic inches. Find, to the nearest cubic inch, the capacities of the two jars, and express the less as a decimal fraction of the other to two decimal places.

6. The figure below represents a lawn drawn to a scale of an inch to 20 yards, the inside portion representing a square flower bed, so that the rest of the figure represents grass. Find the area of the grass, and the cost of fresh turf for it, the turf costing £4 per 1000 pieces, and each piece measuring on the average 3 square feet.

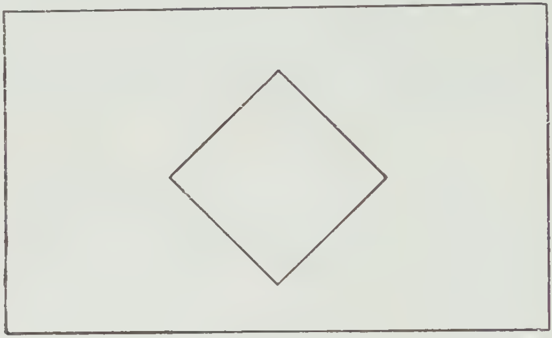


FIG. 40.

PAPER IV.—Time allowed, Two Hours.

You are not restricted to arithmetical methods. For various data see page 372. For full credit the working must be immediately intelligible and the answers given only to the degree of accuracy that is asked for or is justified by the data.

1. AVERAGE ANNUAL IMPORTS OF TEA (1901–2).

	Value in £1000.
United Kingdom	8,683
Russia	4,928
United States	2,095
Holland	753
Australia	734
Canada	657
Other countries	468

From the preceding table calculate the value of tea imported (i) by Russia, (ii) by the United Kingdom, as percentages, to the nearest tenth of the total for all countries.

Write down, after mental calculation only, the percentage imports for the other areas; use these approximate values to check your earlier results.

2. The catchment area of a reservoir is 2 sq. miles. If 65 per cent. of the rainfall does not reach the reservoir, find how many gallons of water will be added to the reservoir from a fall of rain 0·27 in. Give the answer to the nearest million gallons.

3. A certain kind of brass is composed of 25 parts by weight of tin, 160 of copper and 5 of zinc. Taking the price of copper as £60 per ton, zinc £22 and tin £118, find the cost of the materials of a casting which weighs 130 pounds. Give the answer to the nearest shilling.

4. In a recent motor-car race over a distance of $5\frac{1}{2}$ miles the winner had 1 min. 6 sec. start and won by a quarter of a mile from the next car, which had 2 sec. start. The average speed of the winner was 54 miles an hour; find that of the second car.

5. The capacity of a cask in imperial gallons is given by the formula

$$0{\cdot}0014162 \times l \times (\mathrm{D} \times d + \mathrm{M} \times \mathrm{M})$$

when l is the length of the cask, D and d the inside diameters at the ends and M the inside diameter at the bung, all in inches. Find, to the nearest gallon, the contents of a cask 3 ft. 6 ins. long, with inside diameter of 27 ins. at the bung and 23 ins. at each end.

6. The diagram below represents a garden containing four square flower beds surrounded by paths as shown. It is drawn to a scale of 1 in 500. Find (i) the length of the side of the beds in yards, (ii) the area of the paths in sq. yards, (iii) the cost of gravelling the paths at 1*s.* 8*d.* per sq. yard.

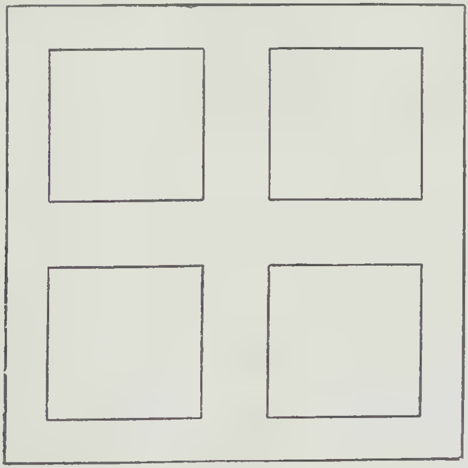


Fig. 41.

PAPER V.—TIME ALLOWED, TWO HOURS.

You are not restricted to arithmetical methods. For various data see page 372. For full credit your work must be immediately intelligible, and your answers given only to the degree of accuracy that is asked for or is justified by the data.

1. A person has £12,000 invested in a business that gives an annual return of 8 %. If he spends £700 a year and adds the rest of his income to his invested capital, how much money will he have in the business at the end of five years?

2. You measure a rectangular block and find its dimensions to be 424, 263·5 and 86 mm. Your measurements are not perfect, but you know that no error exceeds 0·5 mm. Find the limits between which the volume of the block must lie. Find the mean and the difference of these numbers, and find what percentage the difference is of the mean.

3. A quantity of cotton is described in Alexandria as 95 cantars worth 260 piastres per cantar; and in Liverpool as 4 tons 13 cwt., worth $7\frac{5}{16}$ pence per lb. Find (in English units) the difference between these two weights, and between the values of the whole as declared in Liverpool and Alexandria.

1 cantar = 50 kilos. 100 piastres = £E1 = £1. 0*s.* 6*d.*

4. A flat ring is 1·5 cm. thick, its internal and external diameters are respectively 7·8 cm. and 9·5 cm. and it weighs 24 grams. Calculate its volume, and find the weight of 1 c.cm. of the material of which it is made.

5. In a summary of an annual report of an insurance company the income derived from investments is £3,093,028. The investments are grouped as follows:

Mortgages - - - -	£17,782,117
Loans - - - - -	6,073,607
Railway bonds - - - -	34,277,186
Other bonds - - - -	4,940,036
Railway stocks - - - -	2,150,753
Other stocks - - - -	7,680,129

What is the average rate of interest on all the investments? Assuming that the average rate on mortgages and loans is 5·5%, and on bonds 3·4%, what is the average rate received on the railway and other stocks? Answers are to be given to one decimal place.

6. In sinking a shaft it is calculated that the depth sunk every month is $\frac{9}{10}$ of that in the preceding month. If the depth sunk in the first month is 100 feet, find the total depth sunk at the end of 2, 3...6 months.

Use a horizontal line at the foot of your paper as base line; along it mark vertical lines $2\frac{1}{2}$ cm. (=5 small spaces) apart to represent the lapse of 1, 2, 3 ... 6 months. On these vertical lines mark off lengths to represent the total depth sunk at that time, taking 1 small space to represent 10 ft. Let the upper extremities of these lines be shown by points that are easily seen. Join each of these points to the next by straight lines.

7. As you ascend a mountain, taking a barometer with you, the air-pressure diminishes and the barometer consequently falls. If h is your height above sea-level in thousands of feet, and b is the height of the barometer in inches, then, assuming that the height of the barometer at sea-level is 30 in., it is known that $h = 61{\cdot}6\{\log 30 - \log b\}$.

If your barometer stands at 20 in., how high are you above the sea-level?

On the top of a mountain of 15,000 ft., what is the height of the barometer?

PAPER VI.—Time allowed, One Hour.

For various data see page 372. *For full credit your work must be shown and be immediately intelligible, and your results must be given only to the degree of accuracy that is asked for or is justified by the data.*

1. At an election there were 11,391 voters on the register. The successful candidate obtained 5,726 votes and the unsuccessful candidate 4,218 votes. Find what percentage of the voters used their votes, and what percentage of those who actually voted gave their votes for the successful candidate. Give the results to the nearest integer.

2. A servant's wages are £35 a year; find how much would be due to him at the end of 113 days, and how long he would take to earn £12. You may answer the question graphically if you prefer.

3. A newspaper is printed on four sheets, each measuring 112 cms. by 64 cms. How many copies can be printed from a roll of paper which is 5 kilometres long and 112 cms. wide? If the roll contains 0·46 cubic metres of paper, find, to the nearest tenth of a millimetre, the thickness of the paper.

4. In a recent motor race the length of the course was $8\frac{1}{2}$ miles. The winner had 8,404 yards start, and finished 1 mile ahead of the second car. The latter had 2,335 yards start, and passed the post 1 min. 23 sec. behind the winner. The third car, starting from scratch, passed the post 1 min. 42 sec. behind the winner. Find the speed of each car, assuming it to be constant. Give the results to the nearest tenth of a mile per hour.

PAPER VII.

No restriction is made as to method. For full credit your work must be easily intelligible, and answers given only to the number of significant figures that is asked for or is justified by the data.

1. A square courtyard, 35 feet in the side, has to be paved with square tiles 8 inches in the side. How many tiles will be used, and what space will be left uncovered at the edges, if the tiles cannot be divided?

2. How many bricks $9'' \times 4\frac{1}{2}'' \times 3''$ can be carried by a truck whose load is 5 tons? The bricks in question weigh 145 pounds per cubic foot. Give your result to two significant figures.

3. A warship has a radius of action of 1000 miles if steaming at 16 knots, and a radius of action of 2000 miles if steaming at 10 knots. If the radius of action is determined solely by the coal consumption, compare the amounts of coal burned per hour at each speed.

4. Where a railway goes round a curve the outer rail is raised to keep the train on the line. When the curve has a radius of R feet, the distance between the two rails is W feet, and the customary speed of the trains is V miles an hour, the outer rail should be $\dfrac{W \times V^2}{1{\cdot}25 \times R}$ inches higher than the inner rail. Calculate how much higher the outer rail should be on a curve of radius 580 feet, if the rails are 4 ft. $8\frac{1}{2}$ in. apart, and the customary speed is 40 miles an hour.

5. A boy measures up a sheet of lead to see how many bullets he can cast from it. Its thickness lies between 2·5 and 3 millimetres, and its area is shown below (Fig. 42) on a scale of $\frac{1}{10}$. He means to cast bullets 8 mm. in diameter, and knows that the volume of each will be $0{\cdot}52 \times 8^3$ c.mm. How many can he cast?

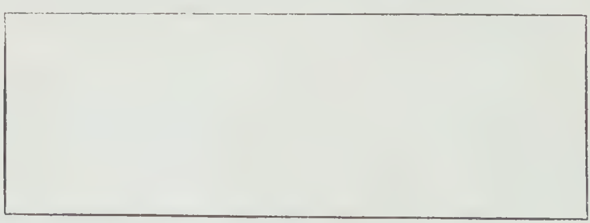


Fig. 42.

6. A piece of corrugated metal measures 10 ft. by 10 ft. A section taken across the corrugations is a wavy curve of the shape indicated in Fig. 43, the portions between *a* and *b*, *b* and *c*, *c* and *d*, etc., being each one quarter of a circle of radius 2 in. If the piece is made of

sheet-metal which, without corrugations, weighs 2·73 lb. to the square foot, find the weight of the corrugated piece. ($\pi = 3{\cdot}14$.)

FIG. 43.

PAPER VIII.—TIME ALLOWED, TWO HOURS.

For various data see page 372. For full credit all your working must be shown and the results be given only to the degree of accuracy that is asked for or is justified by the data.

1. A merchant's total expenses for rent, rates, wages, etc., amount to £16. 5*s.* 6*d.* per week. In addition he has to pay $4\frac{1}{4}$ per cent. annually on a loan of £2650. What must his profits be that there may be left for himself an income of £500 a year?

2. Find the number of cubic feet added to a rectangular room, 15 ft. high, if a bay window is built out from one side, the plan of which drawn to a scale of 1 inch to 1 yard is given below. The area of the figure shown may be found by multiplying half the sum of the two parallel sides by the distance between them; with the letters on the figure the area is

$$\frac{a+b}{2} \times h.$$

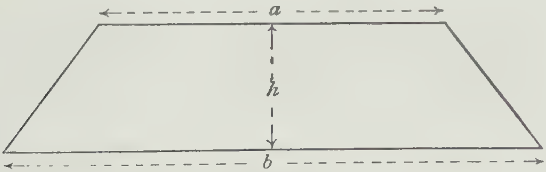


FIG. 44.

3. The lifting capacity of a balloon is equal to the difference between the weight of the gas it contains and the weight of the same volume of air, less the weight of the balloon.

Find the lifting capacity of a spherical balloon 26 ft. in diameter, filled with a gas which is 0·4 times as heavy as air, the weight of the balloon being 1 cwt.

The volume of a sphere is $\frac{\pi d^3}{6}$, where d is the diameter; a pound of air has a volume of 13·14 cubic ft.

4. A vessel steams out from a port at the rate of 20 miles an hour. If a gun was discharged at the port every minute, find at what

intervals the minute gun would be heard on deck, assuming that the vessel's course is always in the same direction. Sound travels 1100 ft. a second.

5. A river holds 23 grains of solid matter in suspension to every gallon. At a certain point the average velocity of the river is 4 miles an hour, and the area of the cross-section of the river is 1400 sq. ft. How many tons of solid matter in suspension are carried past this point in a year?

6. In 1880 the area under crops of all kinds in Great Britain was 48,850,000 acres, and the number of agricultural labourers was 1,200,000. 20 years later the area under crops had diminished to 40,400,000 acres and the number of labourers to 870,000. Find to the nearest tenth the percentage reduction (*a*) of land under crops, and (*b*) of agricultural labourers.

If the decrease for the next 20 years were to be at the same rate, what would be the acreage under crops in 1920? Give the result to the nearest million.

PAPER IX.—Two Hours Allowed.

For various data see the list of Constants printed on page 372. *You are not restricted to arithmetical methods. For full credit results must be given only to the degree of accuracy that is asked for or is justified by the data.*

1. The Cunard Steamship Company started in 1840 with the "Britannia," a wooden paddle steamer, which made her first voyage from Liverpool to New York in 14 days 8 hours. The fastest voyages of the four principal vessels of that company at the time of writing are given in the following table:

	Going West.			Going East.		
	d.	h.	m.	d.	h.	m.
"Lucania," -	5	7	23	5	8	38
"Campania," -	5	9	6	5	9	18
"Etruria,"- -	5	20	55	6	0	37
"Umbria," -	5	22	7	6	1	15

Find by how much per cent. (of the "Britannia's" speed) the "Lucania's" speed on her fastest voyage exceeded the "Britannia's," and by how much the average time taken by these four vessels on their fastest voyages eastward exceeds the average on their fastest voyages westward.

2. A town has a population of 400,000, and a water supply of 30 gallons per head per day is required. A reservoir is provided to hold six months' supply [say, July to December inclusive], with a mean depth of 20 feet. What must the area of the reservoir be? Find the area of the catchment basin if 20 inches of rainfall in the year will yield sufficient water to supply the town.

3. A triangle, whose sides are a, b and c units long, has an area (in the corresponding square units) of

$$\sqrt{\left[\frac{a+b+c}{2}\times\frac{b+c-a}{2}\times\frac{c+a-b}{2}\times\frac{a+b-c}{2}\right]}.$$

Fig. 45 is a plan of a field on a scale of $\frac{1}{2000}$. Divide it into two triangles, and use this formula to find its area. Give the area in hectares and in acres.

Check one of the results by finding the area in some other way.

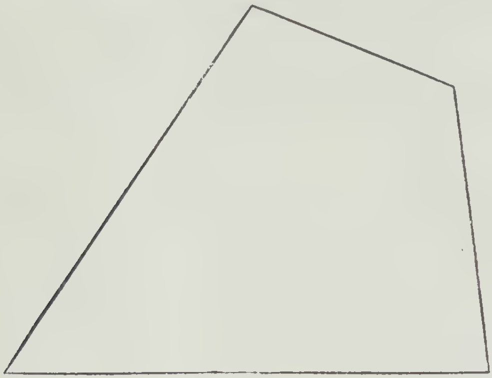


FIG. 45.

4. The population of England and Wales has increased in the successive decennial periods 1801-11, 1811-21, etc., by the following percentages: 14·00, 18·06, 15·80, 14·48, 12·89, 11·90, 13·21, 14·36, 11·65, 12·17, each percentage being reckoned on the population at the beginning of the decennial period concerned. The population was 32,527,843 in 1901; find the population in 1851 and in 1801 to the nearest hundred-thousand.

5. The claims against a bankrupt company are £5000 claimed by the bank for an overdraft, £4000 claimed by A, and £11,000 by other creditors. Moreover A has guaranteed the bank against loss. The available assets are £4000. The case is taken into court, where A's claim against the company is reduced to £2000, and the other claims are allowed in full. How much more is A out of pocket after dis-

charging his guarantee than he would have been if the court had not reduced his claim?

6. On a single line railway, 55 miles long, there are stations at every 5 miles, where it is possible for two trains to pass one another. A fast train starts at noon from one terminus, running at the rate of 50 miles an hour, and stops 2 minutes at the fifth station out. A slow train starts 5 minutes later from the other terminus, running at the rate of 20 miles an hour, and stops 2 minutes at every station. At what station must the slow train lie by for the fast train to pass it? The slow train must be out of the way at least a minute before the fast train is due.

7. Fig. 46 gives in shillings per ton the price of a certain iron ore during 1904 and 1905. What were the highest and lowest prices within this period, and when were they reached? At what other times did the price (1) cease to rise and begin to fall, (2) cease to fall and begin to rise? When was the price rising fastest?

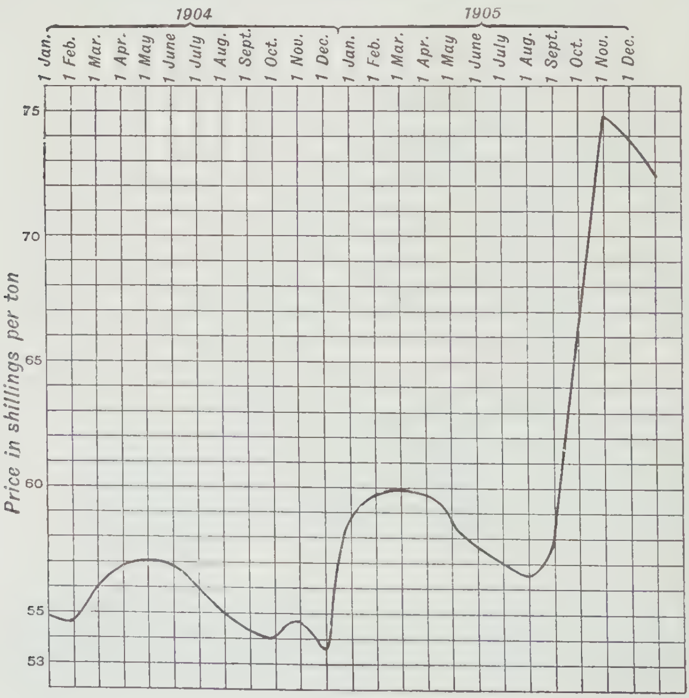


Fig. 46.

PAPER X.—TIME ALLOWED, TWO HOURS.

For various data see page 372. For full credit all your working must be shown and the results be given only to the degree of accuracy that is asked for or is justified by the data.

1. The speed of cars on a motor trial track is to be ascertained by counting the number of equidistant pegs, placed alongside of the track, that are passed in a stated interval of time. At what distance apart ought these pegs to be, so that the number passed in half a minute may approximately express the speed of a car in miles per hour?

If 50 of these pegs are passed in any particular half minute, between what limits may the speed of the car lie?

2. English corn at 33*s.* 6*d.* per quarter of 8 bushels, Russian corn at 37*s.* 9*d.* per quarter, and other foreign corn at 38*s.* per quarter, are blended in the ratio of 5 parts by weight to 3 parts and 2 parts respectively, to make a flour. English corn weighs 63 lbs. to the bushel, Russian 62 lbs. to the bushel, and the other foreign corn 60 lbs. to the bushel. If the price of English corn remains stationary while Russian and other foreign corn falls to 34*s.* $4\frac{1}{2}$*d.*, find to the nearest integer the fall per cent. in the cost of the flour.

3. A farmer wishes to put up a rough shelter for hay consisting of a nearly flat horizontal rectangular roof supported by a brick pillar at each corner. The shelter has to be made large enough to cover 750 cubic yards of hay. Each pillar will cost 1*s.* 3*d.* per foot of height to build, and the roof will cost 1*s.* per square yard. Find what will be the total cost when the height is 2 yds., 3 yds., 4 yds..... Plot your results, and draw through the points plotted a curve which will show the cost of the shelter when the height varies from 2 yds. to 10 yds., and find, approximately, what the height of the shelter should be for a minimum outlay.

4. Copper sheeting, an eighth of an inch thick, is electro-plated with silver on one side, an ounce (avoirdupois) of silver going to the square foot. If copper is 8·95 times as heavy as water, and silver 10·56 times as heavy as water, find the weight of copper and of silver in a plate weighing 100 lbs.

5. The net total expenditure of the Board of Education out of the Parliamentary Vote was £12,226,637 for the year 1904-5, £12,604,048 for the year 1905-6, and £13,165,338 for the year 1906-7. Of this the grants to meet expenditure in respect of Elementary Education for these three years were £10.669,353, £10,829,396 and £11,248,794 respectively. From these data calculate to the nearest integer (a) what percentage of the total expenditure for the three years went towards the cost of Elementary Education; (b) the increase per cent. on the year 1905-6 of the expenditure for Elementary Education for the year 1906-7.

6. The figure below represents a vertical crosswise section AA of a stream of water flowing in a straight horizontal channel, the shaded portion representing water, and the drawing being on the scale of 1 in. to 3 ft. 4 in. Find the average depth of the water in the channel.

Find also to two significant figures the number of gallons of water in a quarter of a mile of the channel.

FIG. 47.

APPENDIX.

PRISM, CYLINDER, PYRAMID, CONE, AND SPHERE.

A **prism** is a solid figure bounded by plane faces, of which all except two are parallel to an axis, and the other two are parallel to each other but not to the axis.

If the ends are at right angles to the axis, the figure is called a **right prism**.

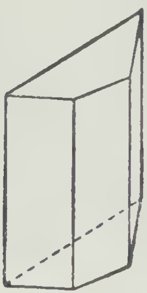


Fig. 48.

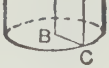


Fig. 49.

A **cylinder** is the solid figure formed by the revolution of a rectangle about one side which remains fixed. The fixed side is called the *axis* of the cylinder.

In both the right prism and the cylinder, *the volume is found by multiplying the area of one end by the length*, as will be proved in the following articles.

The volume of a right prism is equal to the area of its base multiplied by its altitude.

Let ADFK be a right prism, whose edges AF, BG, etc. are at rt. ∠ˢ to its bases ABCDE, FGHKL.

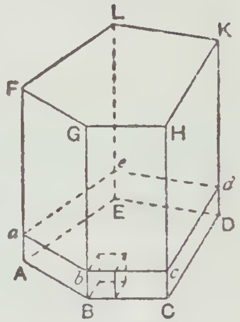


Fig. 50.

Let S be the area of each of its bases, and h its altitude, so that

$$h = AF = BG = CH, \text{ etc.}$$

Take a plane *abcde* ∥ to ABCDE and at unit distance from it.

Then we see that the vol. of the prism bounded by these planes consists of S unit cubes.

But since $AF = h$ units of length, the whole vol. consists of h prisms each equal to AD*da*.

∴ the vol. of the whole prism $= S \times h$.

The volume of a right cylinder, whose height is h and the radius of whose ends is r, $= \pi r^2 h$.

Remembering that the area of each circular end $= \pi r^2$, we may prove this in the same way as the preceding proposition.

Given the area of the end of a right prism, find its length.

$$V = Ah; \quad \therefore \ h = V \div A.$$

Example. In a right prism, whose end is 5 sq. inches and volume 35 cubic inches, the length $= \frac{35}{5}$ inches $= 7$ inches.

In a prism the area of the surface of any lateral face (not one of the ends) = length of the corresponding side of the end × length of prism.

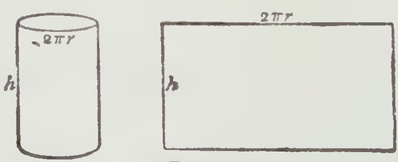


Fig. 51.

Calling the end the base, and the length the height of the prism, we have the following truths:

Area of a lateral face = corresponding edge of base × height;
∴ the lateral surface of a prism = perimeter of base × height.

Similarly, ***the curved surface of a cylinder***
= circumference of base × height = $2\pi rh$.

In fact, the surface might be unrolled into a rectangle measuring $2\pi r$ in one direction and h in the direction perpendicular to this.

In both the prism and cylinder the total surface = lateral surface + the ends.

EXAMPLE. The volume of a cylinder is 1056 cub. metres, and the height 21 metres; find the area of the curved surface, supposing that $\pi = \frac{22}{7}$.

$$\pi r^2 = \text{area of base} = \tfrac{1056}{21} \text{ sq. metres};$$
$$\therefore\ r^2 = \tfrac{1056}{21} \times \tfrac{7}{22} = \tfrac{352}{22} = 16; \quad \therefore\ r = 4.$$
$$\text{Curved surface} = 2\pi rh = \tfrac{44}{7} \times 4 \times 21 = 528 \text{ sq. metres.}$$

If we take $\pi = 3{\cdot}1416$, we get

$$\pi^2 r^2 = \frac{\pi r^2 h \times \pi}{h} = \frac{1056 \times 3{\cdot}1416}{21} = \frac{352 \times 3{\cdot}1416}{7}$$
$$= 157{\cdot}9776;$$
$$\therefore\ \pi r = 12{\cdot}57;$$
$$\therefore\ 2\pi rh = 527{\cdot}9.$$

The volume of brickwork in a cylindrical or rectangular tower may be found as follows:

To find the area of the ground covered by brickwork, subtract the internal area from the whole area occupied. Then multiply this result by the height.

EXAMPLE. What is the volume of brickwork in a cylindrical tower whose external radius is 12 ft., the thickness of brickwork being 2 ft., and the height (including the foundation) being 28 ft.?

$$\text{Area of base} = \pi \,.\, 12^2 - \pi \,.\, 10^2 = \pi(12^2 - 10^2)$$
$$= \pi(12+10)(12-10)$$
$$= 3{\cdot}1416 \times 44 = 138{\cdot}2304 \text{ sq. ft.}$$
$$\text{Volume} = 138{\cdot}2304 \times 28$$
$$= 3870{\cdot}4512 \text{ c. ft.}$$

The volume of material in a cylindrical pipe is found in the same way.

Error in taking $3\frac{1}{7}$ for π.

The value of π is $3{\cdot}14159\ldots$.

$3\frac{1}{7} = 3{\cdot}142857\ldots$.

$\therefore$ the ratio of the error to the true value is very nearly $\cdot0004$; *i.e.* in taking $3\frac{1}{7}$ for the value of π we obtain a result which has an excess of about $\cdot0004$ of the true value.

EXAMPLES XXXIV. a.

PRISM AND CYLINDER.

(Take $\pi = 3{\cdot}1416$ unless otherwise directed.)

1. A cylinder has a 7 m. radius and its curved surface is 3738·504 sq. m. What is its length?

2. The length of a right triangular prism is 8 cm. and its end has sides 1·4, 2·3, 2·7 cm.: find the area of its lateral surface.

3. What quantity of sheet iron is wanted to make a cylindrical tube 2 feet in diameter and 40 feet long?

4. A gasometer covers $\frac{1}{10}$ of an acre and is 30 feet high. How many cubic feet of gas will it hold?

5. If all its linear dimensions were halved, what volume would it contain?

6. The base of a prism has an area of 35 sq. cm., and the volume is equal to 1 litre. What is the height?

7. A well 2 m. in diameter contains 75398·4 litres. What is its depth?

8. Find the volume of a triangular prism whose length is 5 ft. and the sides of the base 6, 8, 10 inches.

9. The area of the section of a boring is 1325 sq. ft. and the excavating machine is driven forward 4 ft. a day. How many cubic yds. of earth are excavated in a day?

10. The curved surface of a cylinder whose radius is 3·5 cm. and length 2·2 cm. is unwrapped into a rectangle and wrapped up to form a new cylinder. What are the dimensions of the new cylinder? [$\pi = 3\frac{1}{7}$.]

11. Find the volume of a cylinder which has a height of 1 m. 7 dm. 5 cm. and a base of radius 4 dm.

12. Find the curved surface of this cylinder.

13. A prism has for base a $\triangle$ whose sides are 3, 4, 5 cm. and height 6 cm. What is its lateral surface?

14. What is its volume?

15. A stone near Damascus is in the form of a square prism 63 ft. long, its ends being 12 ft. square. Find its volume and surface.

16. Find the lateral surface of a triangular right prism whose ends have edges 6·2, 4·5, 5·6 cm. and whose length is 1 dm.

17. Find the volume of a right prism 6 ft. long, whose ends are equilateral triangles of 12 in. sides.

18. Find the total surface of a cylinder whose radius is 5 ft. 3 in and height 1 ft. 2 in.

19. A cub. ft. of copper is drawn into a wire 1000 yds. long. Find the area of a section of the wire.

20. ABCD is the cross section of a right prism 18 in. long. AB=7 in., BC=15 in., AD=24 in., and the angles at A and C are rt. angles. Find the volume and total surface.

21. The base of a right prism 7 ft. high is a trapezium whose equal sides are 13 in. and whose parallel sides are 9 and 19 in. respectively: find its volume in cubic feet.

22. Find, to the nearest cubic ft., the volume of a cylinder 5 ft. high on a circle of 3 ft. radius as base.

23. Find the number of cub. ft. of air in a rectangular building open to the roof, the height of the side walls being 14 ft. and of the gables 21 ft., and the floor being $8\frac{1}{2}$ yds. by 10 yds.

PYRAMID AND CONE.

A pyramid has plane faces and all its faces except one meet in a point called the **vertex**.

The face opposite to the vertex is called the **base**.

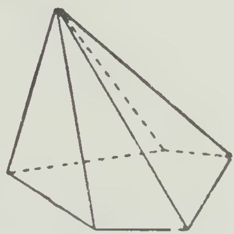


FIG. 52.

A tetrahedron is a pyramid on a triangular base.

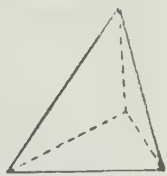


FIG. 53.

A **right circular cone**, or shortly, a **right cone**, is the solid generated by the revolution of a right-angled triangle about one of the sides containing the right angle.

Thus if the △OAB, right-angled at A, revolves about OA, it generates the solid shown in the figure.

FIG. 54.

OA is called the **axis** of the cone, and is perpendicular to the base.

Any line through O on the surface of the cone is called a **generating line**, and the length of such a line from vertex to base is called the **slant height**.

As we shall deal with *right* cones only, we may omit the word '*right*.'

The volume of a pyramid does not depend upon its shape, but only on its base and height; for it can be proved that pyramids standing on the same base and having equal altitudes are equal. From this we proceed to show that a right triangular prism can be divided into three equal triangular pyramids.

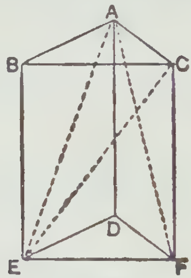


FIG. 56.

The right prism ABCFDE consists of the pyramids EABC, ADEF and EACF.

The pyramid EABC has the base ABC and altitude BE.

" " ADEF has the base DEF and altitude AD.

" " EACF = BACF (on the same base ACF)

= a pyramid whose base is ABC and altitude CF.

Thus each of the three pyramids has for base a base of the prism and for altitude the height of the prism.

$\therefore$ they are equal.

Hence the volume of a triangular pyramid

$= \frac{1}{3}$ of a right prism of the same altitude and base

$= \frac{1}{3}$ *base* $\times$ *height*.

By dividing the base into triangles it is easy to extend this to *any* pyramid.

Since a regular polygon becomes a circle if the number of sides be infinitely increased, the formula applies to a cone also.

i.e. in a right cone the *volume* $= \frac{1}{3}$ base $\times$ height

$$= \tfrac{1}{3}\pi r^2 h,$$

where r = radius of base.

The curved surface of a cone, whose slant height is l and whose base has a radius r, unrolls into a circular sector whose arc is $2\pi r$ and radius l.

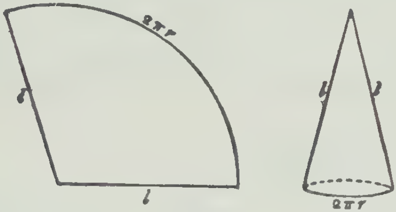


FIG. 56.

Area $= \frac{1}{2}$ ***arc*** $\times$ ***radius*** $= \pi rl$

$= \frac{1}{2}$ ***circumference of base*** $\times$ ***slant height.***

Total area of surface of cone = curved surface + base

$= \pi rl + \pi r^2 = \pi r(l + r).$

EXAMPLE 1. **Find the volume of a pyramid whose base is a square with a side of 6 cm. and whose altitude is 21 cm.**

$$\text{Volume} = \tfrac{21}{3} \times 6^2 = 7 \times 36 = 252 \text{ c. cm.}$$

EXAMPLE 2. **Find the cost of canvas at 3s. a square yard for a conical tent whose height is 9 ft. 4 in., and whose base has a radius of 7 ft.**

$$\text{Slant height} = \sqrt{7^2 + (9\tfrac{1}{3})^2} = \sqrt{7^2 + (\tfrac{28}{3})^2} = \tfrac{35}{3} = 11\tfrac{2}{3}.$$

$$\text{Area of canvas} = 7 \times \frac{35}{3}\pi = \frac{245\pi}{3} = 256{\cdot}564 \text{ sq. ft.}$$

Cost (at $4d.$ a sq. ft.) $= 256{\cdot}564 \times 4 = 1026{\cdot}256d. =$ £4. 5s. 6d. to the nearest penny.

EXAMPLE 3. **A pyramid, whose base is a square of 7 cm. sides, and whose altitude is 8 cm., is cut into two parts by a plane parallel to the base and 6 cm. from it. Find the volumes of the two parts.**

Volume of the whole pyramid $= \frac{1}{3} \times 7^2 \times 8 = 130{\cdot}6667$ c. cm.

Altitude of upper part $= 2 = \frac{1}{4}$ of the whole altitude.

$\therefore$ the side of the base of the upper part $= \frac{1}{4}$ of 7.

Volume of upper part $= \frac{1}{3} \times (\frac{7}{4})^2 \times 2 = 2{\cdot}0417$ c. cm.

Volume of lower part $= 130{\cdot}6667 - 2{\cdot}0417 = 128{\cdot}625$ c. cm.

EXAMPLES XXXIV. b.

PYRAMID AND CONE.

1. Find the volume of a square pyramid whose height is $15\frac{3}{4}$ ft. and base 2·5 by 2·5 ft.

2. Find the volume of a square pyramid whose height is 12·6 cm. the perimeter of the base being 32 cm.

3. Find the lateral surface of a triangular pyramid when each side of the base is 6 in. and each lateral edge 5 in.

4. What is the volume of a triangular pyramid in which the height is 30 cm. and each side of the base 5·5 cm.?

5. The volume of a pyramid is 352 c. cm. and the base is 44 sq. cm. Find the height.

6. Find the surface and volume of a square pyramid whose base is 12 ft. by 12 ft., the bisector of each face (drawn from the vertex of the pyramid) being 10 ft.

7. A pyramid whose volume is 26 c. ft. 1152 c. in. has a height of 5 ft.: find the area of its base.

8. Find the surface and volume of a square pyramid on a base 30 cm. by 30 cm., one of its equal lateral edges being 25 cm.

9. What is the volume of a cone whose height is 18·5 cm., while the diameter of the base is 12 cm.?

10. A wooden pyramid on a square base 6 in. by 6 in. has a height of 8 in.: find the volume and surface.

11. Find the cost of dressing a conical stone spire at $6\frac{1}{2}d.$ per sq. ft., if the circumference of the base is 30 ft. and the slant height 45 ft.

12. The volume of a cone is 19242·3 c. ft. and its base has a diameter of 35 ft. What is the height?

13. Find the volume of a pyramid whose height is 12 in. and whose base is an equilateral triangle of 10 in. side.

14. A symmetrical pyramid 8 ft. high stands on a square base whose side is 12 ft. Find the area of each triangular face.

15. A pyramid on a square base whose side is 12 ft. has each of its slant edges 16 ft. in length: find the altitude and volume.

16. A pyramid stands on a regular hexagonal base whose side is 1 ft. If each slant edge is 20 in., find the altitude and volume.

17. What is the expense of dressing at $6d.$ per sq. ft. the lateral surface of a stone pyramid 10 ft. high standing on a square base whose side is 9 ft.?

18. What quantity of canvas is required for a conical tent of altitude 8 ft., the diameter of the base being 13 ft.?

19. What is the volume of a cone in which the height = the circumference of the base, and the radius of the base is 2·25 cm. $(\pi = 3\frac{1}{7}.)$

20. The diameter of the base of a conical piece of timber is 5·5 cm. and the length of its slant side is 25·75 cm. Find its curved surface.

21. The volume of a pyramid is 19·25 c. ft. and the base is a rectangle measuring 3·75 by 2·8 ft. Find the height.

22. A right-angled isosceles triangle, whose equal sides are each 5 cm., revolves about one of those sides: find the volume and surface of the figure generated.

23. Find the volume and surface generated if the triangle revolves about its hypotenuse.

24. A cylindrical pencil 16 cm. in length has $\frac{1}{8}$ of its length sharpened into a cone. If the diameter be 7 mm., what volume has been cut away? What fraction of the whole volume is this?

25. The height of a conical tent is $7\frac{1}{2}$ ft., and it is to enclose 20 sq. yds. of ground. Find the slant height. $(\pi = 3\frac{1}{7}.)$

26. Having given that the length of an edge of a regular tetrahedron is 4 in., find the total surface in sq. inches correct to three places of decimals.

FRUSTUM OF PYRAMID AND FRUSTUM OF CONE.

Def. A **frustum** of a cone or pyramid is the volume included between the base and a plane parallel to it. For a frustum of either pyramid or cone the

$$\textit{Volume} = \frac{h}{3}\{A + \sqrt{AA_1} + A_1\},$$

where A and A_1 are the areas of the plane ends and h the distance between them. The formula for the volume can be put in a different shape in the case of the conical frustum, since $A = \pi R^2$ and $A_1 = \pi r^2$ where R and r are the radii of the ends.

$$\textit{The volume of a conical frustum} = \frac{\pi h}{3}(R^2 + Rr + r^2).$$

The area of the curved surface of a frustum of a cone
= half the sum of the circumferences of the ends
× slant height of frustum
$= \pi l(R + r)$.

Total surface of frustum of cone = curved surface + area of ends.

Proof of the formula for the volume of a frustum of a cone.

Let EF, the height of the frustum, be h; and let OE be the height of the rest of the cone.

OE and OF are respectively proportional to r and R.

$$\text{i.e. } \frac{OF}{R} = \frac{OE}{r} = m \text{ say};$$

$$\therefore\ OE = mr,\ OF = mR,$$

$$h = OF - OE = m(R - r).$$

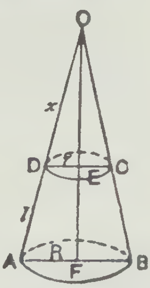


Fig. 57.

Volume of frustum = whole cone − the smaller cone

$$= \frac{1}{3}\mathsf{OF}\pi\mathsf{R}^2 - \frac{1}{3}\mathsf{OE}\pi r^2$$

$$= \frac{\pi}{3}m(\mathsf{R}^3 - r^3)$$

$$= \frac{\pi}{3}m(\mathsf{R} - r)(\mathsf{R}^2 + \mathsf{R}r + r^2)$$

$$= \frac{\pi h}{3}(\mathsf{R}^2 + \mathsf{R}r + r^2).$$

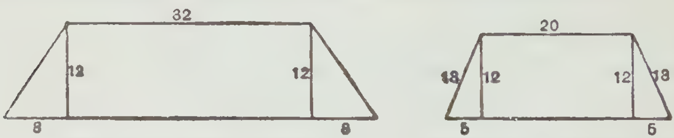
Proof of the formula for the area of the curved surface of a frustum of a cone.

The curved surface of a frustum of a right cone may be divided by generating lines into an infinite number of trapeziums whose common altitude is l (the slant length of the frustum) and whose parallel sides make up the circumferences of the top and bottom of the frustum.

The curved surface = the sum of all these trapeziums

$$= \tfrac{1}{2}l(2\pi\mathsf{R} + 2\pi r) = \pi l(\mathsf{R} + r).$$

EXAMPLE. Find the slant surface of a right pyramidal frustum whose ends are rectangles 32 by 20 and 48 by 30, and whose altitude is 12.



Section perp. to edges, 20 and 30. Section perp. to edges, 32 and 48.

FIG. 58.

Two of the slant faces are trapeziums whose parallel sides are 20 and 30, the distance between them being $\sqrt{8^2 + 12^2}$, *i.e.* 14·422.

The other two are trapeziums whose parallel sides are 32 and 48, the distance between them being $\sqrt{5^2 + 12^2}$, *i.e.* 13.

$$\text{Total area} = 50 \times 14{\cdot}422 + 80 \times 13$$

$$= 721{\cdot}1 + 1040$$

$$= 1761{\cdot}1.$$

EXAMPLES XXXIV. a.

1. What is the volume of a frustum of a square pyramid, the altitude being 10 feet and the sides of the ends 2 ft. 2 in. and 3 ft. 4 in. respectively?

2. Find the volume of a frustum of a cone, the altitude being 3 m. 25 cm. and the diameters of the ends 7 m. and 5 m.

3. The slant height of a kiln is 45 m., the circumference of the bottom is 113 m. 75 cm. and of the top 31 m. 25 cm.; find its curved surface.

4. If a frustum of a square pyramid have its ends 3″ by 3″ and 7·5″ by 7·5″, the height being 18″, what is its volume?

5. A column in the form of a conical frustum 10 ft. high has the radii of its ends 4 and 2·5 inches. Find its volume.

6. If a block of stone in the form of a frustum of a pyramid with square ends taper from a width of 28 to 14 inches in a length of $18\frac{3}{4}$ feet measured perpendicular to the ends, what is the volume?

7. Find the volume of a frustum of a cone if the altitude be 7 ft. and the radii of the ends 4 and 5 ft.

8. The radii of the ends of a conical frustum are 12 and 8 cm. and the area of the curved surface is $20\pi\sqrt{241}$ sq. cm.: find the thickness of the frustum.

9. Find the volume of a frustum of a triangular pyramid 20 cm. in height if the edges of the base and top are 9, 12, 15 cm. and 6, 8, 10 cm. respectively.

10. A frustum tapers from a rectangle 9 by 6 cm. to 3 by 2 cm. The volume is 130 c. cm. Find the thickness.

11. A piece of timber 17 ft. 3 in. long has its section a rectangle. One end measures 3 ft. by 1 ft. 8 in., the other 1 ft. 6 in. by 10 in. What is its volume?

12. A conical frustum, 2 ft. in diameter at the top and 8 ft. at the bottom, has a volume of 88 cub. ft. What is its slant height and its curved surface? ($\pi = 3\frac{1}{7}$.)

13. The circumferences of the ends of a conical frustum are 60 and 38 cm. and the area of the curved surface is 1715 sq. cm. Find the slant height of the frustum and of the cone from which it is cut.

THE SPHERE.

Def. A **sphere** is the solid generated by the revolution of a semicircle about the diameter.

Any plane section of a sphere is a circle. A plane section through the centre is called a **great circle**.

If a cylindrical pipe contain a sphere which fits it exactly, any two planes perpendicular to the axis cut off belts of equal surface from the sphere and the interior of the pipe.

This simple property of the *surface* of a sphere may be expressed as follows:

The surface of a sphere is equal to the curved surface of the circumscribing cylinder; and the surface of any belt of the cylinder between two planes perpendicular to the axis is equal to the surface of the belt of the sphere between the same two planes.

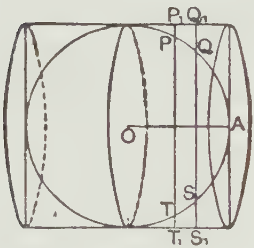


Fig. 59.

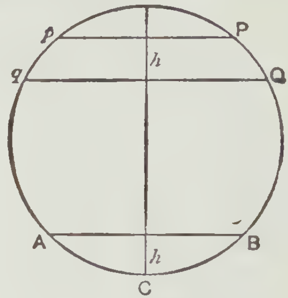


Fig. 60.

The surface of the sphere = curved surface of the cylinder $= 4\pi r^2$.

A belt or zone of a sphere is the part included between two parallel planes, and the height is the distance between them.

A segment or cap is the part cut off by a plane.

If h be the height of a belt such as $PQqp$, or of a cap of a sphere such as ACB, *the curved surface of the belt or cap* $= 2\pi rh$; i.e. *the area is independent of the radius of the base. It only depends on the height h of the belt or cap and r the radius of the sphere.*

Any plane through the centre divides a sphere into two equal parts called *hemispheres.*

The curved surface of a hemisphere $= 2\pi r^2$

$=$ *twice the area of the plane base.*

VOLUME OF A SPHERE.

The surface may be divided into an infinite number of infinitely small polygons. By connecting these with the centre there are formed an infinite number of pyramids, whose vertices are the centre and height ultimately the radius of the sphere.

The volume of the sphere = sum of pyramids

$$= \frac{r}{3} \times \text{sum of bases of pyramids}$$

$$= (\text{ultimately}) \frac{r}{3} \times \text{surface of sphere} = \frac{4}{3}\pi r^3.$$

EXAMPLES XXXIV. d.

SURFACE AND VOLUME OF A SPHERE.

1. Find the surface and volume of a sphere whose radius is 5 cm.

2. Find the surface and volume of a sphere whose diameter is 4·5 m.

3. What would be the cost of gilding the ball on the top of St. Paul's at $3\frac{1}{2}d.$ per sq. inch, supposing its radius to be 3 feet?

4. The volume of a sphere is 94 cub. ft., and its surface is 100 sq. ft.: find the radius.

5. The surface of a sphere is 25 sq. cm.: find the radius.

6. Find the surface and volume of a sphere whose radius is 2·5 m.

7. What is the radius of a sphere whose surface is 2464 sq. ft., if $\pi = 3\frac{1}{7}$?

8. Find the cost of gilding a hemispherical dome 10 ft. in diameter at 2*s.* 1*d.* per sq. ft.

9. A cone has a slant height of 12 cm. and a base whose radius is 3 cm. What is the radius of the sphere whose surface is equal to the curved surface of the cone?

10. How many litres are contained in a hemispherical bowl whose internal diameter is 70 cm.?

11. A sphere, 15 cm. in diameter, is melted and cast into a cone whose base is 20 cm. in diameter. Find its height.

12. The volume of a sphere is 11 c. cm. 754 c. mm. and its surface is 25 sq. cm.; what is its radius?

13. The surface of a sphere is 16 sq. cm.; find the radius.

14. Find the volume of a sphere whose surface is 154 sq. ft., supposing $\pi = 3\frac{1}{7}$.

15. A pontoon consists of a cylinder with hemispherical ends, the length of the cylindrical part being 6 yds. 1 ft. 4 in., and the common radius 1 ft. 4 in. What is the expense of painting at 1*d.* per sq. ft.?

16. Supposing that a cub. ft. of water weighs $62\frac{1}{2}$ lbs. and that lead is 11·4 times as heavy as water, find the weight of a leaden sphere of 3 inches diameter.

17. Find the weight of a leaden sphere of 1 inch diameter. (See Example 16.)

18. If a heavy sphere whose diameter is 4 inches be placed in a cylinder, of 5 inch diameter, which contains water to a depth of 4 inches, how much is the surface of the water raised?

19. What is the weight of an iron spherical shell of 3 inches internal radius, and thickness 1·5 inches; given that a solid iron sphere of 2 inches radius weighs 9 lbs.

20. What difference is there between the inner and outer surfaces of a spherical shell whose external diameter is 6 dm. and thickness 5 cm.?

21. How many litres will it hold?

BELTS OF A SPHERE.

22. Find the curved surface of a belt, 2 cm. in height, cut from a sphere whose diameter is 20 cm.

23. What would be the area if the sphere had a diameter of 17 cm.?

24. Compare the surface of the three zones of the Earth's hemisphere, the frigid extending to $23\frac{1}{2}°$ from the pole, the torrid extending $23\frac{1}{2}°$ from the equator, and the temperate between these.
(Find graphically the respective thicknesses of these zones measured on the polar axis.)

25. Find the whole surface of a segment of altitude 2 ft. cut from a sphere of radius 10 ft.

26. If a sphere have a radius of 8 cm., and the curved surface of a zone be 176 sq. cm., what is the depth of the zone?

27. The height of a segment of a sphere is 1·5 cm., and its curved surface is 42·43 sq. cm. What is the diameter of the sphere?

28. The silk covering of an umbrella forms a portion of a sphere of $3\frac{1}{2}$ ft. radius, the area of the silk being $14\frac{2}{3}$ sq. ft. Find the height of the segment, supposing $\pi = 3\frac{1}{7}$.

29. Find also the area of the ground sheltered from vertical rain when the stick is held upright.

30. Find the surface of a segment 10 ft. in height of a sphere whose radius is 20 ft.

ANSWERS.

EXAMPLES I. a. (p. 6.)

1. 1240. **2.** 1358. **3.** 1019. **4.** 1005. **5.** 13965. **6.** 2732. **7.** 19307. **8.** 1148390. **9.** 24156. **10.** 20152. **11.** 192864. **12.** 187081. **13.** 174891. **14.** 170053. **15.** 470544. **16.** 562308. **17.** 388811. **18.** 337947. **19.** 143695. **20.** 177590. **21.** 56001. **22.** 27748439.

I. b. (p. 7.)

1. 39. **2.** 40. **3.** 9000. **4.** 999. **5.** 9999. **6.** 36. **7.** 8. **8.** 25. **9.** 14. **10.** 306 miles. **11.** 404 feet. **12.** 11 feet. **14.** 462. **15.** 803. **16.** 18. **17.** A majority of 2 the other way. **18.** 53. **19.** 54, 27. **20.** 48, 16. **21.** 35672. **22.** 0. **23.** 1302 yds. **24.** 4305. **25.** 1703. **26.** 1372. **27.** 57, 26. **28.** 66. **29.** A 74, B 56, C 20. **30.** 222.

I. c. (p. 9.)

1. 10845. **2.** 45684. **3.** 274104. **4.** 11214. **5.** 8289. **6.** 223803. **7.** 48275. **8.** 955800. **9.** 573930. **10.** 151680. **11.** 45528. **12.** 136092.

I. d. (p. 11.)

1. 8. **2.** 5. **3.** 5. **4.** 0. **5.** 3. **6.** 6. **7.** 2. **8.** 2. **9.** 1. **10.** 44928, nine-remainder 0. **11.** 6. **12.** 0. **13.** 5. **14.** 7. **15.** 6. **16.** 9. **17.** 8. **18.** 3. **19.** 0. **20.** 4. **21.** and **24.** Wrong. **22, 23, 25** and **26** satisfy the test.

I. e. (p. 12.)

1. 4068. **2.** 64017. **3.** 634896. **4.** 9682. **5.** 351232. **6.** 518256. **7.** 14002564. **8.** 4360092. **9.** 15515721. **10.** 1408008. **11.** 3038638. **12.** 2710456. **13.** 163245852. **14.** 3715998. **15.** 10092429.

I. f. (p. 12.)

1. 340.	**9.** 4995	**17.** 15625.	**25.** 3125.
2. 390.	**10.** 1200.	**18.** 121900.	**26.** 15625.
3. 145.	**11.** 3200.	**19.** 8000.	**27.** 285625.
4. 265.	**12.** 2100.	**20.** 9000.	**28.** 906125.
5. 1240.	**13.** 13200.	**21.** 16000.	**29.** 1600.
6. 3685.	**14.** 4025.	**22.** 60375.	**30.** 2000.
7. 8060.	**15.** 17325.	**23.** 153500.	**31.** 625.
8. 17185.	**16.** 20150.	**24.** 120750.	**32.** 3125.

I. g. (p. 14.)

1. 234.	**3.** 729.	**5.** 6132.	**7.** 2952.
2. 342.	**4.** 2584.	**6.** 3416.	**8.** 7161.

I. h. (p. 14.)

1. 3^2.	**10.** $3^3 \times 2^4 \times 5^2$.	**19.** 10^4.	**28.** 2.	**37.** 125.	**46.** 5.
2. 5^3.	**11.** 7^2.	**20.** 12.	**29.** 1.	**38.** 5.	**47.** 11.
3. 2^4.	**12.** 5^2.	**21.** 13.	**30.** 35.	**39.** 13.	**48.** 2.
4. 7^2.	**13.** 2^3.	**22.** 1.	**31.** 58.	**40.** 30.	**49.** 12.
5. 3^4.	**14.** 2^4.	**23.** 4.	**32.** 2.	**41.** 96.	**50.** 4.
6. 7^3.	**15.** 3^4.	**24.** 22.	**33.** 1.	**42.** 17.	**51.** 6.
7. $3^2 \times 7^2$.	**16.** 2^5.	**25.** 6.	**34.** 100.	**43.** 100.	**52.** 9.
8. $5^2 \times 3^3$.	**17.** 7^3.	**26.** 24.	**35.** 32.	**44.** 9.	
9. $3^4 \times 5^3$.	**18.** 3^5.	**27.** 24.	**36.** 81.	**45.** 3.	

I. k. (p. 17.)

1. 14.	**11.** 24.	**21.** 97.	**31.** 355.
2. 19.	**12.** 21.	**22.** 95.	**32.** 121.
3. 29.	**13.** 6.	**23.** 482.	**33.** 977, Rem^r. 40.
4. 12.	**14.** 12.	**24.** 11.	**34.** 9, Rem^r. 114.
5. 38.	**15.** 8.	**25.** 84, Rem^r. 201.	**35.** 2405, Rem^r. 4.
6. 85.	**16.** 14.	**26.** 268.	**36.** 1037, Rem^r. 27.
7. 2.	**17.** 43.	**27.** 68.	**37.** 23707, Rem^r. 101.
8. 104.	**18.** 121.	**28.** 13.	**38.** 881, Rem^r. 440.
9. 61.	**19.** 5.	**29.** 23, Rem^r. 43.	
10. 169.	**20.** 15.	**30.** 59.	

II. a. (p. 19.)

1. 2*s*. 9*d*.	**6.** 2*s*. 4*d*.	**11.** £1. 0*s*. 3*d*.	**16.** £2. 6*s*. 8*d*.
2. 3*s*. 11*d*.	**7.** 6*s*. 10*d*.	**12.** £2. 1*s*. 8*d*.	**17.** 3*s*. 11*d*.
3. 8*s*. 3*d*.	**8.** 12*s*. 6*d*.	**13.** 8*s*. 4*d*.	**18.** 12*s*. 7*d*.
4. 11*s*. 5*d*.	**9.** 14*s*. 2*d*.	**14.** 4*s*. $5\frac{1}{2}d$.	**19.** 13*s*. 3*d*.
5. 4*s*. 5*d*.	**10.** 16*s*. 2*d*.	**15.** 3*s*. $2\frac{1}{2}d$.	**20.** £1. 1*s*.

21. 5s. 11d.
22. 9s. 6d.
23. 16s. 5d.
24. 3 fr.
25. 28 fr. 60 c.
26. 42 fr. 50 c.
27. 7 fr.
28. 1 fr. 25 c.
29. 27 fr. 6 c.
30. 10.
31. 45.
32. 30.
33. 50.
34. 35.
35. 100.
36. 15s.
37. 28s.
38. 4s. 9d.
39. 6s. 6d.
40. 14s. 6d.
41. £1. 9s. 9d.
42. £6. 5s.
43. £7. 10s.
44. £5. 16s. 9d.
45. 11s. 9d.
46. £8. 15s.
47. £10. 12s. 6d.
48. £15.
49. £13.
50. £22.
51. £37.
52. £79.
53. £151.

II. b. (p. 21.)

1. 253.
2. 82.
3. 126.
4. 81.
5. 12.
6. £1. 2s. 4d.
7. 1s. 2d.
8. 1s. $7\frac{1}{4}d$.
9. £1. 0s. $0\frac{1}{4}d$.
10. £2. 2s. 6d.
11. 49.
12. 103.
13. 5280.
14. 160.
15. 2 yds. 1 ft. 3 in.
16. 2880.
17. 2240.
18. 260.
19. 308.
20. 9 gals. 1 pt.

II. c. (p. 22.)

1. 2089.
2. 3599.
3. 4470.
4. 13686.
5. 14473.
6. 28950.
7. 1439.
8. 1918.
9. £60. 2s. 6d.
10. £23. 15s. 4d.
11. £72. 6s. 1d.
12. £18. 12s. $6\frac{3}{4}d$.
13. £15. 18s. $9\frac{1}{2}d$.
14. £6. 0s. 7d.
15. £1. 10s. $7\frac{1}{2}d$.
16. £9. 2s. 6d.
17. £47. 17s. 6d.
18. 1000.
19. 145.
20. 31.
21. 171.
22. 437.
23. 291.
24. 765.
25. 403.
26. 8067.
27. 10320.
28. 6425.
29. 109.
30. 1 cwt. 2 qrs. 9 lb.
31. 7 tons 3 qrs.
32. 7 tons 15 cwt. 2 qrs. 16 lb.
33. 18700.
34. 569.
35. 9 mls. 6 fur.
36. 247.
37. 1 sq. yd. 3 sq. ft. 28 sq. in.
38. 1470.
39. 18932.
40. 3 ac. 117 sq. yds.
41. 572.
42. 1 sq. yd. 7 sq. ft. 41 sq. in.
43. 9.
44. 56209280.
45. 5 furlongs.
46. 5220.
47. 357180.
48. 7 days 29 min. 23 sec.
49. 488.
50. 1003.
51. 1250.
52. 4 qrs. 7 bus. 2 qts.
53. 7 bus. 1 gall. 1 pt.
54. 3762.
55. 16 qrs. 5 bus. 5 gall.
56. 4453200.
57. 47 tons 16 cwt. 3 qrs.
58. 3 ml. 6 fur. 7 chains.
59. 12 ac. 700 sq. yds.
60. 108 reams 9 quires 17 sheets.
61. 14527.
62. 5786.
63. 60 reams 10 quires 14 sheets.
64. 259344.
65. 38° 14′ 40″.
66. 160° 43′ 26″.
67. 289302.
68. 111.
69. 1599.
70. 3840.
71. £19. 1s. 5d.
72. 13731.

73. 308952.
74. £1. 8s. $4\frac{3}{4}d$.
75. £107. 5s. 7d.
76. 1900.
77. 6730.
78. 263.
79. 5 tons 13 cwt. 2 qrs.
80. 20 ml. 1 yd.
81. 5105.
82. 30 fr. 75 c.
83. 234.
84. 7605.
85. 18004.
86. 2380.
87. 63580.
88. 8070.
89. 150000.
90. 30405.
91. 1 l. 5 dl. 3 cl.
92. 2905.
93. 17 g. 6 dg. 9 cg. 5 mg.
94. 478 dols. 95 cts.
95. 22020.
96. £26. 11s. 6d.
97. 4599.
98. 44352.
99. 10 cwt. 11 lb.
100. 41880.
101. 3828.
102. 4 sq. yds. 1 sq. ft. 104 sq. in.
103. 1 c. yd. 14 c. ft. 1152 c. in.
104. 267900.
105. 86400.
106. 12 bus. 1 gall. 3 qts. 1 pt.
107. 648000.
108. 364500.
109. 121379 sq. ml. 524 ac.
110. 4716800.
111. 86590.
112. £7. 6s. 8d.
113. £3. 19s. 1d.
114. £13. 7s. 11d.
115. £155. 12s. 6d.
116. 185; 703; 847.

III. a. (p. 26.)

1. £21. 4s. $9\frac{1}{2}d$.
2. £23. 7s. $7\frac{1}{4}d$.
3. £10. 3s. $6\frac{1}{4}d$.
4. £121. 17s. 7d.
5. £23. 6s. $10\frac{1}{2}d$.
6. £53. 16s. $3\frac{3}{4}d$.
7. £39. 8s. 2d.
8. £78300. 12s. 5d.
9. £53894. 13s.
10. £13905. 15s. 11d.
11. £20105. 13s. 8d.
12. £15753. 10s. 3d.
13. £27378. 13s. 4d.
14. £11247. 18s. 10d.
15. £71615. 5s. 3d.
16. £146101. 1s. 4d.
17. £625683. 8s. 8d.
18. £572064. 5s. 4d.
19. £2928. 11s. 4d.
20. £21291. 14s. 3d.
21. £378780. 8s. 5d.
22. £384347. 11s. 8d.
23. £3206. 16s. 1d.
24. £31735. 6s. 6d.
25. £169955. 6s.
26. £182497. 2s. 5d.
27. £1200676. 5s. 4d.
28. £128846. 9s. 7d.
29. £116510. 7s. 11d.
30. £340977. 5s. 6d.
31. £331681. 1s. 11d.
32. £121985. 4s. 10d.
33. £128820. 16s. 6d.
34. £6470. 15s. 5d.
35. £6821. 15s. 6d.
36. £2449330. 7s. 4d.
37. £2474090. 7s. 1d.
38. £10408. 4s. 11d.
39. £53215. 14s. 10d.
40. £4588101. 1s. 7d.
41. £372440. 16s. 11d.
42. 109 tons 6 cwt. 10 lb.
43. 23 tons 10 cwt. 1 qr.
44. 126 tons 15 cwt. 3 qrs. 20 lb.
45. 95 yds. 3 in.
46. 61 qrs. 2 bus. 1 gall. 1 qt. 1 pt.
47. 61 ac. 3 ro. 37 sq. po.
48. 11 oz. 17 dwt. 1 gr.
49. 7 tons 7 cwt. 2 qrs. 25 lb. 3 oz.
50. 198 ac. 3 ro. 25 sq. po.
51. 58 sq. yds. 1 sq. ft. 55 sq. in.
52. 40 days 8 hrs. 56 min. 23 sec.
53. 186° 13′ 52″.
54. 1093 miles 59 chains.
55. 2380 miles 28 chains.
56. 136 fr. 40 c.
57. 2333 $ 30 c.
58. 3 Dm. 1 m. 9 dm. 3 cm. 5 mm.
59. 8 Km. 1 Hm. 9 Dm. 3 m.
60. 1 Dg. 1 g. 8 dg. 1 cg. 5 mg.
61. 9 Kg. 3 Hg. 7 Dg. 2 g.
62. 20 Ha. 21 a. 12 ca.

III. b. (p. 29.)

1. £8. 8*s*. 4*d*.
2. £76. 11*s*. 3*d*.
3. £87. 15*s*. 5*d*.
4. £79. 16*s*. 5*d*.
5. £5. 15*s*. 4*d*.
6. £472. 6*s*. 6*d*.
7. £778. 6*s*. 1*d*.
8. £843. 13*s*. 1*d*.
9. £1359. 7*s*. 9*d*.
10. 2 bus. 2 pks. 1 gall. 2 qts. 1 pt.
11. 8 qrs. 5 bus. 3 pks. 1 gall.
12. 4 oz. 6 dwt. 10 gr.
13. 7 lb. 8 oz. 18 dwt. 13 gr.
14. 126 tons 16 cwt. 1 qr. 2 lb.
15. 6 ac. 1 ro. 26 sq. po.
16. 78° 45′ 30″.
17. 9° 36′ 55″.
18. 14 miles 55 chains.
19. 4 mls. 3 fur. 176 yds.
20. 16 hrs. 9 min. 54 sec.
21. 8 Km. 992 m.
22. 3 g. 4 dg. 6 cg.

III. c. (p. 32.)

1. £1. 4*s*. 7*d*.
2. £3. 17*s*. 11*d*.
3. £2. 1*s*.
4. £9. 19*s*. 6*d*.
5. £10. 6*s*. 6*d*.
6. £8. 3*s*. 6*d*.
7. £13. 2*s*. 6*d*.
8. £31. 5*s*.
9. £6. 7*s*.
10. £88.
11. £23. 10*s*.
12. £110.
13. £16. 17*s*. 2*d*.
14. £3. 9*s*. 4*d*.
15. £48. 6*s*.
16. £6. 1*s*. 8*d*.
17. £10. 12*s*. 11*d*.
18. £15. 4*s*. 2*d*.
19. £19. 15*s*. 5*d*.
20. £9. 10*s*.
21. £4. 17*s*. 6*d*.
22. £10.
23. £51.
24. £115.
25. £75.
26. £24. 6*s*. 8*d*.
27. £27. 7*s*. 6*d*.
28. £25. 10*s*.
29. £46. 3*s*. 4*d*.
30. £17. 1*s*. 5*d*.
31. £4. 7*s*.
32. £1. 2*s*.
33. £23.
34. £111.

III. d. (p. 33.)

1. £30. 15*s*. 5*d*.
2. £24. 12*s*. 3*d*.
3. 13 tons 1 cwt. 21 lb.
4. £151. 8*s*.
5. £222. 9*s*. 8*d*.
6. 381 yds. 4 in.
7. £315. 8*s*. 2*d*.
8. £1735. 14*s*. 4*d*.
9. £352. 12*s*. 6*d*.
10. £885. 18*s*. 9*d*.
11. £216. 9*s*. $6\frac{1}{2}$*d*.
12. £1154. 3*s*. 8*d*.
13. 180°.
14. £107. 13*s*. 7*d*.
15. £6119. 1*s*. 4*d*.
16. £7987. 19*s*.
17. 105 ac. 3 ro. 27 sq. po.
18. £402. 6*s*. $11\frac{1}{2}$*d*.
19. 762 yds. 8 in.
20. 69 ac. 1 ro. 8 sq. po.
21. £2419. 7*s*. $4\frac{1}{2}$*d*.
22. £2559. 7*s*. $5\frac{1}{2}$*d*.
23. £2173. 1*s*. 9*d*.
24. 15 miles 15 chains.
25. £352. 8*s*. $1\frac{1}{2}$*d*.
26. £16982. 14*s*. $7\frac{1}{2}$*d*.
27. 114 tons 10 cwt. 1 qr 7 lb.
28. £8526. 18*s*. $4\frac{1}{2}$*d*.
29. 4 m. 4 cm. 6 mm.
30. 118 l. 4 dl. 7 cl.
31. 53 $ 76 c.
32. 69 fr. 35 c.
33. 110 g. 1 dg. 1 cg.
34. 24 Kg. 4 Hg. 8 Dg.
35. £2112. 7*s*. 10*d*.
36. £1375. 11*s*. $2\frac{1}{2}$*d*.
37. £8432. 2*s*. 10*d*.
38. £1857. 8*s*. 10*d*.
39. £6665. 10*s*. 4*d*.
40. £1805. 12*s*.
41. 113 ac. 1 ro. 30 sq. po.
42. £331. 11*s*. $5\frac{1}{2}$*d*.
43. £2767. 3*s*. 9*d*.
44. £10455. 4*s*. 6*d*.
45. 59 tons 6 cwt. 1 qr. 10 lb.
46. 69 mls. 7 fur. 2 ch.
47. £81. 3*s*. 9*d*.
48. £9. 4*s*. 11*d*.
49. 70 ch. 50 links.
50. £953. 11*s*. 3*d*.
51. £35. 15*s*.
52. 75 c.
53. £3. 17*s*. 11*d*.
54. £102. 10*s*.
55. £4. 2*s*. 6*d*.
56. £39. 10*s*. 6*d*.
57. £36. 6*s*.
58. £26. 4*s*. 1*d*.
59. £1. 7*s*. 8*d*.
60. 12 tons 8 cwt. 1 qr. $8\frac{1}{2}$ lb.

61. 359 m.
62. 234 Kg. 4 Hg.
63. 80 miles.
64. 156 l. 1 dl. 4 cl.
65. 85 Kg. 3 Hg. 2 Dg.
66. 14 Ha. 7 a.
67. 5 Km. 7 Hm. 9 Dm. 7 m.
68. 325 fr. 50 c.
69. 23 fr. 80 c.
70. 37 tons 17 cwt. 1 qr. 13 lb.
71. 10 ac. 2 ro.
72. 275 mls. 4 fur. 8 ch.
275 mls. 48 chains.
73. 2733 M. 75 pf.
74. £65. 3*s.* 9*d.*

III. e. (p. 39.)

1. 6*s.* 8*d.*
2. 80.
3. 2*s.* 6*d.*
4. 24.
5. 6.
6. 32.
7. 7*d.*
8. £2. 1*s.* 4*d.*
9. 1 qr. 12 lb.
10. 17*s.*
11. 2*s.* 9*d.*
12. £2.
13. 8.
14. 2*s.* 9*d.*
15. 6.
16. 16.
17. 19*s.* 11*d.*
18. 19*s.* 10*d.*
19. 4 times.
20. 7 times.
21. 13 times.
22. 20.
23. 60.
24. $7\frac{1}{2}d.$
25. 8.
26. 1*s.* 1*d.*
27. 60.
28. 16 gallons.
29. 4*s.* 6*d.*
30. 2*s.*
31. 40 lb.
32. 1320.
33. £1. 2*s.*
34. 13.
35. £1. 6*s.* 9*d.*
36. £6. 2*s.* 4*d.*; £3. 1*s.* 2*d.*
37. 9*s.* 6*d.*; 3*s.* 2*d.*
38. 16.
39. 16.

III. f. (p. 40.)

1. £7. 14*s.* 7*d.*
2. £7. 2*s.* 5*d.*
3. £3. 6*s.* 11*d.*
4. £36. 11*s.* 6*d.*
5. £1. 16*s.* $8\frac{3}{4}d.$
6. £9. 9*s.* 3*d.*
7. £332. 16*s.* $7\frac{1}{2}d.$
8. £105. 12*s.* 2*d.*
9. £9. 19*s.* 7*d.*
10. 1 mile 55 chains.
11. 7 yds. 2 ft. 2 in.
12. £7. 13*s.* $6\frac{1}{4}d.$
13. £109. 10*s.* 2*d.*
14. £2. 7*s.* $8\frac{1}{2}d.$
15. £191. 4*s.* 5*d.*
16. £42. 14*s.* $8\frac{1}{4}d.$
17. £52. 16*s.* 1*d.*
18. 18 tons 3 cwt. 2 qrs. 9 lb.
19. 6 ac. 1 ro. 8 sq. po.
20. 15 cwt. 1 qr. 16 lb.
21. 2 g. 4 dg. 3 cg.
22. 8 m. 1 cm. 6 mm.
23. 7*s.* 10*d.*
24. £1. 14*s.*
25. £1. 19*s.* 7*d.*
26. 1 ac. 2 ro. 35 sq. po.
27. 3*s.* $10\frac{1}{2}d.$
28. £72. 12*s.* $8\frac{1}{2}d.$
29. £709. 17*s.* 11*d.*
30. 1 fur. 2 ch.
31. 37.
32. 1464.
33. 123.
34. 91.
35. 501.
36. 5.
37. 43.
38. 18.
39. 35.
40. 51.
41. 382.
42. 31.
43. 103.
44. 32.
45. 32.
46. 16.
47. 6.
48. 432 days.
49. 152 yds. 3 in.
50. Divide £8207. 5*s.* 4*d.* by $4 \times 4 \times 7$.
51. 514.
52. 18*s.* 6*d.*
53. 7.
54. £57. 3*s.*; £9. 10*s.* 6*d.*
55. 3.
56. 23.
57. 16.
58. 2 m. 2 dm. 9 cm.
59. 68.
60. 2*s.* $9\frac{1}{2}d.$; 90 lb.
61. £16. 18*s.*; £13. 13*s.*
62. 93 bus.
63. 3*s.* 10*d.*; 53 lb.
64. £3. 17*s.* 6*d.*
65. £3. 17*s.* $10\frac{1}{2}d.$

IV. REVISION PAPERS.

IV. a. (p. 42.)

1. 202.
2. 9*s.* 10½*d.*
3. £126. 11*s.* 6*d.*
4. £43,801,204.
5. £774. 2*s.* 3*d.*
6. $31 22 c.
7. 15 pieces ; 1 ft. 3 in.

IV. b. (p. 42.)

1. £48. 17*s.* 7*d.*
2. 2008.
3. 13.
4. 38.
5. 617760.
6. £99. 13*s.* 4*d.*
7. 5 Dm. 7 m. 6 dm. 2 cm. 4 mm.

IV. c. (p. 43.)

1. 1025.
2. $3448 35 c.
3. £1. 2*s.* 8*d.*
4. 24 Km. per hour.
5. 7½*d.*
6. 1 cwt. 1 qr. 1 lb. 1 oz.
7. £321. 15*s.*

IV. d. (p. 43.)

1. One hundred and twenty thousand and five.
2. 7,223,040.
3. 11447.
4. 20 tons 10 cwt. 2 qrs. 19 lb. 14 oz.
5. £62. 12*s.* 0½*d.*
6. £1. 3*s.* 5¼*d.*
7. £2. 16*s.*

IV. e. (p. 43.)

1. 1644 cm.
2. 7.
3. 1400472.
4. 9.
5. 15 ; 49 Kg.
6. 1*s.*
7. 8086, remainder 100.

IV. f. (p. 44.)

1. 493,654, rem. 616.
2. 153,503.
3. £4324. 4*s.* 0¾*d.*
4. £3. 15*s.* 8¼*d.*
5. £2521. 13*s.* 4*d.*
6. £5. 11*s.* 3*d.*
7. £87. 8*s.* 11½*d.*

IV. g. (p. 44.)

1. £41. 13*s.* 4*d.*
2. £9. 12*s.* 7*d.*
3. 181 tons 3 cwt. 1 qr. 4 lb. 8 oz.
4. 93.
5. 7.
6. 5*s.* 9*d.* ; £2. 14*s.* 5*d.*
7. £696. 13*s.* 1*d.*

IV. h. (p. 45.)

1. 92145.
2. 837.
3. 66.
4. 46 ; 2 chains over.
5. 5986.
6. 23 lb. ; rem. 4½*d.*
7. 28 tons 9 cwt. 2 qrs. 16 lb.

IV. k. (p. 45.)

1. 1057.
2. 152 ; rem. 160.
3. 6.
4. 122 tons 16 cwt. 1 qr. 4 lb.
5. 324.
6. 5 Kg.
7. A £175 ; B £181. 4*s.* 2*d.* ; £6. 4*s.* 2*d.*

IV. l. (p. 45.)

1. 3.
2. £34. 14*s.* 3*d.*
3. 200.
4. 101.
5. 8*s.* 7*d.*
6. 24 ac 3 ro. 10 sq. po.; 2 ac. 2 ro. 20 sq. po.
7. £3. 5*s.* $8\frac{1}{2}$*d.*

IV. m. (p. 46.)

1. 15130.
2. Error 8207 for 8307.
3. 3 miles 120 yds.
4. 4 times; rem. 0.
5. £13. 13*s.* 6*d.*
6. 20.
7. £6581. 5*s.*

IV. n. (p. 46.)

1. 11989.
2. 122.
3. 71.
4. £25. 17*s.* 1*d.*
5. £1. 17*s.* 8*d.*
6. 80 yds.
7. 1 Kg. 7 Hg. 3 Dg.

IV. o. (p. 47.)

1. 111,111 oz.
2. £644. 1*s.* $5\frac{1}{2}$*d.*
3. 185400.
4. 326 fr. 4 c.
5. 20; £4. 9*s.* 2*d.* over.
6. £1. 19*s.* $7\frac{1}{2}$*d.*
7. 123 times; $1^{st} - 2^{nd} + 3^{rd}$.

IV. p. (p. 47.)

1. Top row 96; 2^{nd} row 100; 3^{rd} 68, 20; 4^{th} 56; 5^{th} 24, 76.
2. 3 miles 770 yds.
3. £82. 19*s.* 8*d.*
4. £10598. 19*s.* 2*d.*
5. 98.
6. £4. 1*s.* 2*d.*
7. 51.

IV. q. (p. 48.)

1. 5003040.
2. 9.
3. £365. 12*s.* 6*d.*
4. £3. 16*s.* $6\frac{1}{2}$*d.*
5. £4483. 6*s.*
6. 56 and 26 gallons; 4*d.* and 1*s.* 8*d.*
7. £3. 8*s.* 2*d.*

IV. r. (p. 48.)

1. 320024.
2. 8889.
3. £2088. 5*s.* 3*d.*
4. £13. 3*s.* 3*d.*
5. 543.
6. 9*s.* 10*d.*; £5. 18*s.*
7. £1. 0*s.* 4*d.*

V. a. (p. 53.)

1. 2, 3, 5.
2. 7, 11.
3. 3, 3, 5.
4. 2, 2, 2, 3, 3.
5. 2, 2, 3, 7.
6. 7, 13.
7. 5, 19.
8. 2, 2, 3, 11.
9. 2, 11, 11.
10. 3, 17.
11. 3, 5, 5.
12. 2, 3, 3, 7.
13. 2, 3, 19.
14. 3, 3, 3, 5.
15. 11, 13.
16. 41, 43, 47.
17. 61, 67.
18. 97.
19. 101, 103, 107, 109.
20. 4.
21. 5.
22. 4.
23. 3.
24. By 11.
25. By both.
26. By 9.
27. By 11.
28. By 9.
29. By both.

30, 31, and **32** divisible.

34. 3.
35. 6.

36. 4.	**40.** 5, 13.	**44.** 7, 17.	**48.** 11, 3, 7.
37. 3.	**41.** 3, 37.	**45.** 5, 19.	**49.** 3, 7, 7.
38. 7, 13.	**42.** 3, 5, 7.	**46.** 11, 17.	**50.** 7, 11, 13.
39. 3, 19.	**43.** 5, 17.	**47.** 11, 13.	**51.** 13, 17.

V. b. (p. 54.)

1. $3 . 7^2$.	**17.** $11 . 13 . 17$.	**32.** $2 . 3^3 . 11 . 13 . 37$.	**47.** 55.
2. $3^2 . 7 . 5$.	**18.** $2^3 . 5 . 13 . 17$.	**33.** 2.	**48.** 49.
3. $2^3 . 11 . 5$.	**19.** $7 . 11^2 . 13$.	**34.** 2.	**49.** 105.
4. $7 . 41$.	**20.** $2^2 . 3^2 . 7 . 37$.	**35.** 5.	**50.** 108.
5. $2 . 5^4$.	**21.** $3 . 13 . 37$.	**36.** 13.	**51.** 77.
6. $3^4 . 11$.	**22.** $3 . 11 . 13 . 17$.	**37.** 13.	**52.** 98.
7. $7 . 2^4 . 13$.	**23.** $7 . 11 . 101$.	**38.** 17.	**53.** 39.
8. $2 . 5^2 . 13$.	**24.** $3^3 . 5 . 7 . 11$.	**39.** 7.	**54.** 52.
9. $3^2 . 11^2$.	**25.** $2^3 . 3 . 7^2 . 13$.	**40.** 14.	**55.** 126.
10. $2 . 3^6$.	**26.** $2^3 . 7 . 11 . 13 . 19$.	**41.** 21.	**56.** 385.
11. $2 . 13 . 17$.	**27.** $2^3 . 11^2 . 13$.	**42.** 28.	**57.** 264.
12. $2 . 3 . 5^2 . 7$.	**28.** $3 . 2^7 . 11^3$.	**43.** 22.	**58.** 2.
13. $2 . 3^3 . 5^3$.	**29.** $3^3 . 5 . 223$.	**44.** 24.	**59.** 6.
14. $3^2 . 7^2 . 11$.	**30.** $3 . 5 . 7 . 11 . 29$.	**45.** 33.	**60.** 2.
15. $3^2 . 5 . 7 . 11$.	**31.** $2^4 . 5 . 7 . 31$.	**46.** 81.	**61.** 35.
16. $3^2 . 7 . 11^2$.			

VI. a. (p. 56.)

1. 3.	**7.** 24.	**12.** 13.	**17.** 17.	**22.** $5 . 7^2$.
2. 4.	**8.** 7.	**13.** 14.	**18.** 5.	**23.** $3 . 7^2 . 11$.
3. 3.	**9.** 21.	**14.** 25.	**19.** 19.	**24.** $2 . 5^2 . 13$.
4. 7.	**10.** 25.	**15.** 49.	**20.** 17.	**25.** $5 . 7 . 13$.
5. 9.	**11.** 37.	**16.** 32.	**21.** $2 . 5^2$.	**26.** $2 . 29$.
6. 3.				

VI. b. (p. 56.)

1. 330.	**7.** 256.	**12.** 378.	**17.** 7.	**22.** 76923.
2. 99.	**8.** 6.	**13.** 315.	**18.** 2053.	**23.** 11.
3. 155.	**9.** 24.	**14.** 74.	**19.** 142857.	**24.** 91.
4. 315.	**10.** 13.	**15.** 1363.	**20.** 11.	**25.** 63.
5. 294.	**11.** 13.	**16.** 83.	**21.** 373.	**26.** 23.
6. 267.				

VI. c. (p. 58.)

1. 90.	**5.** 42.	**9.** 156.	**13.** 720.	**17.** 2160.
2. 400.	**6.** 102.	**10.** 222.	**14.** 1500.	**18.** 728.
3. 1800.	**7.** 144.	**11.** 60.	**15.** 336.	**19.** 720.
4. 30.	**8.** 180.	**12.** 2520.	**16.** 1260.	**20.** 1680.

21. 26950.
22. 1547.
23. $2.3^2.5^2.7$.
24. $2^5.5.7^2.17$.
25. $2^2.3.5.7.11$.
26. $5.7.11^3.13.101$.
27. $3.5.11.37$.
28. $7^2.11^2.13^2$.
29. $2.5.7.2053$.
30. $2.3^2.5.7.37$.
31. $2^6.3.5.7.13$.
32. $2^2.3.5.7.13.19.29$.
33. $2.3.7^2.11.13.23$.
34. $2^2.3.7^2.17.37$.

VI. d. (p. 59.)

1. 151.
2. 9.
3. 57.
4. 24 feet.
5. 784.
6. After 168 sec.
7. 85, 170, 255, 510.
8. 180 miles.
9. 5 pts.
10. At 99 ft.

VII. a. (p. 65.)

1. 21 sq. in.
2. 45 sq. ft.
3. 27 sq. yds.
4. 21 sq. ft.
5. 144 sq. in.; 1 sq. ft.
6. 144 sq. in.; 1 sq. ft.
7. 18 sq. ft.
8. 21 sq. ft.
9. 70 sq. dm.
10. 270 sq. dm.
11. 500 sq. cm.
12. 2400 sq. cm.
13. 630 sq. cm.
14. 350 sq. mm.
15. 115 sq. dm.
16. 34000 sq. m.
17. 26 sq. chains.
18. 400 sq. chains.
19. 9 ft.
20. 9 ft.
21. 18 ft.
22. 5 chains.
23. 25 dm.
24. 25 m.
25. 43 ft.
26. 19 m. 2 dm.
27. 26 m. 2 dm.
28. 60 ft. 2 in.
29. 11 ft. by 7 ft.
30. 15 ft. 6 in. by 11 ft. 3 in.
31. 400 sq. ft.
32. 540 sq. ft.
33. 54 sq. m.
34. 36 yds.

VII. b. (p. 66.)

1. 240 sq. ft.
2. 91 sq. ft.
3. 3 sq. ft. 54 sq. in.
4. 24 sq. yds. 8 sq. ft.
5. 60 sq. yds.
6. 60 sq. ft.
7. 29 sq. ft. 38 sq. in.
8. 10 sq. yds.
9. 364 sq. ft.
10. 199 sq. ft. 114 sq. in.
11. 209 sq. ft. 54 sq. in.
12. 22 sq. m.
13. 10 ac.
14. 21 ac.
15. 11 ft.
16. 328 sq. m.
17. 12 sq. m.
18. 3 hectares.
19. 1442 m.
20. 111 sq. ft.
21. 23200 sq. ft.; 360 yds.
22. 620 sq. ft.
23. 504 sq. ft.
24. 816 sq. ft.
25. 693 sq. ft.
26. 570 sq. ft.
27. £4. 18*s.*
28. 50 m.
29. 360 m.
30. £3. 5*s.* 4*d.*

VII. c. (p. 69.)

1. 343 c. ft.
2. 1 c. ft. 469 c. in.
3. 64 c. cm.
4. 40 c. dm.
5. 9 in.
6. 125.
7. 864.
8. 19 ft.
9. 936 sq. ft.
10. 5760.

VIII. a. (p. 72.)

1. 5*s*.
2. 3*d*.
3. 9 in.
4. 8*d*.
5. 4 in.
6. 10*d*.
7. 15*s*.
8. 9*d*.
9. 15 cwt.
10. 27 in.
11. $\frac{7}{12}$.
12. $\frac{1}{4}$.
13. $\frac{1}{6}$.
14. $\frac{5}{12}$.
15. $\frac{7}{11}$.
16. $\frac{3}{11}$.
17. $\frac{9}{11}$.
18. $\frac{2}{7}$.
19. $\frac{5}{7}$.
20. $\frac{9}{12}=\frac{3}{4}$
21. P, Q, R.
22. X, Y.
23. V, X, Q, Y, Z.
24. $\frac{1}{24}$.
25. 16.
26. 2.
27. 4.
28. $\frac{8}{24}$.
29. $\frac{16}{24}$.
30. $\frac{6}{24}$.
31. $\frac{18}{24}$.
32. $\frac{4}{24}$.
33. $\frac{20}{24}$.
34. $\frac{1}{2}$.
35. $\frac{1}{6}$.
36. $\frac{2}{3}$.
37. $\frac{2}{3}$.
38. $\frac{1}{3}$.
39. $\frac{1}{2}$.
40. $\frac{6}{25}$.
41. $\frac{3}{5}$.
42. $\frac{2}{6}$ or $\frac{1}{3}$.
43. $\frac{6}{30}$ or $\frac{1}{5}$.
44. $\frac{1}{2}$.
45. $\frac{1}{5}$.
46. $\frac{4}{25}$.
51. 2*s*. 6*d*.
52. 1*s*. 3*d*.
53. 6*s*. 8*d*.
54. 10*s*.
55. 8*s*.
56. 9*s*.
57. 7 in.
58. 16*s*. 8*d*.
59. 12 lb.
60. 5*s*.
61. 7*s*. 6*d*.
62. 13*s*. 4*d*.
63. 1*s*. 8*d*.
64. 7*s*. 6*d*.
65. 14*s*.
66. 4*s*.
67. £1.
68. 2*s*. 6*d*.
69. 8*s*.
70. 7*d*.
71. 2*s*. 11*d*.
72. 1*s*. 3*d*.
73. 1*s*. 3*d*.
74. 160 yds.
75. 1210 sq. yds.
76. 13*s*. 4*d*.
77. 10*s*.
78. 1 cwt. 1 qr.
79. 6*s*. $10\frac{1}{2}$*d*.
80. 2 cwt. 3 qrs. 27 lb.
81. £2. 18*s*.
82. 2 yds. 2 ft. 3 in.

VIII. b. (p. 74.)

1. $\frac{1}{2}$.
2. $\frac{2}{3}$.
3. $\frac{2}{3}$.
4. $\frac{1}{2}$.
5. $\frac{1}{3}$.
6. $\frac{4}{5}$.
7. $\frac{3}{4}$.
8. $\frac{3}{4}$.
9. $\frac{2}{3}$.
10. $\frac{2}{3}$.
11. $\frac{2}{5}$.
12. $\frac{3}{5}$.
13. $\frac{2}{5}$.
14. $\frac{1}{3}$.
15. $\frac{2}{5}$.
16. $\frac{3}{8}$.
17. $\frac{2}{3}$.
18. $\frac{7}{13}$.
19. $\frac{a}{b}$.
20. $\frac{c}{d}$.
21. $\frac{2}{3}$.
22. $\frac{3}{4}$.
23. $\frac{7}{8}$.
24. $\frac{8}{9}$.
25. $\frac{1}{5}$.
26. $\frac{3}{5}$.
27. $\frac{1}{2}$.
28. $\frac{4}{5}$.
29. $\frac{3}{4}$.
30. $\frac{5}{7}$.
31. $\frac{13}{14}$.
32. $\frac{35}{81}$.
33. $\frac{13}{17}$.
34. $\frac{25}{33}$.
35. $\frac{16}{35}$.
36. $\frac{13}{23}$.
37. $\frac{25}{68}$.
38. $\frac{35}{52}$.
39. $\frac{7}{11}$.
40. $\frac{5}{9}$.
41. $\frac{185}{203}$.
42. $\frac{4}{9}$.
43. $\frac{13}{15}$.
44. $\frac{16}{35}$.
45. $\frac{3}{4}$.
46. $\frac{5}{7}$.

VIII. c. (p. 76.)

1. 3.
2. 5.
3. 4.
4. $\frac{5}{5}$.
5. $\frac{10}{5}$.
6. $\frac{14}{7}$.
7. $\frac{33}{11}$.
8. $\frac{60}{12}$.
9. $\frac{42}{7}$.
10. $\frac{28}{7}$.
11. $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.
12. $\frac{16}{15}$.
13. 7.
14. 2.
15. $1\frac{5}{7}$

16. 5 and 6.
17. 3 and 4.
18. $2\frac{1}{3}$.
19. $1\frac{3}{4}$.
20. $3\frac{2}{3}$.
21. $2\frac{2}{5}$.
22. $3\frac{2}{7}$.
23. $3\frac{3}{8}$.
24. $1\frac{10}{13}$.
25. $13\frac{1}{3}$.
26. $3\frac{13}{15}$.
27. $5\frac{11}{17}$.
28. $13\frac{4}{7}$.
29. $1\frac{3}{13}$.
30. $3\frac{16}{113}$.
31. $1\frac{1}{3}$.
32. $\frac{5}{3}$.
33. $\frac{7}{4}$.
34. $\frac{12}{7}$.
35. $\frac{45}{34}$.
36. $\frac{11}{4}$.
37. $\frac{17}{6}$.
38. $\frac{7}{2}$.
39. $\frac{10}{3}$.
40. $\frac{83}{8}$.
41. $\frac{95}{9}$.
42. $\frac{25}{8}$.
43. $\frac{38}{9}$.
44. $\frac{89}{12}$.
45. $\frac{98}{11}$.
46. $\frac{149}{12}$.
47. $\frac{1917}{19}$.
48. $\frac{761}{130}$.
49. $\frac{445}{72}$.
50. $\frac{56}{5}$.
51. $\frac{573}{5}$.
52. $\frac{799}{8}$.
53. $\frac{3640}{401}$.

VIII. d. (p. 80.)

1. 2.
2. 2.
3. 3.
4. 4.
5. 5.
6. $\frac{4}{20}$.
7. $\frac{9}{12}$.
8. $\frac{4}{10}$.
9. $\frac{9}{24}$.
10. 1.
11. $\frac{6}{11}$.
12. $\frac{3}{4}$.
13. $\frac{3}{7}$.
14. $\frac{5}{8}$.
15. $\frac{5}{6}$.
16. $\frac{3}{4}$.
17. $\frac{5}{6}$.
18. $\frac{7}{12}$.
19. $1\frac{1}{6}$.
20. $\frac{1}{2}$.
21. $\frac{5}{6}$.
22. $1\frac{1}{2}$.
23. $1\frac{3}{20}$.
24. $\frac{5}{36}$.
25. $\frac{43}{60}$.
26. $\frac{7}{12}$.
27. $\frac{4}{11}$.
28. $\frac{1}{12}$.
29. $15\frac{7}{10}$.
30. $20\frac{7}{11}$.
31. $22\frac{8}{13}$.
32. $7\frac{1}{8}$.
33. $8\frac{3}{8}$.
34. $8\frac{5}{6}$.
35. $\frac{1}{6}$.
36. $1\frac{11}{12}$.
37. 1.
38. $\frac{37}{100}$.
39. $\frac{61}{100}$.
40. $\frac{137}{1000}$.

VIII. e. (p. 80.)

1. $\frac{48}{60}$.
2. $\frac{25}{90}$.
3. $\frac{8}{148}$.
4. $\frac{60}{114}$.
5. $\frac{33}{1001}$.
6. $\frac{142857}{999999}$.
7. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
8. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{10}$.
9. $\frac{11}{21}$, $\frac{5}{7}$, $\frac{8}{11}$, $\frac{11}{15}$.
10. $\frac{12}{17}$, $\frac{25}{34}$, $\frac{7}{9}$, $\frac{5}{6}$.
11. $\frac{17}{35}$, $\frac{6}{11}$, $\frac{4}{7}$, $\frac{3}{5}$.
12. $2\frac{3}{7}$.
13. $\frac{53}{56}$.
14. $1\frac{2}{3}$.
15. $2\frac{11}{84}$.
16. $2\frac{1}{2}$.
17. $\frac{41}{102}$.
18. $1\frac{49}{128}$.
19. $5\frac{23}{24}$.
20. $\frac{31}{32}$.
21. $6\frac{2}{3}$.
22. $10\frac{3}{4}$.
23. $1\frac{103}{144}$.
24. $3\frac{2}{35}$.
25. $5\frac{43}{50}$.
26. $51\frac{28}{45}$.
27. $17\frac{13}{120}$.
28. $9\frac{11}{15}$.
29. $1\frac{5}{6}$.
30. $5\frac{2}{5}$.
31. $6\frac{25}{42}$.
32. $9\frac{5}{36}$.
33. $\frac{1}{6}$.
34. $\frac{1}{2}$.
35. $1\frac{1}{4}$.
36. $3\frac{49}{52}$.
37. $\frac{87}{100}$.
38. $4\frac{1}{21}$.
39. $3\frac{20}{21}$.
40. $7\frac{7}{120}$.
41. $6\frac{13}{24}$.
42. $2\frac{3}{4}$.
43. $99\frac{99}{100}$.
44. $2\frac{109}{120}$.
45. $\frac{161}{170}$.
46. $\frac{1}{8}$.
47. $\frac{5}{48}$.
48. $7\frac{17}{24}$.
49. $1\frac{29}{56}$.
50. $\frac{1}{35}$.
51. $11\frac{13}{15}$.
52. $6\frac{1}{20}$.
53. $5\frac{49}{60}$.
54. $18\frac{13}{105}$.
55. $\frac{2}{3}$.
56. $\frac{23}{45}$.
57. $\frac{17}{32}$.
58. $\frac{3}{5}$.
59. $\frac{18}{25}$, $\frac{5}{7}$, $\frac{7}{10}$.
60. $\frac{17}{20}$, $\frac{13}{16}$, $\frac{4}{5}$, $\frac{3}{4}$.
61. $\frac{23}{26}$, $\frac{31}{39}$, $\frac{9}{13}$.
62. 6*s.* 8*d.*, 5*s.*, 3*s.* 4*d.*, 15*s.*
63. 15*s.*
64. $\frac{11}{24}$.

VIII. f. (p. 84.)

1. $1\frac{1}{2}$.
2. $10\frac{1}{2}$.
3. $4\frac{4}{5}$.
4. $4\frac{1}{10}$.
5. $\frac{3}{4}$.
6. $4\frac{1}{5}$.
7. $43\frac{4}{7}$.
8. $3\frac{4}{5}$.
9. $70\frac{3}{5}$.
10. $53\frac{3}{5}$.
11. $\frac{3}{20}$.
12. $\frac{1}{5}$.
13. $\frac{5}{7}$.
14. $\frac{1}{40}$.
15. $1\frac{1}{4}$.
16. $\frac{5}{11}$.
17. $\frac{1}{18}$.
18. $\frac{2}{9}$.
19. $\frac{1}{15}$.
20. $\frac{2}{5}$; 8 cwt.
21. $\frac{5}{6}$.

VIII. g. (p. 85.)

1. $\frac{15}{28}$.
2. $\frac{1}{6}$.
3. $\frac{8}{45}$.
4. $\frac{14}{93}$.
5. $\frac{15}{16}$.
6. $3\frac{5}{13}$.
7. $\frac{4}{15}$.
8. $\frac{1}{2}$.
9. $\frac{7}{25}$.
10. 2.
11. $\frac{1}{3}$.
12. $2\frac{11}{15}$.
13. $\frac{1}{13}$.
14. $\frac{1}{18}$.
15. 3.
16. $\frac{18}{145}$.
17. $2\frac{9}{11}$.
18. $1\frac{1}{3}$.
19. $4\frac{1}{6}$.
20. 1.
21. $4\frac{5}{7}$.
22. 4.

VIII. h. (p. 86.)

1. $1\frac{7}{18}$.
2. $\frac{3}{10}$.
3. $\frac{2}{21}$.
4. 9.
5. $\frac{10}{129}$.
6. $\frac{7}{43}$.
7. $2\frac{7}{10}$.
8. $\frac{2}{25}$.
9. $\frac{20}{21}$.
10. $\frac{7}{9}$.
11. $\frac{16}{45}$.
12. $3\frac{3}{20}$.
13. $7\frac{1}{5}$.
14. $2\frac{34}{39}$.
15. 16.
16. $\frac{225}{1024}$.
17. 4.
18. $\frac{8}{9}$.
19. $\frac{4}{5}$.
20. $3\frac{17}{21}$.
21. $\frac{13}{18}$.
22. $1\frac{1}{9}$.
23. $6\frac{13}{27}$.
24. $\frac{71}{143}$.
25. $\frac{95}{143}$.
26. $2\frac{1}{3}$.
27. $3\frac{7}{9}$.
28. $1\frac{1}{9}$.
29. $1\frac{5}{12}$.
30. $\frac{17}{18}$.
31. $1\frac{53}{72}$.
32. $43\frac{31}{48}$.
33. $7\frac{1}{24}$.

VIII. k. (p. 88.)

1. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$.
2. 6, 3, 2 hours.
3. $\frac{3}{4}$.
4. $1\frac{1}{5}$ hours.
5. $\frac{1}{4}$.
6. $\frac{1}{6}$.
7. 18 hours.
8. $\frac{1}{4}$.
9. 11.
10. 4 days.
11. $1\frac{1}{2}$ days.
12. $\frac{3}{4}$.
13. $\frac{1}{12}$.
14. $\frac{5}{8}$; $1\frac{3}{5}$ days.
15. 4 mm.
16. 3 days.
17. 12 days.
18. $\frac{7}{12}$.
19. $\frac{3}{40}$.
20. 20 min.
21. 4 hours.
22. $\frac{133}{720}$.
23. 4 min.
24. 12 days.
25. $\frac{8}{15}$.
26. 28 min.
27. 24 hours.
28. 84 min.
29. A 8, B 12, C 24 hours.
30. $15\frac{3}{4}$ hours.
31. $4\frac{2}{7}$ min.
32. 12 min.

VIII. l. (p. 92.)

1. $\frac{1}{2}$.
2. $\frac{3}{5}$.
3. $\frac{7}{16}$.
4. $\frac{1}{3}$.
5. $\frac{1}{4}$.
6. $\frac{7}{20}$.
7. $\frac{1}{3}$.
8. $\frac{1}{5}$.
9. $\frac{1}{4}$.
10. $\frac{1}{6}$.
11. $\frac{1}{7}$.
12. 7.
13. $\frac{3}{4}$.
14. $\frac{1}{3}$.
15. $\frac{4}{5}$.
16. $\frac{2}{7}$.
17. $\frac{3}{4}$.
18. $\frac{16}{17}$.
19. $5\frac{1}{3}$.
20. $\frac{1}{11}$.

21. $\frac{16}{21}$.
22. $\frac{2}{7}$.
23. $\frac{4}{17}$.
24. $2\frac{11}{14}$.
25. $\frac{7}{25}$.
26. 4.
27. $\frac{1}{2}$.
28. $\frac{101}{113}$.
29. 45.
30. $\frac{2}{5}$.
31. $\frac{59}{68}$.
32. $3\frac{1}{4}$.
33. $\frac{1}{3}$.
34. 4.
35. $13\frac{1}{3}$.
36. 9.
37. $1\frac{2}{5}$.
38. $\frac{1}{3}$.
39. $5\frac{5}{9}$.
40. $3\frac{3}{7}$.
41. $\frac{36}{893}$.
42. 3.
43. $\frac{2}{3}$.
44. $\frac{91}{124}$.
45. $1\frac{2}{3}$.
46. 0.
47. $\frac{1}{50}$.
48. 1.

VIII. m. (p. 95.)

1. 4*s.*
2. 2*s.* 6*d.*
3. 6*s.* 8*d.*
4. 3*s.* 4*d.*
5. 13*s.* 4*d.*
6. 16*s.* 8*d.*
7. £2. 5*s.*
8. 6*s.* 8*d.*
9. £10.
10. £28. 13*s.* 4*d.*
11. 12 cwt.
12. £69. 2*s.* $4\frac{2}{3}$*d.*
13. £28. 2*s.* 6*d.*
14. £33. 15*s.*
15. 10*s.*
16. 1 cwt. 2 qrs. 21 lb.
17. 42 weeks.
18. 505 yds. 1 ft. $0\frac{6}{7}$ in.
19. 5 hrs. 36 min.
20. $2\frac{1}{2}$ cwt.
21. 4 tons 1 cwt.
22. £28. 15*s.* 4*d.*
23. 92 sq. yds.
24. £5. 5*s.* $3\frac{3}{4}$*d.*
25. £8. 10*s.* $8\frac{17}{18}$*d.*
26. 4 hrs. 3 min.
27. 1 lb.
28. $27\frac{1}{2}$ yds.
29. 2 dm. 7 cm.
30. 7 Dg. 5 g.
31. 8 dm. 8 cm. 4 mm.
32. 1 g. 8 dg. 8 mg.
33. 9 fr. 65 c.
34. £1.
35. 125 chains 24 links.
36. 1 ac. 90 sq. yds.
37. 11*s.* $8\frac{3}{4}$*d.*

VIII. n. (p. 97.)

1. $\frac{1}{4}$.
2. $\frac{1}{6}$.
3. $\frac{1}{8}$.
4. $\frac{1}{10}$.
5. $\frac{1}{12}$.
6. $\frac{3}{20}$.
7. $\frac{1}{3}$.
8. $\frac{3}{8}$.
9. $\frac{2}{5}$.
10. $\frac{3}{5}$.
11. $\frac{3}{4}$.
12. $\frac{4}{5}$.
13. $\frac{1}{3}$.
14. $\frac{7}{12}$.
15. $\frac{5}{6}$.
16. $\frac{1}{8}$.
17. $\frac{7}{8}$.
18. $\frac{5}{16}$.
19. $\frac{1}{4}$.
20. $\frac{3}{4}$.
21. $\frac{2}{3}$.
22. $\frac{3}{4}$.
23. $\frac{1}{8}$.
24. $\frac{3}{8}$.
25. $\frac{1}{10}$.
26. $\frac{1}{4}$.
27. $\frac{1}{8}$.
28. $\frac{1}{40}$.
29. $\frac{1}{4}$.
30. $\frac{1}{6}$.
31. $\frac{2}{3}$.
32. $\frac{5}{8}$.
33. $\frac{7}{16}$.
34. $\frac{5}{16}$.
35. $\frac{9}{16}$.
36. $\frac{5}{6}$.
37. $\frac{15}{16}$.
38. $\frac{7}{24}$.
39. $\frac{29}{160}$.
40. $\frac{9}{125}$.
41. $\frac{5}{26}$.
42. $\frac{22}{27}$.
43. $\frac{188}{165}$.
44. $\frac{27}{160}$.
45. $\frac{3}{176}$.
46. $\frac{1}{120}$.
47. $\frac{17}{63}$.
48. $\frac{241}{2240}$.
49. $\frac{9}{16}$.
50. $\frac{15}{112}$.
51. $\frac{21}{31}$.
52. $\frac{18}{125}$.
53. $\frac{17}{80}$.
54. $\frac{30}{37}$.
55. $\frac{3}{5}$.
56. $\frac{1}{2}$.
57. $\frac{859}{720}$.
58. $\frac{49}{15}$.
59. $\frac{7}{240}$.
60. $\frac{247}{360}$.
61. $\frac{1}{5}$.
62. $\frac{63}{320}$.
63. $\frac{11}{12}$.
64. $\frac{11}{10}$.
65. $\frac{21}{40}$.
66. $\frac{105}{64}$.
67. $\frac{7}{240}$.
68. $\frac{8}{5}$.
69. $\frac{9}{16}$.
70. $\frac{3}{2}$.
71. $\frac{21}{110}$.
72. $\frac{13}{64}$.
73. $\frac{10}{3}$.
74. $\frac{11}{18}$.
75. $\frac{9}{200}$.
76. $\frac{64}{5}$.
77. $\frac{1}{14}$.
78. $\frac{39}{64}$.
79. $\frac{1}{8}$.
80. $\frac{81}{640}$.
81. $\frac{37}{56}$.
82. 13 cwt.

VIII. o. (p. 101.)

1. £1. 16s.
2. £350.
3. 70.
4. 100.
5. 70.
6. £10816.
7. 168 galls.
8. 80.
9. $19\frac{1}{2}$ miles.
10. £3.
11. 460 ac.
12. 414 ; £80. 10s.
13. 180.
14. £9760.
15. £235. 19s.
16. £1. 6s.
17. £1008.
18. 115.
19. 588.
20. £380.
21. 2 cm. 7 mm. ; 3 cm.
22. $\left.\begin{array}{l}\text{A } 21\frac{3}{5} \text{ days,}\\ \text{B } 48\frac{3}{4} \text{ days.}\end{array}\right\}$
23. $\left.\begin{array}{l}\text{56 miles per hour,}\\ \text{42 miles per hour.}\end{array}\right\}$

IX. REVISION PAPERS.

IX. a. (p. 102.)

1. $10\frac{94}{105}$.
2. $1\frac{3}{32}$.
3. 11s. $5\frac{7}{8}d$.
4. 1 cwt. 2 qrs. 21 lb.
5. $3\frac{13}{68}$.
6. $\frac{80}{81}$.
7. $\frac{274}{1219}$.

IX. b. (p. 102.)

1. $2^2.3.11.101$, $5^2.101$; H.C.F. 101.
2. $\frac{8}{15}$, $\frac{7}{13}$, $\frac{3}{5}$.
3. 96.
4. 1 : 25.
5. 4000.
6. 3 hrs.
7. 84 ft.

IX. c. (p. 103.)

1. 21.
2. 3 . 5 . 7 . 11.
3. $\frac{27}{35}$.
4. $\frac{38}{63}$.
5. £531. 13s. $8\frac{3}{4}d$.
6. $\frac{35}{44}$.
7. $\frac{2}{5}$.

IX. d. (p. 103.)

1. $13\frac{31}{88}$.
2. $8\frac{13}{20}$.
3. $7\frac{21}{64}$.
4. $7\frac{7}{8}$.
5. 10 days.
6. $\frac{89}{160}$.
7. 10s. ; 2d.

IX. e. (p. 103.)

1. $3.2^2.7.29$.
2. $\frac{31}{8}$; £9439. 10s.
3. $\frac{1}{6}$.
4. £120.
5. 4 min.
6. $\frac{7}{40}$.
7. £9. 18s.

IX. f. (p. 104.)

1. 325.
2. $6\frac{1}{4}$ tons.
3. $\frac{7}{90}$, $\frac{13}{15}$; $\frac{17}{18}$.
4. £10. 1s. 5d.
5. $4\frac{2}{21}$.
6. 35721.
7. $\frac{3}{7}$.

IX. g. (p. 104.)

1. 72 Km. 356 m.
2. 5460.
3. 1 ml. 3 fur. 4 yds.
4. 58.
5. $2\frac{1}{2}$.
6. £50. 13s. 11d.
7. £11700.

IX. h. (p. 104.)

1. 60. 4. £6000. 6. £4. 3s. $1\frac{1}{2}d$.
2. $2^3.3^2.7.13$, $2.3.5.7.11$; L.C.M. $2^3.3^2.5.7.11.13$.
3. $\frac{7}{10}$. 5. $\frac{24}{91}$. 7. 90 c. ft., 3 ft.

IX. k. (p. 105.)

1. 15. 2. 15s. $1\frac{1}{2}d$. 3. 1890. 4. 1. 5. $\frac{225}{418}$. 6. 12s. 6d. 7. 196 chains.

IX. l. (p. 105.)

1. 4d. 2. 125. 3. 19. 4. $\frac{2}{13}$. 5. 14 chains; £95. 14s. 6. 12960. 7. 7 min.

IX. m. (p. 105.)

1. 13. 2. 300. 3. £20. 18s. 6d. 4. $\frac{3}{4}$, $\frac{23}{30}$, $\frac{31}{40}$, $\frac{7}{9}$. 5. $\frac{16}{35}$. 6. 361. 7. £280.

IX. n. (p. 106.)

1. 2520, 2688, 2856. 2. $15\frac{13}{24}$. 3. $\frac{17}{182}$. 4. £336. 5. 137760. 6. $\frac{3}{4}$. 7. £4500.

IX. o. (p. 106.)

1. £30. 9s. 2. 525. 3. $2.3.7^2.11.13$. 4. 16. 5. £1. 5s. $8\frac{1}{2}d$. 6. 780, 468, 520. 7. £2. 4s. 6d.

IX. p. (p. 106.)

1. 24 sec. 2. $12\frac{2}{5}$. 3. $\frac{13}{15}d$.; 135. 4. 550. 5. $\frac{41}{96}$. 6. £112. 7. 51 fr. 75 c.

IX. q. (p. 107.)

1. 2 hrs. 56 min. 2. 159. 3. $2^2.3^2.7.23.29$. 4. $\frac{4}{15}$. 5. 12s. $9\frac{1}{2}d$. 6. £2660. 7. $8\frac{2}{11}$ days.

IX. r. (p. 107.)

1. 5, 7, 9. 2. 47. 3. £4. 17s. 6d. 4. $\frac{7}{30}$. 5. 3388 sq. yds. 6. 280. 7. $\frac{1}{35}$, $\frac{1}{56}$; $2\frac{1}{2}$ days.

IX. s. (p. 107.)

1. 5040. 2. £24. 10s. 3. $2^3.3^2.5.7.11.13$. 4. Greatest $\frac{31}{55}$, least $\frac{19}{35}$. 5. $\frac{3}{4}$. 6. 114 ac. 7. 8 days.

IX. t. (p. 108.)

1. $1972\frac{13}{14}$ tons.
3. $3^2.11.19$, $2^2.3.19^2$; H.C.F. 57.
4. 6*s.* 8*d.*
5. £21. 2*s.* 6*d.*
6. £200.
7. 200 galls.

IX. u. (p. 108.)

1. 7.13; 7.17; 13.17; 1553.
2. $\frac{17}{84}$; 17 : 16.
3. $\frac{3}{11}$.
4. 19*s.* 6*d.*
5. 1600.
6. £968.
7. 8000 sec.; 1680 quarts.

X. a. (*Oral.*) (p. 111.)

1. 4*d.*
2. $1\frac{1}{2}d.$
3. 7*s.* 6*d.*
4. £8.
5. 72 days.
6. 5 days.
7. £1.
8. £4.
9. $\frac{1}{2}d.$, 2*s.* 1*d.*, 5*d.*
10. 6 men, 21 men.
11. £10, £90.
12. 1*s.* 1*d.*, 6*s.* 6*d.*
13. 180 miles, 90 miles.
14. £2.
15. 7 days.
16. 5*s.* 10*d.*
17. 7*s.* 6*d.*
18. £3.
19. $9\frac{1}{2}d.$
20. 4*s.* 6*d.*
21. 10 days.
22. £5. 5*s.*, £10. 10*s.*
23. £10.
24. 5*s.*

X. b. (p. 112.)

1. 16*s.* 6*d.*
2. 4*s.*
3. £12. 10*s.*
4. 12 days.
5. £15.
6. £5. 12*s.*
7. £152.
8. 12*s.* $4\frac{1}{2}d.$
9. £6. 19*s.* 6*d.*
10. 200 miles.
11. 77.
12. 90 lb.
13. £1. 6*s.* 8*d.*
14. 32 days.
15. £325.
16. £19. 17*s.*
17. 29 ft. 2 in.
18. 16*s.* 6*d.*
19. £6. 6*s.*
20. 5 hours.
21. 11264 metres.
22. 16.
23. £14. 5*s.*
24. 7*s.* $10\frac{1}{2}d.$
25. £12. 3*s.*
26. £24. 7*s.* $1\frac{1}{2}d.$
27. 4 men.
28. £419. 15*s.*
29. $52\frac{1}{2}$ miles per hour.
30. $3\frac{1}{2}$ days.
31. £37. 3*s.* $1\frac{1}{2}d.$
32. 11*s.* 9*d.*
33. £5.
34. £25. 10*s.*
35. £1300.
36. £975.
37. £4.
38. 90 days.
39. 720.
40. £57.
41. £800.
42. 15*s.* $7\frac{1}{2}d.$
43. 9*d.*
44. £122. 10*s.*
45. $3\frac{3}{4}$ days.
46. 6 days.
47. £780.
48. 1*s.*
49. 2*s.*
50. 2 days.
51. 1*s.* 2*d.*
52. $3\frac{1}{2}$ days.
53. 2*s.* 6*d.*

XI. a. (*Oral.*) (p. 118.)

1. 3.	**27.** ·0104.	**43.** 7·30502.	**58.** ·407 m.	**73.** ·09 fr.
2. 4.	**28.** 6·7.	**44.** $\frac{7}{10}$.	**59.** 3·08 m.	**74.** £2. 10*s.*
3. 7.	**29.** 60·7.	**45.** $\frac{1}{5}$.	**60.** 3·789 m.	**75.** 4*s.*
14. 3·4.	**30.** ·00008.	**46.** $1\frac{3}{10}$.	**61.** ·084 m.	**76.** 10*s.*
15. 40·7.	**31.** 6·5.	**47.** $1\frac{3}{50}$.	**62.** $\frac{1}{2}$.	**77.** 14*s.*
16. 6·04.	**32.** 69·1.	**48.** $2\frac{7}{20}$.	**63.** $\frac{2}{5}$.	**78.** £0·1.
17. 100·63.	**33.** 53·45.	**49.** $3\frac{203}{500}$.	**64.** $\frac{1}{5}$.	**79.** £0·4.
18. ·304.	**34.** 639·4.	**50.** $\frac{1}{500}$.	**65.** $1\frac{1}{5}$.	**80.** £0·5.
19. ·0005.	**35.** 5·026.	**51.** $1\frac{7}{100}$.	**66.** $3\frac{1}{25}$.	**81.** £0·7.
20. ·86.	**36.** 6934·7.	**52.** $10\frac{203}{5000}$.	**67.** $\frac{1}{4}$.	**82.** £3·1.
21. ·806.	**37.** ·008294.	**53.** $506\frac{1}{125}$.	**68.** $\frac{3}{4}$.	**83.** £5·2.
22. ·0006.	**38.** ·063205.	**54.** ·05 m.	**69.** $1\frac{3}{5}$.	**84.** £3·5.
23. ·63.	**39.** 930·04.	**55.** ·7 m.	**70.** 3·25 fr.	**85.** 8*s.*
24. ·871.	**40.** ·0625.	**56.** ·008 m.	**71.** 2·09 fr.	**86.** 14*s.*
25. ·057.	**41.** ·000621.	**57.** ·053 m.	**72.** ·37 fr.	**87.** 32*s.*
26. ·009.	**42.** ·623578.			

XI. b. (*Oral.*) (p. 120.)

1. 3·66.	**13.** 1.	**25.** ·725.	**37.** ·5.	**49.** 1·8.
2. 4·5519.	**14.** 1.	**26.** ·324.	**38.** ·0009.	**50.** 1·97.
3. ·0898.	**15.** ·13.	**27.** 1·325.	**39.** ·5.	**51.** 4·89.
4. 7·7532.	**16.** 3.	**28.** 2·275.	**40.** 1 2.	**52.** 2·5 m.
5. 379·6081.	**17.** 6.	**29.** ·0101.	**41.** 1·8.	**53.** 2·5 m.
6. 256·5852.	**18.** 5.	**30.** 10·1.	**42.** ·02.	**54.** 6·7 m.
7. 1·4126.	**19.** 4.	**31.** ·0105.	**43.** ·7.	**55.** 8·65 m.
8. ·0112.	**20.** 4.	**32.** 2·62.	**44.** 7·5.	**56.** ·93 m.
9. ·9.	**21.** ·011.	**33.** ·729.	**45.** 9·3.	**57.** 23 dm.
10. ·8.	**22.** ·0011.	**34.** ·64.	**46.** 8·6.	**58.** 28 dm.
11. 1.	**23.** ·1.	**35.** 1·175.	**47.** 3·723.	**59.** 32·5 dm.
12. ·9.	**24.** ·22.	**36.** ·5.	**48.** 1·77.	**60.** 22·7 dm.

XI. c. (p. 121.)

1. 10·767.	**7.** 14.	**13.** £11.	**19.** 2·45.
2. 90·6541.	**8.** 1·01011.	**14.** £8. 8*s.*	**20.** 7·7784.
3. 738·454096.	**9.** 275·7968.	**15.** 6*s.*	**21.** 3·92645.
4. 158.	**10.** 314·9666.	**16.** £18. 10*s.*	**22.** 8·744.
5. ·1111.	**11.** 8·71 m.	**17.** 9*d.*	**23.** 712·3185.
6. 16.	**12.** 12·2 Km.	**18.** 14*s.* 2*d.*	**24.** $1\frac{1}{2}d$.

25. 7·245 shillings.
26. 7 m. 1 dm. 9 cm. 8 mm.
27. 4·765.
28. 0·1295.
29. 3·5 m.
30. 87 cm.
31. 4·386.
32. ·4964.
33. 16·7 dm. = 167 cm.
34. 2·6.
35. 2·1025.
36. 1.
37. 7·815.
38. 5·33.
39. 8s. 6d.
40. $3\frac{1}{2}d.$
41. 29·365 Km.
42. 1·1908 m.
43. 0·17486 Km.
44. 61·8 feet.
45. 156 m. = ·156 Km.

XI. d. *Oral.* (p. 124.)

1. 73·2.
2. 5·42.
3. ·35.
4. 703·2.
5. 56.
6. 530.
7. ·5.
8. 5.
9. 6305.
10. 83900.
11. 72350.
12. 62536·1.
13. 375892.
14. 356·78.
15. ·6.
16. 354.
17. 6000354.
18. 18005000.
19. ·75.
20. ·035.
21. 7·86.
22. ·00065.
23. ·10023.
24. ·356.
25. ·005.
26. ·507.
27. ·35429.
28. 638·96.
29. 52·7.
30. 3·24.
31. ·9625.
32. ·903.
33. ·07261.
34. 8·259604.
35. ·0002.
36. ·0602.
37. ·08256.
38. ·008289.
39. ·000814.
40. 6·2571.
41. ·69143.
42. ·00005.
43. $5·62 \times 10$.
44. $6·0135 \times 10^2$.
45. $1·1374 \times 10$.
46. $7·5 \div 10$.
47. $3·28 \div 10$.
48. $3·79 \div 10^2$.
49. $1·289 \div 10^2$.
50. $3·5 \div 10^3$.
51. $6·2835714 \times 10^5$.
52. $9·380502 \times 10^4$.
53. $6 \div 10^6$.
54. $8·375 \times 10^3$.
55. 35 m. 7 dm.
56. 357 m.
57. 3 dm. 5 cm. 7 mm.
58. 17 Km. 9 Dm.
59. 170 Km. 9 Hm.
60. 1 Hm. 7 Dm. 9 dm.
61. 3·407 m.
62. 34·07 m.
63. 340·7 m.
64. 34·07 dm.
65. 340·7 dm.
66. 3407 dm.
67. 5·006 m.
68. 50·06 m.
69. 500·6 m.
70. 50·06 dm.
71. 500·6 dm.
72. 5006 dm.
73. 5006 mm.
74. 50060 mm.

XI. e. *Oral.* (p. 125.)

1. 5·2.
2. 10·5.
3. 22·8.
4. 1·6.
5. ·18.
6. 1.
7. 2.
8. 6.
9. ·018.
10. 30·05.
11. 24·64.
12. ·039.
13. 3.
14. 1.
15. ·0025.
16. 497·14.
17. 126.
18. 10.
19. 25.
20. 540.
21. 2.
22. 12.
23. 10·8.
24. 9·6.
25. 13 m. 1 dm. 1 cm.
26. 35 Km.
27. 20 m. 1 dm. 6 cm.
28. 56 Km. 3 Hm. 5 Dm.
29. 17 francs.
30. 19 francs 50 centimes.
31. 63 fr. 49 centimes.
32. 105 francs.
33. 11 francs.

XI. f. (p. 127.)

1. 19·11.	**12.** ·0306098.	**23.** ·0096.	**33.** 43·5*d*.
2. 1311·345.	**13.** ·528.	**24.** ·0014.	**34.** 750*d*.
3. 8·32.	**14.** 3·27875.	**25.** ·0000169.	**35.** 300*d*.
4. ·51912.	**15.** ·0000185.	**26.** 4·5.	**36.** 41·25*s*.
5. ·78625.	**16.** ·991.	**27.** ·00001.	**37.** 20·52 cwt.
6. ·04185.	**17.** 3·29375.	**28.** ·000021045.	**38.** 322·08 dwt.
7. 4114·57.	**18.** 10·6463625.	**29.** 2.	**39.** 5341·6 yds.
8. 13238·125.	**19.** ·119922034.	**30.** 260.	**40.** 168·75 sq. ft.
9. ·00000625.	**20.** 183588·357.	**31.** 85.	**41.** 10·05 sq. ft.
10. ·000701454.	**21.** 1·44.	**42.** 156 sq. cm. = ·0156 sq. m.	
11. 4036·9.	**22.** 4·16.	**32.** 122·5*s*.	**43.** 1130·976 inches.

XI. g. *Oral.* (p. 128.)

1. ·06.	**6.** 1.	**11.** ·000008.	**16.** 49000.	**21.** ·2.
2. ·006.	**7.** 2·5.	**12.** ·027.	**17.** 8.	**22.** ·2.
3. ·0002.	**8.** 482.	**13.** ·000027.	**18.** 7.	**23.** ·3.
4. ·208.	**9.** 10.	**14.** ·000027.	**19.** ·0169.	**24.** ·03.
5. 2·5.	**10.** ·008.	**15.** ·5.	**20.** 1·69.	

XI. h. (*Oral.*) (p. 130.)

1. 4·16.	**10.** 0·313.	**19.** 0·012.	**27.** 42.	**35.** 1·25.
2. 2·801.	**11.** 0·28.	**20.** 0·014.	**28.** 1·25.	**36.** ·0125.
3. 1·951.	**12.** 0·126.	**21.** 0·0607.	**29.** 12·5.	**37.** 5.
4. 2·605.	**13.** 0·142.	**22.** 0·00125.	**30.** 125.	**38.** 50.
5. 18·004.	**14.** 0·042.	**23.** 0·0013.	**31.** 1·25.	**39.** 5000.
6. 252·625.	**15.** 0·2556.	**24.** 0·00027.	**32.** 0·125.	**40.** 30·17.
7. 0·63.	**16.** 0·091.	**25.** 62·8.	**33.** ·0125.	**41.** 1·4.
8. 0·45.	**17.** 0·093.	**26.** 4·52.	**34.** 0·125.	**42.** 14.
9. 0·671.	**18.** 0·138.			

XI. k. (p. 133.)

1. 4.	**5.** ·5.	**9.** 25.	**13.** 134·25.	**17.** 30000.
2. 1·6.	**6.** 2·5.	**10.** 1·6.	**14.** 100.	**18.** ·01.
3. 2·5.	**7.** 12·25.	**11.** ·1.	**15.** 100.	**19.** 500.
4. ·07.	**8.** 40.	**12.** 10.	**16.** ·63.	**20.** ·00031.

XI. l. (p. 133.)

1. 2·7.	**5.** 26·5.	**9.** 361500.	**13.** ·2305.	**17.** 3·01.
2. 5·4.	**6.** 26·1.	**10.** ·937.	**14.** ·0902.	**18.** 301.
3. 25.	**7.** 3·16.	**11.** ·00005.	**15.** 12.	**19.** 30100.
4. 301.	**8.** 3·24.	**12.** 427·5.	**16.** ·15.	**20.** ·00301.

21. 68·4.
22. 107·59.
23. ·00125.
24. 3279000.
25. ·000413.
26. 39000.
27. 13·629.
28. 2·645.
29. 154·776.
30. ·826.
31. 20·1028.
32. 7·875.
33. 1648·75.
34. 32·1388.
35. 1·2952.
36. ·060204.
37. 5 dm. 9 cm.
38. 5·675 Km. = 5 Km. 6 Hm. 7 Dm. 5 m.
39. 5 fr. 41 c.
40. 3·24 miles per hour.
41. 12672.
42. 2·539975.
43. 12·625 ft.
44. 10·75 inches.
45. ·004 inches.
46. ·914375 m.
47. 85.
48. 95·6 ft.
49. 1·252.
50. 11·15.
51. ·1216.
52. ·000061.
53. ·0307.
54. 36·87.
55. ·000578.
56. ·0013888.
57. ·0000027.
58. ·000457.
59. ·0000128.
60. ·1104.
61. 6·477.
62. ·02418.
63. 206·24.
64. 116·75.
65. 3·64.
66. ·002.
67. ·00048.
68. ·06864.
69. 1·0413.
70. 105.
71. 1·02.
72. ·3.
73. 25·24.
74. ·008775.
75. ·0625.
76. 160.
77. 2·1.
78. ·1503.
79. 7·53.

XI. m. (p. 137.)

1. ·5.
2. ·75.
3. ·625.
4. ·0625.
5. ·1015625.
6. 3·375.
7. 4·3125.
8. 125·04.
9. ·008.
10. ·0008.
11. ·0046875.
12. 3·01640625.
13. 3·125.
14. 17·046875.
15. ·02.
16. 19·04.
17. 19·0625.
18. 3·03125.
19. ·0234375.
20. 1·125.
21. 2·6.
22. 48·25.
23. 69·12.
24. 2167·96875.
25. ·222....
26. ·176....
27. ·159....
28. 19·419....
29. 4·846....
30. 5·023....
31. 6·571....
32. 9·523....
33. 15·379....
34. 17·204....
35. $6\frac{1}{4}$.
36. $3\frac{21}{200}$.
37. $14\frac{3}{250}$.
38. $16\frac{16}{125}$.
39. $17\frac{8}{25}$.
40. $3\frac{1}{16}$.
41. $23\frac{13}{25}$.
42. $\frac{25}{32}$.
43. $2\frac{1}{2}$.
44. $2\frac{1}{2}$.
45. $\frac{1}{15}$.
46. $17\frac{11}{17}$.
47. $2\frac{9}{20}$.
48. $\frac{7}{20}$.
49. 19.
50. $\frac{9}{11}$.
51. $\frac{4}{5}$.
52. 2·06.
53. 374·8.
54. 22·5.
55. 2·408.
56. 2·8.
57. ·0025.
58. ·015625.
59. 27·92.
60. ·88125.
61. ·0024.
62. 1168·375.
63. ·0000173.
64. 3·748.
65. 66·9128.
66. 1·486.
67. ·0979875.
68. 180·24.
69. 100·4.
70. 22·64.
71. 30·24.

XII. a. (p. 141.)

1. ·04.
2. ·633.
3. 7·672.
4. 9400.
5. 10300.
6. 961·073.
7. 961·1.
8. ·063.
9. 93602000.
10. 94000000.
11. 17·274.
12. 24·13.
13. 15.
14. 6·83.
15. 12·624.
16. 2·237.
17. 10·56.
18. ·090.
19. ·51.
20. 1974.
21. 9·286.
22. ·00028.
23. ·022.
24. 47 m. 94 cm.
25. £707.

26. £125.
27. 5 fr. 53 c.
28. 4536 grams.
29. 45359 grams.
30. 45·4 grams.
31. 39·37 in.
32. 3·3 ft.
33. 32·8 ft.
34. 328 ft.
35. 39370 in.
36. 3281 ft.
37. ·00057 miles.
38. ·009.
39. ·02.
40. ·0004.
41. ·116.
42. $\frac{1}{2500}=$ ·0004.
43. $\frac{6}{50000}=$ ·00012.
44. $\frac{21}{40000}=$ ·00053.
45. $\frac{42}{1000}=$ ·042.
46. $\frac{175}{8000}=$ ·022.
47. $\frac{121}{36000}=$ ·0034.

XII. b. (p. 144.)

1. 3·62.
2. 3·618.
3. 3·6176.
4. 5·33.
5. 5·73.
6. 6·82.
7. 6·8178.
8. 1·037.
9. 1·0367.
10. 4·138.
11. 4·1384.
12. ·1.
13. ·100.
14. ·10000.
15. ·628.
16. ·33.
17. ·3300.
18. ·7458.
19. ·067.
20. ·0133.
21. ·141.
22. 1·901.
23. ·032.
24. ·20988.
25. ·00139.
26. ·00040.

XII. c. (*Oral.*) (p. 147.)

1. 3.
2. -3.
3. 6.
4. -6.
5. $2a$.
6. $-2a$.
7. $-6a$.
8. $2a$.
9. $-8x$.
10. $-4x$.
11. $-10x$.
12. $-2x$.
13. $-6x$.
14. $-x$.
15. $-5x$.
16. $-2x$.
17. $-16x$.
18. 0.
19. 20 miles south from the starting point; 120 miles.
20. 3 miles west of the starting point; 43 miles.

XII. d. (p. 149.)

1. To one decimal place $\pm$ ·01.
2. ,, ,, $\pm$ ·01. $\frac{1}{1600}$.
3. ,, ,, $\pm$ ·015.
4. ,, ,, $\pm$ ·01.
5. ,, ,, $\pm$ ·01. $\frac{1}{6}$.
6. ,, ,, $\pm$ ·0015.
7. To two decimal places $\pm$ ·0015.
8. ,, ,, $\pm$ ·00015.
9. ·04 in., ·02 in., $\frac{1}{206}$, $\frac{1}{41}$.
10. To one decimal place. ·0018 approximately.
11. To two decimal places.
13. ·23.
14. 3280·87 feet.
15. ·914 m.

XII. e. (*Oral.*) (p. 153.)

1. £0·5.
2. £0·25.
3. £0·1.
4. £0·05.
5. £0·4.
6. £0·85.
7. £0·125.
8. £0·75.
9. ·25 of £2.
10. ·5 of £3.
11. ·75 of £4.
12. ·25 of 1 qr.
13. ·75 of a ton.
14. ·07 of a franc.
15. ·25 of 2 francs.

16. ·25 of 10*s*.
17. ·375 of £1.
18. ·25 of 1 oz. Troy.
19. ·35 of 1 oz. Troy.
20. ·175 of 2 oz. Troy.
21. £1·05.
22. ·525 of £2.
23. 5*s*.
24. 15*s*.
25. 19*s*.
26. 12*s*. 6*d*.
27. 14*s*. 6*d*.
28. 1*s*. 6*d*.
29. 13*s*.
30. 8*s*.
31. 14*s*.
32. 8*s*. 6*d*.
33. 16*s*. 6*d*.

XII. f. (p. 154.)

1. £0·3875.
2. £0·2625.
3. £0·675.
4. £0·94375.
5. ·8625 of £2.
6. ·784375 of £2.
7. £0·465625.
8. £0·728125.
9. £2·709375.
10. ·939375 of £5.
11. £0·790625.
12. 1·834375 of £2.
13. ·58984375 of £4.
14. £0·303125.
15. ·7796875 of £10.
16. ·63265625 of £20.
17. ·55 of £25.
18. ·8 of 13*s*. 9*d*.
19. ·125 of £1. 4*s*. 10*d*.
20. ·75 of £1. 6*s*. 8*d*.
21. ·0625 of £4. 2*s*.
22. ·03125 of £1. 16*s*. 8*d*.
23. ·08 of £13. 2*s*. 6*d*.
24. ·125 of a ton.
25. ·3125 of a ton.
26. ·625 of 3 tons.
27. ·015625 of a ton.
28. ·0015625 of a lb. Troy.
29. ·0078125 of a lb. Troy.
30. ·053125 of a mile.
31. ·71875 of a mile.
32. ·75 of a guinea.
33. ·846 of £1.
34. ·333 of 10*s*.
35. ·782 of £1.
36. 2·177 of £1.
37. ·669 of £5.
38. ·732 of £2. 10*s*.
39. ·476 of £3. 10*s*.
40. ·675 of £5. 10*s*. 4*d*.
41. ·528 of £1.
42. ·365 of £10. 6*s*.
43. ·080 of £10.
44. ·190 of a mile.
45. ·002 of a mile.
46. 13*s*. 6*d*.
47. 6*s*. 9*d*.
48. 5*s*. 3*d*.
49. 14*s*. $6\frac{3}{4}$*d*.
50. £3. 13*s*. $4\frac{1}{2}$*d*.
51. 1*s*.
52. £9. 4*s*. 6*d*.
53. 3*s*. 9*d*.
54. £3. 11*s*. 9*d*.
55. £1. 4*s*. $4\frac{1}{2}$*d*.
56. £1. 10*s*. 10*d*.
57. £7. 12*s*. 1*d*.
58. 12 cwt. 2 qrs.
59. 1 cwt. 2 qrs. 14 lb.
60. 4 lb. 3 oz.
61. 6*s*. 6*d*.
62. £1. 10*s*. $11\frac{1}{2}$*d*.
63. £2. 13*s*. $6\frac{1}{2}$*d*.
64. £2. 4*s*. 5*d*.
65. £2. 15*s*. 3*d*.
66. 3 yds. 1 ft. 7 in.
67. 11*s*.
68. 11*s*.
69. £1.
70. £7.
71. £1. 11*s*. 3*d*.
72. £1. 5*s*.
73. 13*s*.
74. 15*s*.
75. 1 cwt.
76. 9*s*.
77. 6*s*. 10*d*.

XII. g. (*Oral.*) (p. 156.)

1. 14*s*.
2. 10*s*.
3. 18*s*.
4. 1*s*.
5. 3*s*.
6. 15*s*.
7. 9*s*.
8. 19*s*.
9. 7*s*.
10. 12*s*. 6*d*.
11. 4*s*. 6*d*.
12. 14*s*. 6*d*.
13. 5*s*. 6*d*.
14. 8*s*. 6*d*.
15. 11*s*.
16. 3*s*. 6*d*.
17. 19*s*. 6*d*.
18. 16*s*. 6*d*.
19. £0·2.
20. £0·35.
21. £0·45.
22. £0·125.
23. £0·225.
24. £0·425.
25. £0·175.
26. £0·275.
27. £0·475.
28. £0·625.
29. £0·825.
30. £0·675.
31. £0·775.
32. £0·875.
33. £0·975.

XII. h. (*Oral.*) (p. 157.)

1. £0·019.
2. £0·028.
3. £0·038.
4. £0·041.
5. £0·044.
6. £0·023.
7. £0·049.
8. £0·009.
9. £0·117.
10. £0·167.
11. £0·525.
12. £0·221.
13. £0·408.
14. £0·392.
15. £0·333.
16. £0·071.
17. £0·319.
18. £0·169.
19. £0·264.
20. £0·244.
21. £0·480.
22. £0·613.
23. £0·420.
24. £0·665.
25. £0·734.
26. £0·561.
27. £0·498.
28. £0·678.
29. £0·872.
30. £0·790.
31. £0·077.
32. £0·493.
33. £0·943.
34. £0·999.
35. $6\frac{1}{2}d$.
36. $9\frac{1}{4}d$.
37. $3\frac{1}{2}d$.
38. 1*s*. $3\frac{1}{2}d$.
39. $11\frac{1}{4}d$.
40. 1*s*. $9\frac{1}{4}d$.
41. 10*s*. $6\frac{3}{4}d$.
42. 6*s*. $3\frac{1}{4}d$.
43. 4*s*. 3*d*.
44. 6*s*. $3\frac{1}{4}d$.
45. 14*s*. $6\frac{1}{4}d$.
46. 18*s*. $10\frac{3}{4}d$.
47. 14*s*. 1*d*.
48. 16*s*. $10\frac{3}{4}d$.
49. 8*s*. $7\frac{3}{4}d$.
50. 1*s*. $2\frac{1}{2}d$.
51. 1*s*. $7\frac{3}{4}d$.
52. 1*s*. $9\frac{1}{2}d$.
53. $4\frac{3}{4}d$.
54. 1*s*. $4\frac{3}{4}d$.
55. 3*s*. $7\frac{1}{4}d$.
56. 4*s*. $9\frac{1}{2}d$.
57. 7*s*. $4\frac{3}{4}d$.
58. 15*s*. $9\frac{1}{2}d$.
59. 3*s*. $1\frac{1}{2}d$.
60. 5*s*. $3\frac{1}{4}d$.
61. 6*s*. 2*d*.
62. 1*s*. $4\frac{1}{4}d$.
63. 7*s*. $8\frac{1}{2}d$.
64. 4*s*. $5\frac{1}{2}d$.
65. 16*s*. $11\frac{1}{4}d$.
66. 6*s*. $5\frac{1}{4}d$.
67. 2*s*. 4*d*.
68. 13*s*. $11\frac{1}{2}d$.
69. 19*s*. 10*d*.
70. 19*s*. $11\frac{3}{4}d$.

XII. k. (p. 158.)

1. 2·1 inches.
2. 3·9 grammes.
3. 616 yds.
4. 568·46 sq. m.
5. 4·7244816 in.
6. 3·024 lb.
7. $13\frac{1}{2}$ hours.
8. 149 lb. per sq. in.
9. 3·308 hours, 3 hours, $3\frac{1}{2}$ hours.
10. £1. 1*s*. 10*d*. = £1·092.
11. 7·92 feet.
12. 298 ; 0·11 ft.
13. 1·6.
14. 5 sec.
15. 0·11 mm.
16. 10 yds. 2 ft.
17. ·00234375 of a mile.
18. £3. 3 fl. 6·25 cents.
19. £519. 4*s*.
20. 173·6875.
21. 929·03 sq. cm.
22. 43·32 sq. m. = 4332 sq. dm.
23. 4·9375 cwt. £13. 4*s*. 2*d*.
24. 8 cm. ; $\frac{2}{673}$.
25. 40 lengths ; 23 cm. remainder.
26. 14 ft. ; 373 revolutions.
27. 6 min. 16 sec.
28. 420 feet.
29. 7 sq. ft.
30. 785 lb.
31. 3·2375 tons. £3. 12*s*. 10*d*.
32. 39·65 miles ; 24 miles per hour.
33. 5·9 miles per hour.
34. ·0063 ft.
35. ·0762.
36. The former is the more valuable by 1*s*. 4*d*.

XII. l. (p. 161.)

1. 66·125.	5. ·678.	8. ·167.	11. ·35.	14. 1·81.
2. 78·213.	6. 13·462.	9. ·718.	12. 74·90.	15. 2·90.
3. 35·490.	7. ·200.	10. ·454.	13. 19·91.	16. 1·80.
4. ·075.				

XII. m. (p. 163.)

1. 29·15.	5. 28·132.	9. 24·6907.	13. 42711000.	17. 31·6.
2. ·153.	6. 23·32.	10. ·43.	14. 2180 millions.	18. ·00316.
3. 9·012.	7. 2·705.	11. 3·021.	15. 309.	19. 20·93.
4. 2·9493.	8. 1·9669.	12. 19·78.	16. 4072.	20. 384·613.

XII. n. (p. 166.)

1. ·087006.	7. ·000517.	13. ·857.	18. 1·894.	23. ·00062.
2. ·2796.	8. 23·96.	14. ·00184.	19. 27·4116.	24. ·000053.
3. 1·709.	9. ·0003.	15. ·537.	20. 14·662.	25. ·046.
4. 8·83253.	10. 6·074.	16. 4·655.	21. 10·0342.	26. ·00097.
5. 56·313.	11. 29·09.	17. ·144.	22. ·235.	27. ·0032.
6. ·1311.	12. 25·557.			

XIII. (p. 168.)

1. 2 days. 7. 45 days. 11. On Wednesday.
2. $16\frac{1}{2}$ min. from the time when the first tap was opened.
3. $\frac{21}{22}$ of a day. 8. 16*s*. 12. 4 days.
4. $8\frac{2}{3}$ days. 9. A $14\frac{2}{5}$ days, B 24 days, C 18 days. 13. 50 minutes.
5. 12 min. after the taps are turned off. 14. $5\frac{395}{551}$ min. (=5·72 nearly).
6. 18 days. 10. 12 min. from the time when the first tap was opened.

XIV. a. (p. 171.)

1. 27.	9. 54.	17. 5027.	25. 16.	33. ·5.	41. 29.
2. 36.	10. 127.	18. 1001.	26. 35.	34. 1·6.	42. 31.
3. 45.	11. 341.	19. 5230.	27. 105.	35. ·1.	43. 73.
4. 72.	12. 623.	20. 3089.	28. 33.	36. ·04.	44. 43.
5. 25.	13. 201.	21. 5.	29. 22.	37. ·35.	45. 1·1.
6. 35.	14. 3023.	22. 9.	30. 63.	38. 5·4.	46. ·9.
7. 105.	15. 3509.	23. 8.	31. 55.	39. 17.	47. ·012.
8. 42.	16. 8231.	24. 15.	32. 54.	40. 19.	48. ·07.

XIV. b. (p. 174.)

1. 4·26.	5. 150·7.	9. ·0054.	13. ·02.	17. ·13.
2. 2·05.	6. 62·3.	10. ·0035.	14. ·063.	18. ·041.
3. 1·602.	7. 50·27.	11. 1·001.	15. ·05.	19. ·012.
4. 13·75.	8. ·201.	12. ·0105.	16. ·016.	20. ·012.

21. 4·796.	**24.** 89·11.	**27.** ·018.	**30.** $13\frac{1}{2}$.	**33.** $18\frac{3}{11}$.
22. 26·29.	**25.** 28·18.	**28.** $2\frac{1}{2}$.	**31.** $3\frac{4}{7}$.	**34.** $24\frac{23}{25}$.
23. 8·31.	**26.** ·058.	**29.** $3\frac{1}{4}$.	**32.** $15\frac{7}{8}$.	**35.** $37\frac{2}{27}$.

XIV. c. (p. 176.)

1. 27 sq. ft.	**4.** 35 sq. yds.	**7.** $59\frac{1}{2}$ yds.
2. 1 sq. ft. 99 sq. in.	**5.** 440 sq. yds.	**8.** $1\frac{1}{8}$ ac.
3. ·825 ac.	**6.** 76 yds.	

XIV. d. (p. 179.)

1. 7·07 in.	**3.** 7·071 ft.; 7 ft. 1 in.	**5.** 10 ft. 7 in.
2. 7 chains 62 links.	**4.** 16 ft.	**6.** $4\frac{1}{2}$ sq. ft.

XIV. e. (p. 180.)

1. ·577.	**5.** 4·236.	**9.** 3·732.	**13.** ·101.	**17.** 1·932.
2. ·447.	**6.** 2·732.	**10.** ·072.	**14.** 4·160.	**18.** 3·732.
3. ·756.	**7.** ·944.	**11.** 17·944.	**15.** ·295.	**19.** ·318.
4. 2·414.	**8.** ·172.	**12.** 15·937.	**16.** 1·618.	**20.** ·697.

XV. a. (p. 183.)

1. £2. 8*s.*	**11.** £31. 19*s.*	**21.** £39. 5*s.*	**31.** £19. 13*s.* 4*d.*
2. £2. 8*s.* 6*d.*	**12.** £7.	**22.** £12. 12*s.* 6*d.*	**32.** £164.
3. £2. 9*s.* 6*d.*	**13.** £7. 10*s.*	**23.** £50. 17*s.* 6*d.*	**33.** £46.
4. £3. 7*s.* 6*d.*	**14.** £7. 2*s.*	**24.** £39. 7*s.* 6*d.*	**34.** £48. 13*s.* 4*d.*
5. £1. 1*s.*	**15.** £7. 18*s.*	**25.** £52.	**35.** £80.
6. £1. 1*s.* 3*d.*	**16.** £23.	**26.** £107. 6*s.* 8*d.*	**36.** £80. 1*s.* 8*d.*
7. £2. 6*s.* 3*d.*	**17.** £31. 10*s.*	**27.** £44. 13*s.* 4*d.*	**37.** £13. 3*s.* 4*d.*
8. £3. 5*s.* 6*d.*	**18.** £79. 15*s.*	**28.** £54.	**38.** £15. 8*s.* 4*d.*
9. £5.	**19.** £16.	**29.** £21. 16*s.* 8*d.*	**39.** £11. 11*s.* 8*d.*
10. £7. 10*s.*	**20.** £16. 2*s.* 6*d.*	**30.** £86. 10*s.*	**40.** £9. 18*s.* 4*d.*

XV. b. (p. 184.)

1. £43. 17*s.* 6*d.*	**9.** £776. 19*s.* $3\frac{3}{4}d.$	**17.** £11113. 12*s.* $10\frac{1}{2}d.$
2. £218. 5*s.*	**10.** £12387. 10*s.*	**18.** £14407. 19*s.* 5*d.*
3. £230. 12*s.* 6*d.*	**11.** £9880.	**19.** £2984. 18*s.* $8\frac{1}{2}d.$
4. £368. 17*s.* 6*d.*	**12.** £2922. 12*s.* 6*d.*	**20.** £32316. 8*s.* $4\frac{1}{2}d.$
5. £125. 6*s.* 8*d.*	**13.** £16521. 3*s.* $4\frac{1}{2}d.$	**21.** £19832. 16*s.* 8*d.*
6. £637. 15*s.* 9*d.*	**14.** £46903. 4*s.* 9*d.*	**22.** £19045. 11*s.* $10\frac{1}{4}d.$
7. £401. 14*s.* $4\frac{1}{2}d.$	**15.** £51806. 5*s.*	**23.** £12626.
8. £484. 10*s.*	**16.** £73072. 2*s.* 6*d.*	**24.** £26422. 2*s.* $11\frac{1}{4}d.$

25. £29724. 11*s*. 6*d*.
26. £1782. 17*s*. $10\frac{1}{4}d$.
27. £1879. 13*s*. 3*d*.
28. £5466. 19*s*. 3*d*.
29. £368. 7*s*. 4*d*.
30. £3956. 6*s*. 9*d*.
31. £2504. 6*s*. 5*d*.
32. £5928. 10*s*.
33. £16934. 3*s*. $10\frac{1}{2}d$.
34. £1275. 11*s*. 6*d*.
35. £1553. 18*s*. $7\frac{1}{2}d$.
36. £33042. 6*s*. 9*d*.
37. £23451. 12*s*. $4\frac{1}{2}d$.
38. £5969. 17*s*. 5*d*.

XV. c. (p. 185.)

1. £12. 15*s*. 11*d*.
2. £11. 18*s*. 3*d*.
3. £15. 8*s*.
4. £26. 6*s*. 5*d*.
5. £72. 1*s*. 1*d*.
6. £4. 9*s*. 5*d*.
7. £730. 1*s*. 7*d*.
8. £2. 15*s*. 11*d*.
9. £142. 0*s*. 10*d*.
10. £222. 11*s*. 10*d*.
11. 8 m. 9 dm. 4 cm.
12. 390·525 kilos.
13. £46. 0*s*. 11*d*.
14. £35. 5*s*. 5*d*.
15. £17. 18*s*. 9*d*.
16. £13552. 1*s*.
17. £4037.
18. £5939. 1*s*. 3*d*.
19. £741. 17*s*.
20. £72187. 10*s*.

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XVI. a. (p. 186.)

1. £9453. 2*s*. 6*d*.
2. 15 centimes.
3. (1) $\frac{11}{91}$, (2) $\frac{40}{47}$.
4. ·8006, 1·4.
5. 2·71875.; 17*s*. $7\frac{1}{2}d$.
6. £18. 8s. 0d.
7. £2410. 1*s*. 6*d*.

XVI. b. (p. 187.)

1. 8640.
2. 15*s*. $1\frac{1}{2}d$.
3. (1) 2, (2) $\frac{3}{4}$.
4. (1) 2·4, (2) 17*s*. $6\frac{1}{4}d$.
5. 19*s*. 9*d*.
6. £7. 18*s*. 3*d*.
7. 1001.

XVI. c. (p. 187.)

1. 931475, 968734.
2. 4 fr. 70 centimes.
3. (1) 2, (2) $1\frac{1}{9}$.
4. The set of 9 hens.
5. ·042024, 10·82.
6. ·87222.
7. 295·8 Km., 382·8 Km.

XVI. d. (p. 188.)

1. 2520, 2688, 2856.
2. (1) $\frac{1}{105}$, (2) $\frac{5}{76}$.
3. 31240, ·6671875.
4. £62. 10*s*.
5. 6*s*.
6. £44.
7. £153684.

XVI. e. (p. 188.)

1. 561 kilog.
2. (1) $1\frac{131}{135}$, (2) 0.
3. £3570.
4. 17·666, 2*s*. 4*d*.
5. 10*d*.
6. 14 men.
7. 220 yds., 70 yds.

XVI. f. (p. 188.)

1. 10*s*. 7*d*.
2. ·000008.
3. 31 lb. 4 oz. 16 dwt. 16 gr.
4. $\frac{9}{100}$, ·96510417.
5. £96.
6. £1. 18*s*. 8*d*., 5*s*. 4*d*.
7. £6. 10*s*.

XVI. g. (p. 189.)

1. 2·112 m.
2. £664. 1*s.* 3*d.*
3. 6 sq. yds. 6 sq. ft.
4. 6 hours.
5. 17*s.* 8*d.*, 1*s.* 7*d.*
6. £322. 10*s.*
7. 60 of each sort.

XVI. h. (p. 189.)

1. 42 miles.
2. (1) $\frac{47}{72}$, (2) $\frac{7}{16}$.
3. £2. 2*s.* 7*d.*
4. £61. 10*s.*
5. £4. 17*s.* 6*d.*
6. (1) 12·7143, (2) £10. 8*s.* 4*d.*
7. 193 feet.

XVI. k. (p. 190.)

1. 288.
2. $\frac{2}{9}$ of a gall. is the greater by $\frac{1}{72}$ of a quart.
3. 10*s.* 8*d.*
4. ·23968.
5. 1092 min.; 78, 84, 91 revolutions.
6. £7. 4*s.*
7. £14. 17*s.* 6*d.*

XVI. l. (p. 190.)

1. $\frac{47}{84}$*d.*, $7\frac{5}{6}$*d.*
2. (*a*) £2395, (*b*) £4. 6*s.* $7\frac{1}{2}$*d.*
3. ·107, ·64625.
4. 5 ft. 9 in.
5. $3\frac{1}{2}$ days.
6. 33 yds., £22. 7*s.*
7. £78. 15*s.*

XVI. m. (p. 190.)

1. 35.
2. He gained £8. 9*s.* 9*d.*
3. 105.
4. 25 min. 12 sec. from the start.
5. 102 yds. 6 in.
7. 48.

XVI. n. (p. 191.)

1. $8\frac{1}{2}$ sec.
2. £27. 2*s.* 1*d.*
3. 2*s.* 5*d.*
4. £421.
5. £126.
6. 462, 4·62.
7. 46 yds.

XVI. o. (p. 191.)

1. 19*s.* 2*d.*
2. 66 pieces; 12 cm. left over.
3. £1802. 3*s.* 1*d.*
4. £26. 19*s.* $0\frac{1}{2}$*d.*
5. 182 yds.
6. £17333. 6*s.* 8*d.*

XVI. p. (p. 192.)

1. $15\frac{2}{5}$.
2. 1·1960.
3. 554 fr. 75 centimes.
4. £30. 8*s.*
5. £7. 4*s.*
6. £1. 0*s.* $1\frac{1}{4}$*d.*
7. 24 men.

XVI. q. (p. 192.)

1. $3\frac{1}{4}$*d.*
2. 99 feet.
3. 40 times.
4. 1·37.
5. 1·549 sq. in.
6. £45. 16*s.* 8*d.*
7. £17. 14*s.* 3*d.*

XVI. r. (p. 193.)

1. ·3125.
3. $\frac{2}{3}$.
4. £1. 5*s*. 8*d*.
5. £8. 12*s*. 6*d*.
6. £1184.
7. 3675 sq. ft.

XVI. s. (p. 193.)

1. £1367. 10*s*.
2. 32·64772.
3. 5 min. 31 sec.
4. 96.
5. ·07725.
6. 6 m. 3 dm. 2 cm.
7. 16072, 16473.

XVI. t. (p. 194.)

1. 82, remainder 1·24 in.
2. £455. 14*s*.
3. 1·97.
4. £478. 10*s*.
5. 3 lb. 2 oz. 6 drs.
6. 1668·7 m.
7. 2004.

XVI. u. (p. 194.)

1. (*a*) 4·334375, (*b*) 5·184.
2. 20 feet.
3. 1611 m.
4. 34.
5. ·3140625 of a ton.
6. £839. 13*s*. $5\frac{1}{4}$*d*.
7. 2·78.

XVI. v. (p. 194.)

2. ·648698.
3. £4441.
4. 4*s*. 2*d*., 4*s*.
5. £9·86875, £9. 8 fl. 6 c. $8\frac{3}{4}$ f.
6. £1343. 6*s*. $10\frac{1}{2}$*d*.
7. One mile.

XVII. a. (p. 196.)

1. 178 sq. ft. 72 sq. in.
2. $9\frac{2}{5}$ ac.
3. £5. 4*s*. 2*d*.
4. 12*s*.
5. 80 sq. yds.; £10. 2*s*. 6*d*.
6. 423.
7. 121 yds.
8. 22 yds.
9. $82\frac{1}{2}$ ft., $27\frac{1}{2}$ ft.
10. 52 yds. 2 ft.; £9. 4*s*. 4*d*.
11. 10 rows; 270 tiles.
12. 170 sq. ft.
13. 98 sq. yds.
14. 8 ac. 3 ro. 21 sq. po.
15. £3. 15*s*.
16. £4. 12*s*. 7*d*.
17. 78408.
18. 2 ft. 1 in.
19. 17 sq. m. 8608 sq. cm.
20. 309·84 sq. ft.
21. 9*s*. 7*d*.
22. £7. 1*s*.
23. £16. 19*s*. 2*d*.
24. £4. 13*s*. 11*d*.
25. 18*s*. 11*d*.
26. 1325 sq. ft.
27. 3 sq. ft., 36 sq. in.
28. 10 sq. ft.
29. £1. 13*s*. 9*d*.
30. 11 pieces.
31. 15*s*. 10*d*.
32. £14. 1*s*. 8*d*.
33. 158 m.
34. 3*s*. 9*d*.
35. $14\frac{1}{2}$ sq. ft.

XVII. b. (p. 199.)

1. 512 c. ft.
2. 384 sq. ft.
3. 144 c. ft., 192 sq. ft.
4. 108440·28 c. cm., 0·10844028 c.m.
5. £1. 12*s*. 8*d*.
6. 80.
7. 100.
8. 10.
9. $14128\frac{1}{3}$ c. ft.
10. £4. 1*s*. 8*d*.
11. 126 c. in., 163 sq. in.
12. 16800 litres.
13. 44 c. ft. 1584 c. in.; £13. 2*s*. 6*d*.
14. 81 sq. cm.
15. 1452 c. cm.
16. 27.
17. 10·752 in.
18. $8\frac{4}{7}$ ft.
19. 4 ft. 8 in.

20. 6906·25 lb.
21. 1·6 ft.
22. 64800.
23. 26 ft. by 13 ft.
24. 51 tons.
25. 5000.
26. $3\frac{1}{5}$ ft.
27. 6 ft. 2 in.
28. 401 lb.
29. $2\frac{3}{7}$ ft.
30. 424286 tons.
31. 23000.
32. 2 c. ft. 1536 c. in.; 1560 lb.
33. 2·75 c. ft.
34. 128.
35. 216 c. in.
36. 19200.

XVII. c. (p. 204.)

1. 3·14 in.
2. 3 yds.
3. 98.
4. 720.
5. 10 feet.
6. 188,000 miles per sec.
7. 6 miles.
8. 14 sq. ft.
9. ·79 sq. in.
10. 64 sq. ft.
11. 7 sq. ft. 10 sq. in.
12. 12 yds. 1 ft.
13. 17·7 ft.
14. ·63 in.
15. 118 sq. ft.
16. 15·60 sq. ft.

XVIII. (p. 206.)

1. 46.
2. 199.
3. 56.
4. 88·6.
5. 11·54.
6. 2·51875.
7. 14 yrs. 4 months.
8. $20\frac{1}{6}$ miles.
9. 10 stone 13 lb.
10. 12 stone 3 lb.
11. 1·8 in.
12. 15·6.
13. (1) 11·9. (2) 7·6.
14. $15\frac{1}{2}$ miles per hour.
15. 1*s*. $11\frac{3}{4}d$.
16. 4*d*. per lb.
17. £573,458,000.
18. 157; 87 per cent.
19. £876,508.
20. £657,000.
21. 56, 26 gallons, 4*d*., 1*s*. 8*d*.
22. The *second* is the higher by about $4\frac{3}{4}d$. a day.
23. 51°.

XIX. a. (p. 208.)

1. 8*s*. 6*d*.
2. 6*s*. 8*d*.
3. 8 men.
4. £83. 6*s*. 8*d*.
5. 5 days.
6. £64. 10*s*. 4*d*.
7. £3264.
8. 22 days.
9. 54 men.
10. $\frac{9}{91}$ths.
11. £2. 1*s*. 3*d*.
12. 1016.
13. 4 men
14. 2*s*. $7\frac{1}{2}d$.
15. £156.
16. £201. 14*s*. 11*d*.
17. £27. 14*s*. 9*d*.
18. £294.
19. 16*s*.
20. 143 bushels.
21. £1. 16*s*. 6*d*.
22. £103. 16*s*. 8*d*.
23. £3. 18*s*. $5\frac{1}{2}d$.
24. 30 c. ft.
25. 14*s*. 9*d*.
26. £29. 1*s*. 3*d*.
27. 7*s*. 10*d*.
28. £863.
29. 106 miles.
30. $\frac{5}{16}$.
31. 4 extra.
32. $8\frac{1}{2}d$.
33. 14 days.
34. £22. 4*s*. 2*d*.
35. 9*s*. 3*d*.
36. 1*s*. 3*d*.
37. (1) 9*s*. 2*d*. (2) 11*s*.
38. £635. 10*s*.
39. £625.
40. 41.
41. £46. 4*s*. $10\frac{1}{2}d$.
42. 34 boys.
43. 1*s*. 11*d*.
44. £35. 8*s*. 6*d*.
45. $9\frac{1}{2}d$.
46. 49.
47. £5. 5*s*.
48. £2. 8*s*. 6*d*.
49. £18. 1*s*. 8*d*.
50. $\frac{1}{6}$, $\frac{5}{12}$.
51. 9.
52. £3. 14*s*.

XIX. b. (p. 212.)

1. 22 horses.
2. 15 men.
3. £26. 1*s*. 4*d*.
4. 14 men.
5. 2 days.
6. 480 miners.
7. £163. 6*s*. 8*d*.
8. 500 reams.
9. 175 men.
10. 47 tons 17 cwt. 66 lb.
11. 8 ft.
12. £873. 12*s*.
13. 8 weeks.
14. £83. 11*s*. 10*d*.
15. 2268 c. ft.
16. 660 yds.
17. $1\frac{1}{2}$ minutes.
18. 9 ft. 4 in.
19. 8 hours a day.
20. 13*s*. 4*d*.
21. $48\frac{3}{4}$ days.
22. $15\frac{1}{2}$ cwt.
23. 33 men.
24. £18.
25. $10\frac{1}{4}d$.
26. 9 men.
27. £25. 10*s*.
28. 720.
29. £666.
30. $42\frac{3}{7}$.
31. 15 days.
32. $2\frac{5}{11}$ c. ft.
33. 345.
34. 10 ft. 10 in.
35. 18 weeks.
36. £750.
37. £16.
38. 27 men.
39. 5 days.
40. 33*s*.
41. 40 men.

XX. a. (p. 218.)

5. $-2, -1{\cdot}5, 0{\cdot}5, 1{\cdot}2, 1{\cdot}8, 2{\cdot}7, 3{\cdot}1$.
7. (3, 3).
8. 6·7.
9. $y=3, 11, 19, 27, 35$.
10. $y={\cdot}4, 1{\cdot}6, 2{\cdot}8, 4, 5{\cdot}2, 6{\cdot}4$.
11. $y=-5, -2{\cdot}5, 0, 2{\cdot}5, 5$.
12. 1·4.
13. 1·41.
14. 1·7.
15. 1·73.
16. 33 feet.

XX. c. (p. 228.)

1. 4*s*. 5*d*., 2*s*. 6*d*., 1*s*. 10*d*., 34, 45, 58 oranges.
2. 2*s*. 8*d*., 3*s*. 7*d*., 7*s*. 7*d*., 9, 16, 37 lb.
3. 12, 18, 44 miles. 2, $2\frac{1}{2}$, $3\frac{3}{5}$ hours.
4. 40 francs. 33*s*. 6*d*.
5. 61 cm., 20 in.
6. 22, 45, 17 miles.
7. 16, 21·5, 36·5 approx.
8. In the first hour he walks 5 miles, in the next 2 hours he walks 5 miles. He then rests for half an hour, and in the next half hour walks 3 miles; in the next half hour he rests, and in the last half hour walks 3 miles.
9. 9, 10, 5 miles.
10. $14\frac{1}{2}$, 18 miles. 3 p.m., 2.30 p.m.
11. 15 miles away at 3 o'clock.
12. In 1·7 hours, 17 miles from P.
13. 5 miles. 20 min., 29 min.

XX. d. (p. 233.)

1. £4. 18*s*., £5. 14*s*.
2. 33·7° at 3 a.m., 36·7° at 9 p.m.
4. 4700 feet. 205°.
5. 1·3, 1·6, 1·9.
6. 90 lb., 280 lb.
7. 2000, 1450.
8. 217 million £.
9. 22,300 in 1898; 16,500 near the end of 1893.

XXI. a. (p. 236.)

1. $\frac{1}{3}$.
2. $\frac{3}{4}$.
3. $\frac{3}{7}$.
4. $\frac{1}{3}$.
5. $\frac{2}{3}$.
6. $\frac{3}{5}$.
7. $\frac{1}{5}$.
8. $\frac{3}{7}$.
9. $\frac{3}{4}$.
10. $\frac{1}{4}$.
11. $\frac{1}{6}$.
12. $\frac{5}{7}$

13. $\frac{10}{1}$. 16. $\frac{25}{9}$. 18. $\frac{5}{6}$. 20. $\frac{7}{50}$. 22 $\frac{4}{9}$. 24. $\frac{4}{5}$.
14. $\frac{4}{5}$. 17. $\frac{1}{5}$. 19. $\frac{3}{1}$. 21. $\frac{2}{1}$. 23. $\frac{6}{7}$. 25. $\frac{7}{1}$.
15. $\frac{1}{8}$.

XXI. b. (p. 236.)

1. $\frac{5}{8}$. 7. $\frac{2}{1}$. 13. $\frac{5}{4}$. 19. $\frac{3{\cdot}14}{1}$ in each case.
2. $\frac{1}{4}$. 8. $\frac{1}{240}$. 14. $\frac{27}{46}$. 20. 4·3, 4·4, 4·6, 4·9.
3. $\frac{12}{19}$. 9. $\frac{1}{2}$. 15. $\frac{10}{1}$. 21. 1·03. 24. 1·45.
4. $\frac{6}{5}$. 10. $\frac{7}{13}$. 16. $\frac{1}{6}$. 22. 10·5. 25. 153 lb.
5. $\frac{55}{63}$. 11. $\frac{11}{100}$. 17. $\frac{40}{99}$. 23. 13·5. 26. 1·4 oz.
6. $\frac{5}{1}$. 12. $\frac{1}{3}$. 18. $\frac{62}{75}$.

XXI. c. (p. 241.)

1. $x=1\frac{1}{3}$. 15. 7. 25. $x=2\frac{2}{3}$. 35. 2 : 7.
2. $x=1\frac{1}{2}$. 16. 33·8. 26. $x=21$. 36. 2 : 1.
3. $x=14$. 17. 9. 27. $x=2\frac{2}{3}$. 37. 4 : 3.
4. $x=10$. 18. 25. 28. $x=8$. 38. 8 : 1.
5. $x=2\frac{1}{2}$. 19. $\frac{1}{3}$. 29. $x=1\frac{1}{5}$. 39. 2 : 1.
6. $x=\frac{1}{4}$. 20. $\frac{3}{22}$. 30. $x=18$. 40. 2 : 1.
7. $x=18\frac{3}{4}$. 21. $x=3$. 31. 4 : 7. 41. 6.
8. $x=\frac{ac}{b}$. 22. $x=32$. 32. 4 : 7. 42. 3.
9. $x=\frac{ab}{c}$. 23. $x=\frac{1}{4}$. 33. 4 : 3. 43. $1\frac{1}{2}$.
10. $x=\frac{2}{5}$. 24. $x=1\frac{1}{5}$. 34. 2 : 1. 44. 2.
11. 10. 45. 35°, 60° Réaumur. 19°, 44° Centigrade.
12. $1\frac{2}{3}$. 46. 108°, 174° Fahr. 18°, 39° Centigrade.
13. $\frac{2}{3}$. 47. 113°, 138° Fahr. 2°, 70° Réaumur.
14. $13\frac{1}{2}$.

XXII. a. (p. 245.)

1. 279, 372. 12. £1. 18*s*. 4*d*., £2. 17*s*. 6*d*., £4. 6*s*. 3*d*.
2. 781, 355. 13. £2680, £1072, £670. 22. £7. 5*s*., £6. 0*s*. 10*d*.
3. 339, 452, 565. 14. £221, £468, £97. 10*s*. 23. 4*s*. 4*d*.
4. 555, 370. 15. 11·2 lb. 24. £175. 8*s*., £219. 5*s*., £328. 17*s*. 6*d*.
5. £1. 4*s*., £2. 8*s*., £3. 12*s*., £4. 16*s*. 25. £216. 13*s*. 6*d*.
6. £45, £20. 16. £63. 14*s*. 26. 39, 88.
7. £8. 7*s*. 6*d*., £26. 16*s*. 17. £124. 16*s*. 6*d*. 27. £351, £416.
8. £17. 10*s*., £21, £20. 18. £554. 16*s*. 3*d*. 28. £22. 10*s*., £18.
9. 6*s*. 8*d*., £1, £2. 6*s*. 8*d*. 19. £11. 11*s*., £5. 15*s*. 6*d*., £3. 17*s*.
10. £1, £1. 8*s*., 14*s*. 20. £3126. 13*s*. 4*d*., £1675.
11. £2. 11*s*., £1. 5*s*. 6*d*., 17*s*. 21. £643. 15*s*. 29. £2860.
30. A £154. 19*s*. 9*d*., B £120. 10*s*. 11*d*.

XXII. b. (p. 248.)

1. $4d.$ per lb.
2. £3. $2s.$
3. $3s.$ per lb.
4. $6s.$ $6d.$
5. $5d.$ per lb.
6. $1\frac{1}{4}d.$
7. 6 gallons.
8. 2 : 5.
9. One-third of the coin is alloy.
10. 18 lb. of the first kind, 6 lb. of the second.
11. $6s.$ per lb.
12. 1 : 3.
13. $11\frac{1}{4}$, 45, $63\frac{3}{4}$ pints.

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XXIII. a. (p. 249.)

1. 3 m. 93 cm.
2. £41. $12s.$
3. £4. $1s.$ $3\frac{1}{4}d.$
4. 4 min.
5. 24, 38, 59, 66.
6. 3 lb. of the first to 4 lb. of the second.
7. 19 min. past one.

XXIII. b. (p. 249.)

1. $\frac{5}{13}$, $\frac{8}{13}$, $\frac{3}{13}$. 14 ft. $7\frac{1}{2}$ in.
2. 193·12 sq. cm.
3. 45.
4. (1) ·143, (2) 19·673.
5. $18s.$ $9d.$
6. 47, 63, 69.
7. £63, £218. $8s.$, £138. $12s.$

XXIII. c. (p. 250.)

1. 47 sq. m. 54 sq. dm.
2. 6 dm.
3. 37.
4. 81 men.
5. £611. $9s.$ $4\frac{1}{2}d.$
6. 4.30 p.m.
7. £1050.

XXIII. d. (p. 250.)

1. 64·453125 tons.
2. £42. $7s.$ $11d.$
3. $15\frac{3}{4}$ min. to 12.
4. $11\frac{1}{4}$ miles.
5. £451. $10s.$
6. £4. $7s.$
7. A, £140. B, £145.

XXIII. e. (p. 251.)

1. 937. ·02268 in.
2. £360.
3. 32 mm.
4. 20 days.
5. £3. $2s.$ $2d.$
6. 50·803 Kg.
7. £960, £1440, £480.

XXIII. f. (p. 251.)

1. ·03125 of $10s.$, ·0025.
2. £2. $2s.$ $3\frac{1}{2}d.$
3. £186. $13s.$ $4d.$, £166. $13s.$ $4d.$, £146. $13s.$ $4d.$
4. $3\frac{3}{8}$ gallons.
5. ·8165.
6. $45\frac{1}{3}$ miles per hour.
7. (1) 20 miles. (2) 12.48 p.m., 3.24 p.m

XXIII. g. (p. 252.)

1. 393 lb. 10 oz.
2. ·946875 of £6.
3. 8 hours a day.
4. £9. $0s.$ $10d.$
5. 72095.
6. £40. $15s.$ $10d.$, £63. $0s.$ $10d.$
7. 135.

XXIII. h. (p. 252.)

1. ·76 litre.
2. 48 miles.
3. £96. 6*s*.
4. 90 Chinamen.
5. 7·746.
6. 15 miles per hour.
7. 13 c. ft. 1404 c. in. 16355 lb.

XXIII. k. (p. 253.)

1. $128\frac{1}{4}$ sq. ft.
2. $8\frac{1}{2}$ hours a day
3. ·2540 dm.
4. £1. 18*s*. 3*d*.
5. 6·4 in.
6. $7\frac{1}{2}d$.
7. 12 miles from the start.

XXIII. l. (p. 253.)

1. 335·18.
2. 2*s*. $9\frac{1}{4}d$.
3. £3. 15*s*. 4*d*.
4. 1 ton 14 cwt. 2 qr. 24 lb.
5. $10\frac{1}{2}$.
6. 13·13.
7. 1·8 c. in., 2·9 c. in., 52·5 c. cm.

XXIII. m. (p. 254.)

1. 2904 tons.
2. 2 cwt.
3. £5. 2*s*. $4\frac{1}{2}d$.
4. £1. 0*s*. 4*d*.
5. 24 secs., $6\frac{6}{19}$ secs.
6. 6 gallons.
7. 10 hours.

XXIII. n. (p. 254.)

1. ·265.
2. 1,136,073,600 millions.
3. $27\frac{1}{2}$ miles.
4. $42\frac{1}{2}$ miles.
5. 46.
6. ·037320.
7. ·3183, 1·1 in.

XXIII. o. (p. 255.)

1. £280.
2. 19·5, ·08125.
3. £2904. 4*s*.
4. £495. 10*s*.
5. 3 : 2.
6. £5. 13*s*. 4*d*., £36. 16*s*. 8*d*.
7. At 1.50.

XXIII. p. (p. 255.)

1. (1) $3\frac{3}{7}$. (2) 7*s*. $1\frac{1}{2}d$.
2. 20 lb. Troy.
3. £3. 13*s*. 4*d*.
4. 4918 rupees.
5. 10*d*.
6. 93 men.
7. 24·3 in.

XXIII. q. (p. 256.)

1. 770 times.
2. 343 c. ft.
3. 1060 grams.
4. 24, 40.
5. 93,000,000 miles. Possible error = 400,000 miles.
6. 55. The answer might be any number from 53 to 57.
7. £695. 13*s*. 3*d*.

XXIII. r. (p. 257.)

1. 21805.
2. £614. 15*s*. 4*d*.
3. Height 12 ft., length 16 ft.
4. 170·18 sq. in.
5. $\frac{48}{55}$ min.
6. 3*s*. $7\frac{3}{4}d$.
7. 166 ft. per min.

XXIII. s. (p. 257.)

1. 792·30661 ac.
2. 48 ft.
3. 15 hours.
4. £86. 13*s.* 4*d.*
5. 150 pupils.
6. 1127 c. in.
7. 514,000 gallons. $1\frac{1}{2}$ ft. per sec.

XXIII. t. (p. 258.)

1. £338.
2. £3. 16*s.*
3. £120.
4. £6911.
5. $83\frac{1}{3}$ c. cm.
6. $22\frac{1}{2}$ miles.
7. $5\frac{2}{3}$ hours.

XXIII. u. (p. 258.)

1. 1048.
2. 215 oz.
3. 5027 sq. yds.
4. ·000001.
5. 2*s.* $11\frac{1}{2}$*d.*
6. £70. 5*s.* 10*d.*
7. A's income £1305, B's £1044, C's £1160.

XXIII. v. (p. 259.)

1. 365.
2. £478. 4*s.*
3. 400 eggs.
4. £1. 0*s.* $0\frac{1}{4}$*d.*, $4\frac{1}{4}$*d.*
5. Between $25\frac{5}{7}$ and $27\frac{3}{11}$ miles per hour.
6. 8 kg. 262 g.
7. $54\frac{3}{8}$ lb.

XXIV. a. (p. 262.)

1. 40.
2. 40 %.
3. 15.
4. 15 %.
5. 2·6 %.
6. $\frac{1}{25}$.
7. $\frac{1}{10}$.
8. $\frac{1}{4}$.
9. $\frac{3}{5}$.
10. $\frac{1}{8}$.
11. $\frac{2}{5}$.
12. $\frac{1}{3}$.
13. $\frac{3}{4}$.
14. $\frac{1}{16}$.
15. $\frac{3}{8}$.
16. $\frac{7}{20}$.
17. $\frac{3}{40}$.
18. $\frac{2}{3}$.
19. 5 %.
20. 25 %.
21. 20 %.
22. $16\frac{2}{3}$ %.
23. $6\frac{1}{4}$ %.
24. $8\frac{1}{3}$ %.
25. 60 %.
26. ·24.
27. ·13.
28. ·03.
29. ·075.
30. ·3875.
31. $\frac{13}{10}$.
32. $\frac{23}{20}$.
33. 52.
34. 360.
35. 900.
36. 40.
37. 5 %.
38. 20 %.
39. $12\frac{1}{2}$ %.
40. $7\frac{1}{2}$ %.
41. £3·7, *i.e.* £3. 14*s.*
42. £40. 6*s.*
43. £61. 5*s.*
44. £176.
45. 51.
46. 3.
47. 9.
48. 2·17.
49. 2.
50. £12. 17*s.*
51. £3. 14*s.*
52. £1. 8*s.* 6*d.*
53. £1. 12*s.* 3*d.*
54. 2*s.* $1\frac{1}{2}$*d.*
55. 13*s.* 9*d.*
56. £3. 7*s.* 8*d.*
57. £2. 3*s.* $1\frac{1}{2}$*d.*
58. 18*s.* 3*d.*
59. 60.
60. 75.
61. 165.
62. 32 %.
63. 88.
64. 102.
65. 3000.
66. 93 : 100.
67. $\frac{117}{100}$.
68. 1250.
69. £17. 16*s.*
70. 5 %.
71. 600.
72. 12.

73. 68.
74. 120
75. 77 %.
76. 10 %.
77. 20 %.
78. £95.
79. 5 %.
80. £297.

XXIV. b. (p. 267.)

1. 3·33 %.
2. 12·5 %.
3. 9·375 %.
4. 16·67 %.
5. 9·02 %.
6. 3·55 %.
7. 1·52 %.
8. £18. 9s. 4d.
9. £1. 17s. 9d.
10. £6. 6s. 4d.
11. £6. 17s. 5d.
12. £87. 12s.
13. £68. 19s.
14. £3. 5s. $7\frac{1}{2}$d.
15. 527.
16. 5·36 %.
17. £20. 12s. 6d.
18. $1\frac{1}{2}$ %.
19. £222,118. 15s. 11d.
20. $2\frac{1}{2}$ %.
21. 34·63 %.
22. 57·16 %.
23. 12·16 %.
24. 11·09 %.
25. 5·23 %.
26. 13·06 %.
27. 8·26 %.
28. 13s. 6d.
29. 1200.
30. A 200, B 328
31. £24000.
32. £22000.
33. 60 %.
34. 60000.
35. 7 %.
36. 66·8 %.
37. 71·5 %.
38. 3·01 %.
39. 11·2 %.
40. 14·21 %.

XXV. a. (p. 272.)

1. £2.
2. £9.
3. £6.
4. £4.
5. £2.
6. £3.
7. £1.
8. 11s.
9. 2s.
10. £3.
11. £56.
12. £45.
13. £48.
14. £13.
15. £90.
16. £4. 10s.
17. £40. 15s.
18. £9. 15s. 6d.
19. £23. 11s. 6d.
20. £15. 12s.
21. £11. 14s.
22. £23. 8s. 10d.

XXV. b. (p. 273.)

1. £117. 10s.
2. £32. 16s.
3. £348. 9s.
4. £97. 4s.
5. £441.
6. £37. 9s. 3d.
7. £41. 15s.
8. £118. 3s. $4\frac{1}{2}$d.
9. £76. 12s. $1\frac{1}{2}$d.
10. £277. 5s. 4d.
11. £47. 14s. 3d.
12. £14. 18s. 1d.
13. £126. 3s. 6d.
14. £439. 11s. 2d.
15. £36. 1s. 5d.
16. £28. 2s. 4d.
17. £212. 17s.
18. £27. 7s.
19. £551. 19s. 2d.
20. £1484. 8s. 9d.
21. £3516. 16s. 4d.
22. £833. 9s. 10d.
23. £741. 5s. 11d.
24. £370. 15s.
25. £1995. 6s. 11d.
26. £205. 11s. 4d.
27. £270. 0s. 9d.
28. £926. 3s. 2d.
29. £75. 3s. 4d.
30. £917. 6s. 3d.
31. £225. 15s.
32. £3. 4s. 8d.
33. £3. 10s. 8d.
34. £3. 6s. 1d.
35. £14. 3s. 2d.
36. 15s. 2d.
37. £7. 7s. 7d.
38. £94. 19s. 6d.
39. £417. 3s.
40. £13. 12s. 9d.
41. £137. 5s. 6d.
42. £12. 0s. 10d.
43. £33. 9s. 7d.
44. £25. 10s. 5d.
45. £13. 18s.
46. £4. 1s. 8d.
47. £4. 18s. 3d.
48. £3. 15s. 9d.
49. £4. 10s.

XXV. c. (p. 277.)

1. 5 yrs.	**5.** 7 yrs.	**9.** 4 yrs.	**13.** 2 yrs.	**17.** 5 %.
2. 4 yrs.	**6.** 3 yrs.	**10.** 5 yrs.	**14.** 2 yrs.	**18.** 7 %.
3. 4 yrs.	**7.** 7 yrs.	**11.** 2 yrs.	**15.** $3\frac{1}{2}$ %.	**19.** 6 %.
4. 6 yrs.	**8.** 3 yrs.	**12.** 1 yr.	**16.** 12 %.	**20.** £113. 10*s.*

XXV. d. (p. 278.)

1. 2 yrs.	**13.** 4 %.	**25.** $5\frac{1}{2}$ %.	**37.** £165.
2. $4\frac{1}{4}$ %.	**14.** 2 yrs.	**26.** £3420. 18*s.* 9*d.*	**38.** £657.
3. 8 yrs.	**15.** £666. 13*s.* 4*d.*	**27.** $3\frac{1}{4}$ %.	**39.** £331. 17*s.* 6*d.*
4. £560.	**16.** $2\frac{1}{3}$ yrs.	**28.** £733. 6*s.* 8*d.*	**40.** $3\frac{3}{4}$ %.
5. 6 %.	**17.** 4 %.	**29.** 30 %.	**41.** $4\frac{1}{2}$ %.
6. £455.	**18.** £412. 10*s.*	**30.** £455.	**42.** £110. 10*s.*
7. $3\frac{1}{2}$ yrs.	**19.** $3\frac{1}{4}$ yrs.	**31.** $4\frac{1}{2}$ %.	**43.** $4\frac{1}{2}$ %.
8. 6 %.	**20.** £292.	**32.** £275.	**44.** £16. 16*s.* $10\frac{1}{2}d$
9. £450.	**21.** $12\frac{1}{2}$ %.	**33.** $8\frac{1}{3}$ %.	**45.** 3 years.
10. $12\frac{1}{2}$ yrs.	**22.** $4\frac{1}{2}$ %.	**34.** $3\frac{1}{2}$ %.	**46.** $3\frac{1}{8}$ %.
11. 5 %.	**23.** $2\frac{5}{6}$ yrs.	**35.** 25 years.	**47.** £1857.
12. £187. 10*s.*	**24.** £1275.	**36.** 150 days.	

XXV. e. (p. 285.)

1. £330. 15*s.*	**18.** £21. 1*s.* 6*d.*	**35.** £1183. 5*s.* 9*d.*
2. £811. 4*s.*	**19.** £216,653.	**36.** £800.
3. £926. 19*s.* 5*d.*	**20.** £499. 18*s.* 8*d.*	**37.** £6000.
4. £6945. 15*s.*	**21.** £439. 4*s.*	**38.** £175.
5. £424. 7*s.* 2*d.*	**22.** £25. 13*s.* 5*d.*	**39.** £160.
6. £1568. 6*s.* 5*d.*	**23.** £3. 1*s.* 6*d.*	**40.** £533. 6*s.* 8*d.*
7. £2249. 14*s.* 7*d.*	**24.** £97. 16*s.* 11*d.*	**41.** £500.
8. £1033. 16*s.* 4*d.*	**25.** £259. 14*s.* 11*d.*	**42.** £2811. 18*s.*
9. £2575. 18*s.* 9*d.*	**26.** £174. 2*s.* 10*d.*	**43.** 1*s.* 8*d.*
10. £307. 10*s.*	**27.** £192. 17*s.* 8*d.*	**44.** £625.
11. £390. 4*s.*	**28.** £510. 18*s.* 8*d.*	**45.** £2. 3*s.* 10*d.*, £2. 13*s.* 4*d.*
12. £139. 1*s.* 10*d.*	**29.** £430. 11*s.* 3*d.*	**46.** £1414.
13. £1077. 10*s.* 8*d.*	**30.** £1340. 2*s.* 6*d.*	**47.** 4.
14. £36. 15*s.* 9*d.*	**31.** £706. 5*s.* 6*d.*	**48.** £16. 8*s.*
15. £1083. 5*s.* 3*d.*	**32.** £796. 12*s.*	**49.** £32. 0*s.* 10*d.*
16. £2762. 16*s.* 4*d.*	**33.** £456. 19*s.* 6*d.*	**50.** £8. 10*s.* 3*d.*
17. £14. 9*s.* 11*d.*	**34.** £592. 5*s.* 10*d.*	**51.** £2. 1*s.* 7*d.*

XXVI. a. (p. 289.)

1. £1150.	**4.** £3900. 10*s.*	**7.** £1350.	**10.** £836. 14*s.* 3*d.*
2. £6575.	**5.** £420.	**8.** £3520.	**11.** £475. 16*s.* 8*d.*
3. £510.	**6.** £6000	**9.** 243. 2*s.* 6*d.*	**12.** £35. 5*s.*

13. £37. 16*s*.
14. £20. 1*s*. 4*d*.
15. £12. 16*s*.
16. £16. 10*s*. 9*d*.
17. £1. 5*s*.
18. £4. 1*s*. 8*d*.
19. £6.
20. £30. 16*s*.
21. £1405. 6*s*. 8*d*.
22. £2379. 3*s*. 4*d*.
23. £4840. 16*s*. 8*d*.
24. £35. 15*s*. 5*d*.
25. £3. 0*s*. 6*d*.
26. £11. 15*s*. 7*d*.
27. 13*s*. 5*d*.
28. 5*s*. 11*d*.

XXVI. b. (p. 291.)

1. 4 %.
2. 3 %.
3. In 6 months.
4. £675. 15*s*.
5. 3*s*. 6*d*.
6. £513.
7. In 8 months.
8. £1057. 10*s*.
9. £7320.
10. 4 %.
11. 3 %.
12. 5 %.
13. £328. 18*s*. 7*d*.
14. £350.

XXVI. c. (p. 294.)

1. £8. 8*s*.
2. £11. 6*s*. 10*d*.
3. £2. 8*s*. 10*d*.
4. £8. 0*s*. 1*d*.
5. £12. 12*s*.
6. £1. 15*s*.
7. £76. 3*s*. 1*d*.
8. £102. 0*s*. 1*d*.
9. £261. 6*s*.
10. £28. 13*s*. 9*d*.

XXVII. (p. 298.)

3. 1·17.
4. ·94.
5. 25 % gain.
6. 20 % loss.
7. 50 % gain.
8. 20 % loss.
9. 5 % gain.
10. 20 % gain.
11. $12\frac{1}{2}$ % gain.
12. $8\frac{1}{3}$ % gain.
13. 30 % loss.
14. $12\frac{1}{2}$ % loss.
15. £25. 6*s*.
16. £36. 15*s*.
17. £129.
18. 10 % gain.
19. 25 % gain.
20. 20 % loss.
21. 12 % gain.
22. 10 % loss.
23. 20 % gain.
24. 20 % loss.
25. £130.
26. £83. 12*s*.
27. £309.
28. £30. 18*s*.
29. £191. 8*s*.
30. £49. 15*s*.
31. £72.
32. £3. 16*s*.
33. 111 : 100.
34. 28 : 25.
35. 6 : 5.
36. 5 : 4.
37. 23 : 20.
38. 217 : 200.
39. 19 : 20.
40. 93 : 100.
41. 4 : 5.
42. 3 : 4.
43. 7 : 8.
44. 23*s*.
45. £2. 14*s*.
46. £5. 6*s*.
47. 7*s*.
48. 3*s*. 2*d*.
49. £5. 15*s*.
50. £2. 12*s*. 6*d*.
51. 9*s*. 3*d*.
52. £11. 10*s*.
53. 25 %.
54. 5 %.
55. 25 %.
56. $16\frac{2}{3}$ %.
57. 20 %.
58. $7\frac{1}{2}$ %.
59. 9 %.
60. $33\frac{1}{3}$ %.
61. £46. 10*s*.
62. $13\frac{1}{3}$ % gain.
63. $21\frac{3}{7}$ %.
64. $6\frac{2}{3}$ % loss.
65. 25 %.
66. $22\frac{2}{9}$ %.
67. £80.
68. 21*s*.
69. £53. 2*s*. 6*d*.
70. £1.
71. 16*s*. 8*d*.
72. £66.
73. 10 %.
74. 7 for 4*d*.
75. $6\frac{2}{3}$ %.
76. 50 %.
77. $45\frac{5}{8}$ %.
78. 3*s*. 10*d*.
79. 10*s*. $11\frac{1}{4}$*d*.
80. $11\frac{3}{8}$ %.
81. Add 8*d*.
82. $10\frac{5}{7}$ %.
83. 21*s*. a ton.
84. 2*s*. 6*d*.; $12\frac{1}{2}$ %.
85. £5.
86. $9\frac{1}{11}$ %.
87. Increased from 8 to 10 %.
88. £12.
89. 3*s*.
90. 4 %.
91. 23·45 % approx.
92. $12\frac{1}{2}$ %.
93. $16\frac{2}{3}$ %.
94. £315.
95. 21·3 % approx.
96. 5*s*. $2\frac{1}{2}$*d*.
97. 1*s*. 6*d*. per lb.

98. £40.
99. £5.
100. 1s. 5d.
101. £36.
102. 3 lb. of coffee to 1 lb. of chicory.
103. £84.
104. The company gained 6 %.
105. 16 %.
106. £56250.
107. 80 : 11.
108. $28\frac{4}{7}$ %.
109. 2·4 %.
110. $15\frac{15}{19}$ %.
111. 25s.
112. 13 gallons of the cheaper to 5 gallons of the dearer.
113. £17. 10s.
114. 7s. $1\frac{1}{2}$d.; $5\frac{5}{19}$ %.

XXVIII. a. (p. 308.)

6. £330.
7. £360.
8. £44.
9. £720.
10. £552.
11. £260.
12. £570.
13. £15.
14. £36.
15. 30.
16. 20.
17. 20.
18. £26.
19. £21.
20. £96.
21. £42.
22. £75.
23. £5.
24. £840.
25. £880.
26. £70.
27. £380.
28. £210.
29. £720.
30. £30.
31. £21.
32. £12.
33. £15.
34. £24. 10s.
35. £3.
36. £8. 10s.
37. £4. 10s.
38. £9.
39. £15.
40. £85.
41. £18.
42. £132.
43. £22.
44. 17s. 6d.
45. £11. 5s.
46. £30.
47. £32.
48. £12.
49. £8.
50. £90.
51. £56.
52. £22. 10s.
53. £27.
54. £8.
55. £7.
56. £1. 10s.
57. 5 %.
58. 5 %.
59. $3\frac{1}{2}$ %.
60. 4 %.
61. $3\frac{1}{3}$ %.
62. $3\frac{3}{4}$ %.
63. 4 %.
64. £400.
65. 5.
66. 14.
67. The 2nd.
68. Equal.
69. The former.
70. The 2nd.
71. 75.
72. 80.
73. 200.
74. 300.
75. 125.
76. 40.

XXVIII. b. (p. 310.)

1. £637, £21.
2. £1225, £50.
3. £7990. 10s., £294.
4. £832, £35. 15s.
5. £187, £8. 10s.
6. £2856. 10s., £128. 1s.
7. £1686. 8s., £61. 4s.
8. £283. 8s., £13. 1s. 7d.
9. £37. 10s.
10. £168.
11. £88. 6s. 8d.
12. £60. 10s.
13. £25.
14. £20. 12s. 6d.
15. £520.
16. $3\frac{1}{3}$.
17. $3\frac{9}{17}=3·529$, *i.e.* £3. 10s. 7d.
18. $2\frac{6}{7}=2·857$, *i.e.* £2. 17s. 2d.
19. $3\frac{91}{103}=3·883$, *i.e.* £3. 17s. 8d.
20. $3\frac{4}{7}=3·571$, *i.e.* £3. 11s. 5d.
21. The latter.
22. The latter.
23. Equally paying.
24. $3\frac{1}{8}$, $3\frac{1}{3}$, $3\frac{11}{48}$.

25. The 2^{nd}, £15000.
26. 1^{st} income £120, 2^{nd} income £117.
27. A gain of £360.
28. A gain of £444.
29. A loss of £20. 5*s*.
30. A gain of £78. 15*s*.
31. A gain of £272.
32. A gain of £25. 7*s*. 6*d*.
33. A loss of £20. 8*s*.
34. A gain of £42. 1*s*. 6*d*.
35. A gain of £3.
36. 97.
37. $90\frac{1}{2}$.
38. $97\frac{3}{4}$, £105.
39. 85.
40. A gain of £12. 10*s*.
41. A loss of £5. 2*s*. 6*d*.
42. A loss of £3.
43. A gain of £7.
44. A gain of £61. 4*s*. 4*d*.
45. £715.

XXVIII. c. (p. 314.)

1. £955. 10*s*., £91.
2. £144. 7*s*. 6*d*., £6. 3*s*. 2*d*.
3. £58. 10*s*.
4. £66. 10*s*.
5. £7453. 2*s*. 6*d*.
6. (1) £23125 stock; (2) a decrease of £67. 10*s*.
7. A gain of £9.
8. £37. 10*s*.
9. The latter by £10 a year.
10. £530. 2*s*.
11. £108 a year.
12. £108.
13. No change; the income is £156.
14. 168.
15. 10 per cent.
16. $133\frac{1}{3}$.
17. £8850.
18. $11\frac{1}{9}$ per cent.
19. £14400; $83\frac{1}{3}$.
20. A gain of £50 a year.
21. £285. 8*s*.
22. £3. 13*s*. 9*d*.
23. None.
24. 3·3109 per cent.
25. £4207. 10*s*., £5049.
26. £1780, £1940.

REVISION PAPERS.

XXIX. a. (p. 316.)

1. 7 fr. 75 c.
2. 27.
3. £40. 4*s*. 4*d*.
4. £11. 13*s*. 4*d*.
5. 60,000.
6. 12 %.
7. 80.

XXIX. b. (p. 316.)

1. 75 Km. 22 m.
2. 15*s*.; 9*s*.
3. Army 50 %, Navy 16 %, C.S. 13 %.
4. £273. 3*s*. 8*d*.
5. £1350.
6. £502.
7. £116. 13*s*. 6*d*.

XXIX. c. (p. 317.)

1. £6.
2. 144 fr.
3. £888. 10*s*., £44. 10*s*., £555.
4. £5. 3*s*. 6*d*.
5. $4\frac{1}{2}$ %.
6. 24 %.
7. £20; £66. 13*s*. 4*d*.

XXIX. d. (p. 317.)

1. 2805 yds.
2. 12 %.
3. 100 days.
4. 4 %; £213. 6*s*. 8*d*.
5. £63, £73. 10*s*., £94. 10*s*., £294.
6. He gains $3\frac{1}{2}$ %.
7. £5376. 5*s*.

XXIX. e. (p. 318.)

1. 2·21 lb.
2. £5. 3*s*. 4*d*.
3. £1275.
4. 1*s*. 8*d*.
5. £44. 18*s*. 8*d*.
6. 14*s*.
7. £200. 16*s*. 3*d*.

XXIX. f. (p. 318.)

1. ·62137 mile, ·00363 mile.
2. 225 : 256.
3. £1440, £1440, £1440, £2400, £2400.
4. £937. 10*s*.
5. £45. 13*s*. 1*d*.
6. £1,000,000.
7. $104\frac{3}{8}$.

XXIX. g. (p. 319.)

1. 28·16 sq. Km.
2. $2\frac{1}{2}$ years.
3. 14725.
4. £8250.
5. £375.
6. 4*s*. 2*d*.
7. 113 : 64.

XXIX. h. (p. 319.)

1. £112. 12*s*. 6*d*.
2. £1. 7*s*. 4*d*.
3. £1075. 15*s*. 9*d*.
4. £2500, £7500, £5000.
5. $\frac{3}{4}$*d*. each.
6. £539.
7. £4154. 6*s*. 8*d*.

XXIX. k. (p. 320.)

1. 200 : 243.
2. 12 min. past 5.
3. £7. 1*s*. 9*d*.
4. £194. 5*s*., £224. 14*s*.
5. $3\frac{1}{2}$ %.
6. £41. 14*s*. decrease.
7. £23. 14*s*.

XXIX. l. (p. 320.)

1. 1087 fr. 66 c.
2. £18. 7*s*.
3. $4\frac{1}{2}$ %.
4. £5446. 12*s*. 6*d*.
5. 7 %.
6. 24 %.
7. £4000 stock, $3\frac{7}{16}$ %.

XXIX. m. (p. 321.)

1. 3·01.
2. B £44. 9*s*., C £55. 11*s*. 3*d*., D £122. 4*s*. 9*d*.
3. $18\frac{2}{9}$ %.
4. £1634. 3*s*. 4*d*., £2258. 6*s*. 8*d*.
5. $3\frac{1}{2}$ yrs.
6. £40.
7. £3 increase.

XXIX. n. (p. 321.)

1. £1268. 15*s*.
2. £1255. 6*s*. 8*d*.
3. 48 lb.
4. £6012. 18*s*. 4*d*.
5. 2*s*. 8*d*.
6. £5. 2*s*. 11*d*.
7. £1578. 10*s*.

XXIX. o. (p. 322.)

2. £2,306,941. 7*s*. 10*d*.
3. £7066. 13*s*. 4*d*.
4. £485. 8*s*. 4*d*.
5. 42
6. £81. 12*s*.
7. £21000.

XXIX. p. (p. 323.)

1. 37,800.
2. $14\frac{2}{17}$ days.
3. £429. 15*s*., £334. 5*s*.
4. £75; 4 %.
5. £384.
6. £9. 6*s*. 8*d*.
7. £6480.

XXIX. q. (p. 323.)

1. 63·012.
2. £4. 4*s*., £3. 12*s*., £3. 10*s*.
3. $3\frac{1}{3}$ %.
4. £197. 17*s*. 4*d*.
5. 15 : 14.
6. 3 : 8.
7. £26. 13*s*. 4*d*.

XXIX. r. (p. 324.)

1. 280·5 Km., 367·5 Km.
2. £17. 10*s*., £70, £157. 10*s*.
3. 8*s*. 5*d*., $2\frac{7}{9}$ %.
4. The shares £3550, £3124, £2982 would have become £3536, £3128, £2992.
5. $4\frac{1}{2}$ %.
6. £8638.
7. £9000 stock.

XXIX. s. (p. 324.)

1. £45. 13*s*. 3*d*.
2. 495 : 128.
3. £5187.
4. £1,657,803.
5. 3 hrs.
6. £1593. 15*s*., £1406. 5*s*.
7. £12000.

XXIX. t. (p. 325.)

1. 999·647 Kg.
2. £172. 2*s*. 8*d*.
3. £2245.
4. £41. 13*s*. 4*d*.
5. 3 %.
6. £846.
7. £29500.

XXIX. u. (p. 325.)

1. 929.
2. 4 wks.
3. 19*s*. 3*d*.
4. 125.
5. 1st class, a gain of $3\frac{1}{2}$ %; 2nd class, neither gain nor loss; 3rd class, a loss of 2 %.
6. £55. 19*s*. $9\frac{1}{2}$*d*.
7. $145\frac{1}{4}$.

XXIX. v. (p. 326.)

1. 32 ft., 24 ft.
2. $16\frac{56}{199}$ days.
3. 551368.
4. Town 70 %, Country 30 %.
5. 6 fr. 60 c.
6. £350.
7. $79\frac{1}{2}$.

XXIX. w. (p. 326.)

1. 1022·2 fr.
2. 11 men and 11 boys.
3. £661. 11*s*. 3*d*.
4. £91. 12*s*. $4\frac{1}{2}$*d*., £246. 5*s*. $1\frac{1}{2}$*d*.
5. £280, 4 %.
6. £11500 stock.
7. £100000.

XXX. a. (p. 328.)

1. $32\frac{8}{11}$ min. past 6.
2. $16\frac{4}{11}$ min. past 6.
3. $10\frac{10}{11}$ min. to 7.
4. $5\frac{5}{11}$ min. past one.
5. $16\frac{4}{11}$ min. past one.
6. $21\frac{9}{11}$ min. to two.
7. 24 min. past one, or $3\frac{3}{11}$ min. to two.
8. 29.
9. 28.
10. $16\frac{4}{11}$ and $27\frac{3}{11}$ min. past 10.
11. 4.32 p.m.
12. 2 a.m. Tuesday.
13. 7.30 p.m.
14. 3 strokes remain. 5 secs.

XXX. b. (p. 330.)

1. 88 fr. 34 c.
2. £14. 15*s*. 10*d*.
3. 22 fr. 19 c.
4. £28. 10*s*. 6*d*.
5. 251 fr. 31 c.
6. £65. 18*s*. 6*d*.
7. 975·61 yen.
8. 1559 marks 25 pf.
9. 204 m. 26 pf.
10. $427\frac{1}{2}$ roubles.
11. £123. 14*s*. 3*d*.
12. £21. 12*s*. 1*d*.
13. £209. 17*s*. 11*d*.
14. 640 fr. 80 c.
15. £11. 5*s*.
16. £833. 6*s*. 8*d*.
17. 31527 Nap. = £25000.
18. £1. 15*s*. 10*d*.
19. He gains £11. 5*s*.
20. £13. 9*s*. 5*d*.
21. £76. 13*s*. 4*d*.
22. 405 roubles 30 kopecks.
23. 427 fr. 85 c.
24. £3. 10*s*.
25. 119·25 marks.
26. 18*s*. 11*d*.
27. £1 = 20·49 marks.
28. 47·83 florins.
29. 3 marks 96 pf.
30. 3049·83 dollars.
31. £742. 10*s*.
32. £69. 10*s*. 3*d*.

XXX. c. (p. 333.)

1. 42·9 yds.
2. 10 yds.
3. A wins by $\frac{1}{7}$ of a mile.
4. The autocar leads by $234\frac{2}{3}$ yds.
5. $11\frac{63}{92}$ secs.
6. A $54\frac{9}{22}$, B 57, C 60 secs.
7. 107 yds.
8. 5.
9. 15 secs.
10. $190\frac{2}{3}$ ft.
11. 45 miles per hour.
12. 264 ft.
13. 12 miles.
14. 15 minutes.
15. 1.16 p.m.
16. 264 feet.
17. 396 ft., 27 miles per hour.
18. 90 miles.
19. $26\frac{7}{8}$ miles.
20. In $3\frac{1}{2}$ hours, 28 and 35 miles from their starting points.
21. $28\frac{3}{4}$, $36\frac{1}{4}$ miles from their starting points.
22. 3.33 p.m., 171 miles.
23. 45 miles.
24. One minute.
25. B wins by $9\frac{1}{3}$ yds.
26. 8 hours.
27. In $43\frac{1}{3}$ min.
28. $5\frac{5}{9}$ yds.
29. $9\frac{207}{352}$, $9\frac{3}{8}$ miles per hour.
30. Every 4 min.
31. In 2 miles, 24 min. after the tricyclist passes the milestone.

32. At $38\frac{2}{11}$ min. past 10, $24\frac{6}{11}$ miles from London.
33. 36 miles.
34. 3200.
35. 168 lb.
36. 2 : 5.
37. $18\frac{486}{743}$ miles per hour.
38. A £9. 16*s*., B £12. 12*s*.
39. $3\frac{317}{421}$ per cent.
40. £113. 12*s*. 9*d*.
41. £303. 15*s*.
42. A pays B £10. 8*s*. 4*d*.
43. 21·74 per cent. in England, 21·92 per cent. in France.
44. £14000, £20000, £6000.
45. 52 per cent.
46. £141.
47. 25 per cent.
48. 44 per cent.
49. 1·0557 correct to 4 decimal places.
50. 200 : 243.
51. £4. 1*s*.
52. $18\frac{2}{9}$ per cent.
53. 113 grains.
54. 42, 32 min.
55. £14400 at $83\frac{1}{3}$.
56. 3*s*. 4*d*.

XXXI. a. (p. 339.)

1. a.	**6.** a.	**11.** a^2.	**16.** $a^{\frac{2}{3}}$.	**21.** 5^3.
2. a.	**7.** a^5.	**12.** $a^{\frac{5}{3}}$.	**17.** $a^{\frac{9}{4}}$.	**22.** $5^{\frac{3}{2}}$.
3. a^2.	**8.** 1.	**13.** a^2.	**18.** 2^4.	**23.** $10^{\frac{2}{3}}$.
4. a.	**9.** 10.	**14.** $a^{\frac{3}{2}}$.	**19.** $2^{\frac{3}{2}}$.	**24.** 10^{-4}.
5. $a^{\frac{1}{4}}$.	**10.** $a^{\frac{3}{2}}$.	**15.** $a^{\frac{1}{4}}$.	**20.** $2^{\frac{5}{2}}$.	**25.** $10^{\frac{3}{4}}$.

XXXI. b. (p. 345.)

1. 2.	**6.** $\bar{1}$.	**10.** 1·3010.	**14.** 5·3010.	**18.** 5·3736.
2. 3.	**7.** $\bar{4}$.	**11.** 3·3010.	**15.** $\bar{3}$·3010.	**19** $\bar{1}$·3736.
3. 0.	**8.** $\bar{3}$.	**12.** $\bar{1}$·3010.	**16.** ·3736.	**20.** $\bar{3}$·3736.
4. 4.	**9.** 1.	**13.** $\bar{4}$·3010.	**17.** 2·3736.	**21.** 5·3736.
5. $\bar{1}$.				

XXXI. c. (p. 347.)

1. 3, 5, 0, −2, −4.	**13.** $\bar{8}$·3855.	**23.** 3, 0, $\bar{3}$.
2. ·6990, 2·6990, 1·3980.	**14.** $\bar{5}$·3980.	**24.** 3·2454, ·2454, $\bar{3}$·2454.
3. 2, 3, 1, $\bar{1}$, $\bar{2}$.	**15.** $\bar{1}$·8474.	**25.** 5·6956.
4. 5.	**16.** $\bar{1}$·5796.	**26.** ·4592.
5. 1, log 6, 2 log 2, log 3.	**17.** $\bar{1}$·4615.	**27.** 3·5474.
6. 3, 1, 2, 4, $\bar{3}$, $\bar{1}$, 0, $\bar{5}$, 1.	**18.** 13.	**28.** 1·5474, 5·5474, $\bar{2}$·5474.
9. 1·2068.	**19.** 3·4134, ·4134, 5·4134, $\bar{3}$·4134.	
10. $\bar{3}$·9356.	**20.** 7.	**29.** 2.
11. $\bar{3}$·4042.	**21.** 8.	**30.** 1·4150.
12. 2·4925.	**22.** 2035, 2035, 176, 176, 1308.	

31. 3·4150.	**34.** ·4245.	**37.** 412·4.	**39.** 24·28.
32. 2·4232.	**35.** 2·4245.	**38.** 2·428.	**40.** 242·8.
33. 3·4245.	**36.** $\bar{3}\cdot4245$.		

XXXI. d. (p. 348.)

1. 4·5663, 2·5663.	**13.** ·0984.	**24.** ·2012.	**35.** £24.
2. 3·1951, $\bar{2}\cdot1951$.	**14.** 41·04.	**25.** ·04082.	**36.** £1014.
3. 2·6320, 6·6320.	**15.** ·076.	**26.** ·001605.	**37.** £159.
4. 3·3250, $\bar{3}\cdot3250$.	**16.** 9·234.	**27.** 1·276.	**38.** £60.
5. 2·5872, $\bar{2}\cdot7568$, 6·8832, $\bar{1}\cdot8836$.		**28.** 62·09 c. dm.	**39.** 14·2 yrs.
6. 2921.	**17.** 8·352.	**29.** 8·277.	**40.** 17·7 yrs.
7. 1·46.	**18.** ·5903.	**30.** 12·54.	**41.** £70.
8. 14·04.	**19.** ·00052.	**31.** 2·99.	**42.** 26.
9. 2·448.	**20.** ·0357.	**32.** 1118 sq. yds.	**43.** £8040.
10. ·1376.	**21.** ·00231.	**33.** 9·58 cm.	**45.** £8318.
11. ·4650.	**22.** 1·350.	**34.** £99.	**46.** 1308.
12. 191·4.	**23.** £3. 9*s*. 6*d*.		

XXXII. a. (p. 351.)

1. $\cdot\dot{0}7692\dot{3}$.	**4.** $\cdot\dot{2}3076\dot{9}$.	**7.** $4\cdot01\dot{9}2\dot{5}$.	**10.** $7\cdot0\dot{0}24691135\dot{8}$.
2. $\cdot\dot{3}8461\dot{5}$.	**5.** $1\cdot\dot{3}170\dot{7}$.	**8.** $5\cdot72\dot{6}1904\dot{7}$.	**11.** $9\cdot0180\dot{5}$.
3. $\cdot\dot{7}6923\dot{0}$.	**6.** $3\cdot0\dot{2}9\dot{6}$.	**9.** $10\cdot\dot{0}4761\dot{9}$.	

XXXII. b. (p. 353.)

6. $\frac{3}{11}$.	**10.** $\frac{4}{165}$.	**14.** $2\frac{1}{54}$.	**18.** $6\frac{242}{375}$.	**22.** $12\cdot\dot{0}4\dot{1}$.
7. $\frac{4}{11}$.	**11.** $3\frac{17}{330}$.	**15.** $\frac{9}{37}$.	**19.** $\frac{30}{41}$.	**23.** $37\cdot2\dot{3}\dot{4}$.
8. $\frac{23}{90}$.	**12.** $\frac{1}{7}$.	**16.** $\frac{1}{74}$.	**20.** $\frac{7}{13}$.	**24.** $2982\cdot5\dot{6}5\dot{7}$.
9. $1\frac{31}{90}$.	**13.** $\frac{3}{7}$.	**17.** $3\frac{3}{185}$.	**21.** $3\cdot\dot{7}$.	

REVISION PAPERS.

XXXIII. a. (p. 354.)

1. 180.	**4.** 760 fr. 72 c.	**6.** $3\frac{3}{4}$%.
2. £125. 17*s*. 6*d*.	**5.** £1270. 10*s*.	**7.** 74 fr. 50 c.
3. 8 mm.		

XXXIII. b. (p. 354.)

1. ·0875 sq. inch.	**4.** 13*s*. 6*d*.	**6.** 100 yards.
2. 8 tons 5 cwt. 20 lb.	**5.** £68. 3*s*. 10*d*.	**7.** £6. 14*s*. 2*d*.
3. 137.		

XXXIII. c. (p. 355.)

1. 55·68.
2. $\frac{4}{15}$.
3. £1003.
4. 4 yrs.
5. 25 %.
6. 11.54 a.m.
7. £136. 10*s.*

XXXIII. d. (p. 355.)

1. (1) +150 lb., (2) −127·8 lb.
2. 3 : 7.
4. C's = $\frac{91}{99}$ of B's.
5. 10·954.
6. 44 %.
7. £2840.

XXXIII. e. (p. 356.)

1. £196. 2*s.* 5*d.*
2. 4 fr. 70 c.
3. £38. 18*s.* 11*d.*
4. £1. 1*s.* 8*d.*
5. 3*s.* $8\frac{1}{2}$*d.*
6. 768 yds.
7. 12 miles an hour.

XXXIII. f. (p. 356.)

1. 282 fr. 51 c.
2. £97. 11*s.* 6*d.*
3. 6 days.
4. 42 half-crowns, 126 florins, 168 shillings.
5. $2\frac{1}{2}$ ft.
6. A 12, B 24, C 18 hrs.
7. 19·25.

XXXIII. g. (p. 356.)

2. ·302.
3. 1581·1 Kg.
4. 1408 yds.
5. £103. 11*s.* 10*d.*
6. 352 ft., 36 miles an hour.
7. £$1431\frac{1}{4}$ stock.

XXXIII. h. (p. 357.)

1. ·00121 yd.
2. 20 Ha. 56 a. 92 ca., 154269 Hl.
3. £11. 18*s.* 8*d.*
4. £77. 5*s.*
5. Horse £24, cow £12.
6. 10 minutes.
7. A £4077, B £2718, C £1812, D £1208.

XXXIII. i. (p. 357.)

1. 65 c.
2. 931475, 968734.
3. 12000.
4. 6*s.* 3*d.*
5. $21\frac{9}{11}$ min. past 4.
6. $3\frac{1}{2}$ %.
7. £1220.

XXXIII. j. (p. 358.)

1. £3. 4*s.* 11*d.*
2. 420, 840, 1260.
3. 40·79.
4. 25 %.
5. £90. 8*s.*
6. £1349. 9*s.* 9*d.*
7. $\frac{1}{11}$ hr.; A is 1·2 yd. behind B.

XXXIII. k. (p. 358.)

1. 3 fr. 20 c.
2. ·9646.
3. ·01470.
4. 5*s.*
5. A £150, B £250.
6. 35*s.*, 42*s.*
7. £450 stock.

XXXIII. l. (p. 359.)

1. £148. 7*s.* 10*d.*
2. 3474 m. (2895 strips required).
3. $4\frac{1}{4}$ %.
4. 15 miles an hour.
5. £7840.
6. 385.
7. £40.

XXXIII. m. (p. 359.)

1. £144. 17*s*.
2. 396·9 Kg.
3. A £324, B £648, C £810, D £972, E £1296.
4. 701·5 yds. nearly.
5. $6\frac{1}{4}$ %; £1149. 6*s*.
6. $1\frac{1}{4}$ lb.
7. £494. 7*s*. 6*d*.

XXXIII. n. (p. 360.)

1. 106·30 m.; 116·25 yds.
2. 27·07.
3. 3·2 acres.
4. 89·46.
5. £3.
6. $3\frac{4}{7}$.
7. 140.

XXXIII. o. (p. 360.)

1. £15. 9*s*. 8*d*.
2. 45 days.
3. £221. 16*s*. 4*d*.
4. 60 sixpences, 30 shillings, 12 half-crowns.
5. £2298. 12*s*.; $6\frac{1}{4}$ %.
6. £16830 in each.
7. The former; 369 : 374.

XXXIII. p. (p. 361.)

1. 285 × 35.
2. £33. 9*s*.
3. 47 fr. 75 c.
5. £3920.
6. 17 : 14.
7. £1530.

XXXIII. q. (p. 361.)

1. 1.
2. 1·20.
3. 10 loads; £2. 5*s*.
4. 12 miles an hour.
5. 390 by four-figure logarithms.
6. £45, £90, £112. 10*s*., £135, £180.
7. £1875.

XXXIII. r. (p. 362.)

1. 98·4, 49·2 yds.; product 4841·28 sq. yds.
2. ·836.
3. 289 fr.
4. $2\frac{1}{8}$ days.
5. £5.
6. $40\frac{1}{2}$, 45 miles an hour.
7. $70\frac{5}{8}$.

XXXIII. s. (p. 362.)

1. 3253.
2. 2*s*. 4*d*.
3. 32.
4. £7. 19*s*.
5. 33 seconds.
6. 12 ft. 4 in.
7. $87\frac{1}{2}$.

XXXIII. t. (p. 363.)

1. 26·2386 lb.
2. 12 miles from B.
3. 4 : 5.
4. 323.
5. $3\frac{1}{8}$.
6. £391, £529, £1311.
7. 77 shares; income £92. 8*s*.

XXXIII. u. (p. 363.)

1. 11 thousand.
2. ·02 % too great.
3. 1458 yds.
4. 23 : 5.
5. £787. 8*s*.
6. $1\frac{3}{4}$ %.
7. 3 miles.

XXXIII. v. (p. 364.)

1. 3 decimal places.
2. £3. 19*s*. 5*d*.
3. £22. 1*s*. 10*d*., £7. 1*s*. 9*d*.
4. 1*s*. $1\frac{1}{2}$*d*.
5. £1375, £775, £350.
6. 180 miles.
7. £7590.

XXXIII. w. (p. 365.)

1. £3. 6*s*. 8*d*.
2. $16\frac{2}{3}$.
3. £146. 13*s*. 4*d*.
4. 1 of water to 10 of milk.
5. 1·7 m.
6. 450.
7. 128.

XXXIII. x. (p. 365.)

1. 3292·8 l.
2. 27.
3. $7\frac{1}{17}$ minutes to 10.
4. £25.
5. £1025.
6. £8. 6*s*. 8*d*.
7. £142. 10*s*.

XXXIII. y. (p. 366.)

1. 914 yds. 1 ft. 7 in.
2. 1·25 lb.
3. 324 : 1159.
4. 4 %.
5. 2·225229 ; 1 place. Maximum error ± ·0017.
6. 24 miles.
7. A 24*s*., B 18*s*.

XXXIII. z. (p. 366.)

1. 27.
2. 160.
3. C wins by ·893 yd.
4. 140 miles.
5. 151 yds.
6. $16\frac{1}{2}$ %.
7. 1*s*.

XXXIII. aa. (p. 367.)

1. $\sqrt{10}$, *i.e.* 3·162....
2. ±1·35 %.
3. £2. 10*s*.
4. 20 : 7 ; 5*s*. $1\frac{1}{2}$*d*. per oz.
5. 78·86, 6·42, 5·86, 3·91, 4·95.
6. 400 miles.
7. $112\frac{1}{2}$.

XXXIII. bb. (p. 368.)

1. $39^2 = 1521$.
2. A mile in $5\frac{7}{11}$ minutes.
3. 17*s*. 10*d*.
5. 1*s*. 4*d*. a day.
7. £12000.

XXXIII. cc. (p. 368.)

1. 9261000.
2. 4, 2, 2.
3. 45*s*., 63*s*.
4. A 16 days, B 8, C 16.
5. 120.
6. $5\frac{1}{2}$ min.
7. The faster £63333. 6*s*. 8*d*., slower £16666. 13*s*. 4*d*.

XXXIII. dd. (p. 369.)

1. 8 yds.
2. £2666. 13*s*. 4*d*.
3. 41·28, 18·72 gall.
4. 38 lb.
5. £1200 in land, £960 in railway stock.
6. £9261.
7. 61·6 tons.

XXXIII. ee. (p. 369.)

1. 4 of copper to 1 of tin.
2. 235.
3. 8.
4. £3488. 16*s.* 3*d.*
5. 350 yds.
6. 5 to 4.
7. 87 days approximately.

XXXIII. ff. (p. 370.)

1. 2699 oz.
2. 84, 76.
3. 91·43.
4. £640.
5. £1780, £1940.
6. Inflow 400 gall. per minute; each outlet drains 75 gall. per minute.
7. $13\frac{13}{17}$ past 5; $21\frac{2}{3}$ miles from A.

XXXIII. gg. (p. 371.)

1. Error = ·00029 correct to 5 decimal places.
2. $46\frac{2}{3}$ miles an hour.
3. A 19, B 17 tons.
4. 44·88 %.
5. £818. 8*s.*
6. £28. 12*s.*
7. £19. 2*s.* 6*d.*

Paper I. (p. 373.)

1. £1641.
2. 1244, 1498. 20 %.
3. 287 lb. 159*s.* to the nearest shilling.
4. £36. 17*s.*; 6*s.*
5. 81 millions.

Paper II. (p. 374.)

1. (*a*) correct; (*b*) 76 cm.; (*c*) 22 ft. per sec.; (*d*) correct to 2 decimal places; (*e*) 148·3 lb.
2. 551·3 lb.
3. The last is the fastest.
4. 105 acres.
5. £1. 13*s.* 4*d.*

Paper III. (p. 375.)

1. £1. 12*s.* 3*d.*
2. 58, 18. +42·7 %; −55·1 %.
3. 29·3 sq. in. nearly.
4. 25·08 francs.
5. 24 c. in., 69 c. in., ·34.
6. 1648 sq. yds., £19. 15*s.* 6*d.*

Paper IV. (p. 376.)

1. (i) 26·9 %; (ii) 47·4 %.
2. 3 millions.
3. Tin 18*s.*, copper 59*s.*; zinc 1*s.*
4. 62·4 miles per hour.
5. 75 gallons.
6. 11 yds., 598 sq. yds., £49. 16*s.* 8*d.*

Paper V. (p. 378.)

1. £13525 to the nearest £.
2. Maximum 9·694 c. dm., minimum 9·523 c. dm., mean 9·61 c. dm., difference 0·17 c. dm., 1·8 %.
3. 56 lb. nearly. £64. 3*s.* 9*d.*
4. 34·65 c. cm., 0·69 gram.
5. 4·2 %, 4·6 %.
6. 90, 81, 72·9, 65·61, 59·05 ft.
7. 10850 ft.; 17 in.

Paper VI. (p. 379.)

1. 87 %, 58 %.
2. £10. 16*s.* 9*d.* to the nearest penny. 125 days to the nearest day.
3. 1953 copies. 0·1 mm. thick.
4. 26·2, 43·4, 49·8 miles per hour.

Paper VII. (p. 380.)

1. 2704. A width of 2 in. all round the edge.
2. 1100.
3. 16 : 5.
4. 10·39 in. approx.
5. Between 1926 and 2312.
6. 303·1 lb. approx.

Paper VIII. (p. 381.)

1. £1458. 18*s.* 6*d.*
2. 202 c. ft. to the nearest c. ft.
3. 308 lb. to the nearest lb.
4. Every 61·64 secs.
5. 2,366,000 tons to the nearest thousand.
6. (*a*) 17·3 % ; (*b*) 27·5 % ; 33 millions.

Paper IX. (p. 382.)

1. 170 %, 2 hrs. $4\frac{1}{4}$ min.
2. 407 acres ; $15\frac{1}{4}$ sq. miles.
3. 2 acres, ·81 hectare.
4. 17,900,000. 8,900,000.
5. £$244\frac{4}{9}$.
6. The slower train lies by at its second station out.
7. Highest 74*s.* 8*d.* Nov. 1, 1905. Lowest 53*s.* 6*d.* last week of Nov. 1904.
   (1) May 1, 1904 ; last week of Oct. 1904 ; Mar. 1, 1905.
   (2) 22nd Jan. (about), 1904 ; last week of Sept. 1904 ; Aug. 1, 1905.
   Rising fastest from Sept. 1 to Nov. 1, 1905.

Paper X. (p. 385.)

1. 44 feet apart. Between 49 and 50 miles per hour.
2. 5 %.
3.

Height in yds.	2	3	4	5	6	7	8	9	10
Cost	£20. 5*s.*	£14. 15*s.*	£12. 7*s.* 6*d.*	£11. 5*s.*	£10. 15*s.*	£10. 12*s.* 2*d.*	£10. 13*s.* 9*d.*	£10. 18*s.* 4*d.*	£11. 5*s.* 0*d.*

Height for minimum cost, 7 yds. 1 ft.

4. 98·94 lb. copper, 1·06 silver.
5. 86 %, 4 %.
6. 17 in. 140,000 gallons.

APPENDIX.

XXXIV. a. (p. 331.)

1. 85 m.
2. 51·2 sq. cm.
3. 251·328 sq. ft.
4. 130680 c. ft.
5. 16335 c. ft.
6. 28·57 cm.
7. 24 m.
8. 1440 c. in.
9. $196\frac{8}{27}$ c. yds.
10. radius = ·35 cm.; length = 22 cm.
11. 879·648 c. dm.
12. 439·824 sq. dm.
13. 72 sq. cm.
14. 36 c. cm.
15. 9072 c. ft.; 3312 sq. ft.
16. 163 sq. cm.
17. 2·598 c. ft.
18. 211·665 sq. ft.
19. ·048 sq. in.
20. 4212 c. in.; 1656 sq. in.
21. $8\frac{1}{5}$ c. ft.
22. 141 c. ft.
23. $13387\frac{1}{2}$ c. ft.

XXXIV. b. (p. 334.)

1. 32·8125 c. ft.
2. 268·8 c. cm.
3. 36 sq. in.
4. 130·986 c. cm.
5. 24 cm.
6. 384 sq. ft.; 384 c. ft.
7. 16 sq. ft.
8. 2100 sq. cm., 3968·6 c. cm.
9. 697·435 c. cm.
10. 96 c. in.; 138·528 sq. in.
11. £18. 5*s*. $7\frac{1}{2}d$.
12. 60 ft.
13. 173·2 c. in.
14. 60 sq. ft.
15. 13·564 ft.; 651·072 c. ft.
16. 16 in.; 1995·32 c. in.
17. £4. 19*s*. to the nearest shilling.
18. 248·9 sq. ft.
19. 75 c. cm. approx.
20. 222·465 sq. cm.
21. 5 ft. 6 in.
22. 130·9 c. cm.; 189·6 sq. cm.
23. 92·6 c. cm.; 111·071 sq. cm.
24. (1) 513·13 c. mm.; (2) $\frac{1}{12}$.
25. 10·65 ft.
26. 27·713 sq. in.

XXXIV. c. (p. 336.)

1. 76·759 c. ft.
2. 235·4 c. m.
3. 3262·5 sq. m.
4. 526·5 c. in.
5. 2·345 c. ft.
6. 59·5486 c. ft.
7. 447·15 c. ft.
8. 15 cm.
9. 760 c. cm.
10. 5 cm.
11. 50·3125 c. ft.
12. 5 ft.; 78·57 sq. ft.
13. 35 cm.: 95·45 cm.

XXXIV. d. (p. 338.)

1. 314·16 sq. cm.; 523·6 c. cm.
2. 63·617 sq. m.; 47·71 c. m.
3. £237. 10*s*. 1*d*.
4. 2·82 ft.
5. 1·41 cm.
6. 78·54 sq. m.; 65·45 c. m.
7. 4 yds. 2 ft.
8. £16. 7*s*. 3*d*.
9. 3 cm.
10. 89·80 litres.
11. 16·875 cm.
12. 1·41 cm.
13. 1·128 cm.
14. $179\frac{2}{5}$ c. ft.
15. 15*s*. 4*d*. to the nearest penny.
16. 5·829 lbs.
17. ·216 lb.
18. 1·71 in.
19. 72·14 lbs.
20. 34·558 sq. dm.
21. 65·45 litres.
22. 125·664 sq. cm.
23. 106·814 sq. cm.
24. 8 : 40 : 52.
25. 238·762 sq. ft.
26. 3·501 cm.
27. 9·004 cm.
28. 8 in.
29. 13·27 sq. ft.
30. 1256·64 sq. ft.